MAGNETIC HELICITY OF OSCILLATING CORONAL LOOPS

V.Krishan[1] and E.R.Priest[2]

[1] Indian Institute of Astrophysics, Bangalore 560034, India

[2] Univ. of St. Andrews, Mathematical Institute, St.Andrews KY 169 SS, Fife, Scotland

The dynamics of the velocity and magnetic field in a coronal loop is studied using ideal MHD equations and the Chandrasekhar-Kendall representation. The complete dynamics is described by a set of infinite, coupled nonlinear ordinary differential equations which are first-order in time for the expansion coefficients of the velocity and magnetic field. Here, the coronal loop plasma is represented by a superposition of the three [n = 0 = m; n = m = 1 and n = m = 1] lowest-order C-K functions. This system, when perturbed linearly from its equilibrium state exhibits sinusoidal oscillations. The frequency S of these oscillations is given by (Krishan et al 1988):

$$S^2 = A\eta_a^2 + B\eta_b^2 + C\eta_c^2 \tag{1}$$

where A, B and C are constants and η's are the equilibrium amplitudes of velocity field which are also equal to magnetic field amplitudes. The three quadratic invariants of this system are

$$\text{the total energy } E = 2[\lambda_a^2\eta_a^2 + \lambda_b^2\eta_b^2 + \lambda_c^2\eta_c^2]$$

$$\text{the magnetic Helicity } H_m = \lambda_a\eta_a^2 + \lambda_b\eta_b^2 + \lambda_c\eta_c^2 \tag{2}$$

and the cross helicity H_c becomes equal to the total energy under the conditions of equilibrium $\vec{V} = \vec{B}$, which is an aligned Alfvenic state. λ's are the characteristic wave-vectors of the three modes. From equations (1) and (2), we found that the frequency S can be expressed in a very simple form as

$$S^2 = \frac{A H_m}{\lambda_a} \tag{3}$$

Here λ_a, where $a \equiv (0,0)$ mode, can be expressed in terms of the ratio of poloidal ψ_p to toroidal ψ_t magnetic flux; λ_b and λ_c are numerical values

E. R. Priest and V. Krishan (eds.), Basic Plasma Processes on the Sun, 256–257.

related to zeros of Bessel functions (Montgomery et al (1978)) obtained from

$$(\psi_t/\psi_p) = -\frac{R}{L}\frac{|\lambda_a|}{\lambda_a}\frac{J_0'(\lambda_a R)}{J_0(\lambda_a R)}$$

$$\frac{2\pi R}{L}\gamma_b\, J_1'(\gamma_b R) + \lambda_b\, J_1(\gamma_b R) = 0$$

and $$\frac{2\pi R}{L}\gamma_c\, J_{-1}'(\gamma_c R) + \lambda_c\, J_{-1}(\gamma_c R) = 0$$

where $\lambda = \left[\gamma^2 + (2\pi R)^2/L^2\right]^{1/2}$ and (L,R) are the length and radius of the cylindrical plasma loop. By measuring the periods of oscillating loop prominences often observed in coronograph movies, one has now a way of estimating magnetic helicity which eludes any direct measurement.

References

Krishan,V. Berger,M. and Priest,E.R., 1988 in "Solar and Stellar Coronal Structure and Dynamics" proceedings of the ninth Sacramento peak summer symposium Aug. 1987.

Montgomery,D., Turner,L. and Vahala G. : 1978 Phys. Fluids 21, 757.