

Low cost method for subarcsecond testing of a right angle prism

A. K. Saxena, MEMBER SPIE

L. Yeswanth

Indian Institute of Astrophysics
Bangalore, 560034 India

Abstract. This paper describes a method for testing a right angle prism, or any other optical component having a right angle, to subarcsecond accuracy. The method makes use of Haidinger fringes and is simple to use. Unlike autocollimation and Fizeau interferometer methods, it does not involve an expensive optical setup and has no aperture restriction. The sensitivity of the method can be suitably modified by changing the test setup parameters.

Subject terms: prism testing; Haidinger fringes.

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CONTENTS

1. Introduction
2. Optical setup
3. Theory
 - 3.1. Calculation of errors
 - 3.1.1. Right angle and pyramidal errors
 - 3.1.2. Base angle and pyramidal errors
4. Illustration
 - 4.1. Measurements using the Fizeau interferometer
 - 4.1.1. Right angle error
 - 4.1.2. Optical path difference (OPD) error
 - 4.2. Measurements using the present method
 - 4.2.1. Right angle error
 - 4.2.2. Pyramidal error
 - 4.2.3. Base angle error
 - 4.2.4. OPD error
5. Discussion
6. Remarks
7. Acknowledgments
8. References

1. INTRODUCTION

High precision making of cube corner and right angle prisms poses a great problem in terms of testing the errors in the angles and the optical path difference (OPD) inside the prism. The errors become intolerable when these prisms have to be used in the path of instruments such as the Michelson interferometer, coherence interferometer, etc. During fabrication of a right angle prism, one requires measurement of the base angle, right angle, and pyramidal errors, and in some cases even the OPD changes due to inhomogeneity in the glass. The autocollimation method¹ and the Fizeau interferometric methods² are commonly known methods for such measurements. Measurement accuracies possible with the autocollimation method are of the order of 1 to 2 arcsec, but there is no control on the OPD errors. The Fizeau interferometer is an alternative choice, where the accuracies can be further improved and OPD errors can be conveniently esti-

mated. This interferometer requires high precision optical components like a beamsplitter and a reference flat of similar size to the prism under test and therefore becomes an expensive choice. Choosing among the above three types of errors becomes diagnostically a cumbersome process.

In this paper we suggest a method for testing prisms that is capable of achieving similar accuracies (subarcsecond) to Fizeau interferometry, but involving a very simple optical setup and less expensive optical components. The method is more direct in terms of interpretation of the base angle, right angle, and pyramidal errors. OPD errors can also be checked.

2. OPTICAL SETUP

The optical setup necessary is similar to that of the Haidinger interferometer.² In this arrangement (Fig. 1) a laser beam focused through a diaphragm plate is allowed to fall on the test prism. The diaphragm plate is an optically flat plate with the central pinhole at an angle of 45° to the surface.³ This plate has been introduced for the purpose of increasing the arm length and therefore the sensitivity of the interferometer, and for the convenience of recording. Sensitivity can be increased either by increasing the distance between the diaphragm plate and the prism or by increasing the distance between the diaphragm plate and the recording plane, or both. In the recording plane the center of the diffraction pattern due to the diaphragm hole is a conveniently available fixed reference point for the measurements.

3. THEORY

A right angle prism can have two errors, namely, right angle error ϵ and error due to the orientation of the hypotenuse face of the prism. The second error has two components, namely, the pyramidal error p and the base angle error δ . In a perfect right angle prism, the hypotenuse face must be parallel to the plane containing the vertex line of the right angle and should be normal to the bisector of the right angle. If the hypotenuse face is not parallel to the plane containing the vertex line but is perpendicular to the bisector plane of the right angle, the error is termed the pyramidal error. If the hypotenuse face is parallel

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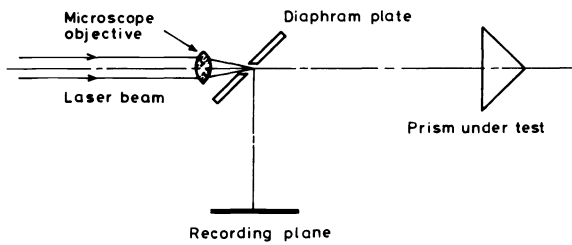


Fig. 1. Optical setup for the suggested method of prism testing.

to the plane containing the vertex line but not normal to the bisector plane of the right angle, it results in the base angle error, one of the base angles becoming less than and the other becoming greater than 45°.

A divergent beam of light incident on a parallel plate of glass will give rise to two reflected beams, one reflected from the front surface and the other reflected from the back surface of the parallel plate. These two beams interfere to give a system a concentric circular fringes known as Haidinger fringes. A right angle prism with half its hypotenuse face masked, as shown in Fig. 2(a), can be considered as a parallel plate of thickness equal to the length of the hypotenuse face. The prism will give rise to two reflected beams, one due to front reflection from the unmasked portion of the hypotenuse face and the other due to internal reflection from the masked portion of the hypotenuse face. These two reflected beams will interfere and form Haidinger fringes. The location of the center of the fringe pattern is a measure of the prism errors.

A glass plate of thickness t having a wedge angle A will produce two images of a point source S , as shown in Fig. 3. The two images will lie along a straight line at an angle β with the original line, and the center of the fringe pattern will lie on this line since it is the line of maximum path difference. The displacement d of the center of the fringe pattern with respect to the point source S is related⁴ to the wedge angle A , expressed by

$$A = \frac{dt}{2Nr(Nr + t)}, \quad (1)$$

where r is the distance between the point source and the front surface of the parallel plate and N is the refractive index of the parallel plate. If $t \ll r$, then Eq. (1) can be reasonably approximated as

$$A = \frac{dt}{8N^2r^2}. \quad (2)$$

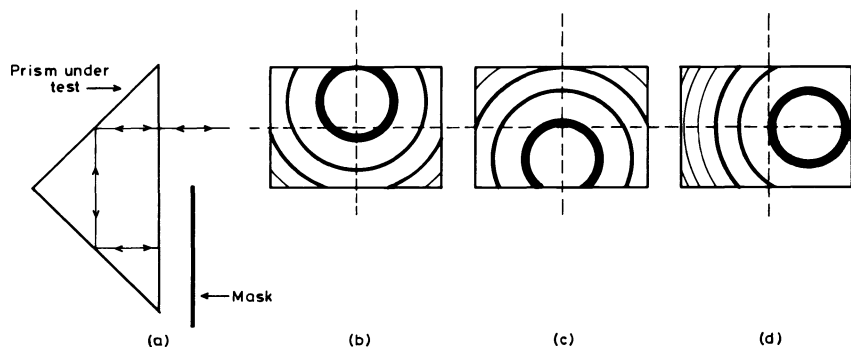


Fig. 2. (a) Setting of the prism for right angle and pyramidal error determination and corresponding fringe center shift for (b) obtuse angle, (c) acute angle, and (d) pyramidal error.

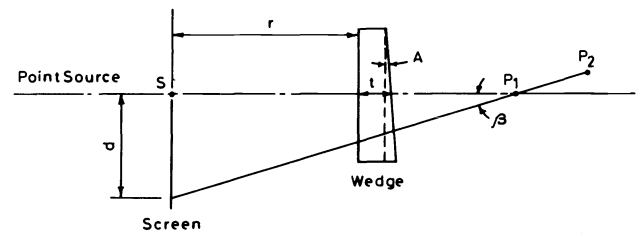


Fig. 3. Geometry of the optical setup for a glass plate of thickness t .

3.1. Calculation of errors

3.1.1. Right angle and pyramidal errors

Right angle error ϵ and pyramidal error p of a right angle prism are tested in the manner shown in Fig. 2. If for a prism of hypotenuse face length t the displacement of the center of the fringe pattern is d , and if we restrict to small angle approximation ($\sin\theta = \theta$, $\cos\theta = 1$), the total angle error will be given by

$$A(\epsilon, p) = \frac{dt}{2N^2r^2M}, \quad (3)$$

where M is a constant that depends on the number of reflections the beam undergoes inside the prism. For a right angle prism tested in the manner shown in Fig. 2, the value of M is 4. Angle A has two components, one in the direction of the right angle vertex line of the prism (Y-axis direction) and the other at a right angle to it (X-axis direction). The component in the X-axis direction dx is a measure of the right angle error ϵ , and the component in the Y-axis direction dy is a measure of the pyramidal error p . These errors are given by

$$\epsilon = \frac{tdx}{8N^2r^2}, \quad (4)$$

$$p = \frac{tdy}{8N^2r^2}. \quad (5)$$

In this method of testing, measurement of the right angle error ϵ is not affected by the base angle error δ of the prism (Fig. 4).

The sign of the right angle error is positive (i.e., obtuse angle) if the center of the fringe system shifts away from the mask, as shown in Fig. 2(b). A shift in the opposite direction corresponds to negative right angle error (i.e., acute angle).

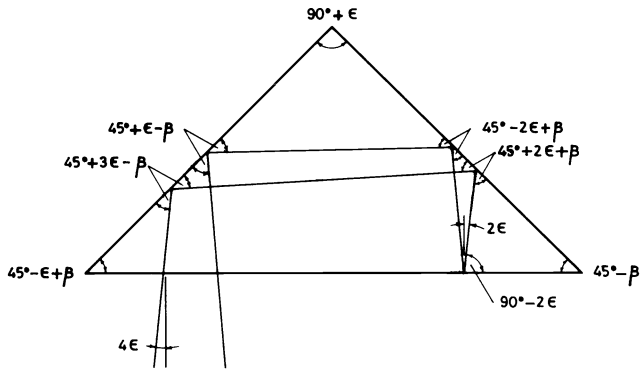


Fig. 4. Effect of internal reflections on right angle and pyramidal errors.

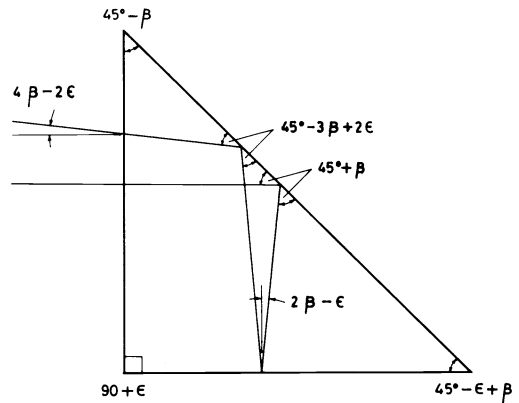


Fig. 6. Effect of internal reflections on pyramidal and base angle errors.

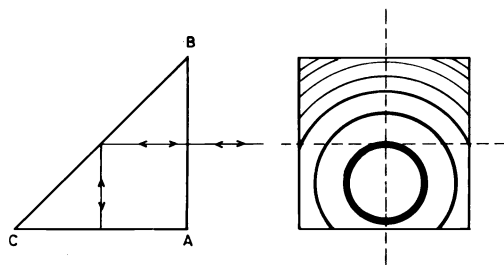


Fig. 5. Setting of the prism for measurement of base angle and pyramidal error.

3.1.2. Base angle and pyramidal errors

Base angle error δ and pyramidal error p of a right angle prism are tested in the manner shown in Fig. 5. Interference takes place between the beams reflected from the faces AB and AC. In this case, the prism acts as a parallel plate of thickness equal to the length t' of the side AB or AC. The total angle error $A(\delta, p)$ as a result of the base angle error and the pyramidal error is given by

$$A(\delta, p) = \frac{d't'}{2N^2r^2M} \tag{6}$$

where the effect of the right angle error has to be taken into account. The value of M in this case also remains 4 (Fig. 6).

If dx' and dy' are the components of d' along the X-axis and Y-axis, respectively, then the base angle error is given by

$$\delta = \frac{t'dx'}{8N^2r^2} - \frac{\epsilon}{2} \tag{7}$$

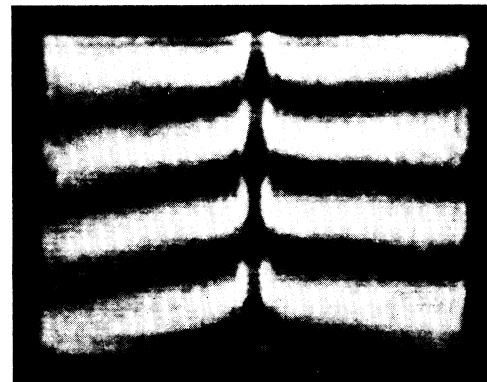
The center of the fringe system shifts toward the base angle, which has a negative error. The pyramidal error p is given by

$$p = \frac{t'dy'}{8N^2r^2} \tag{8}$$

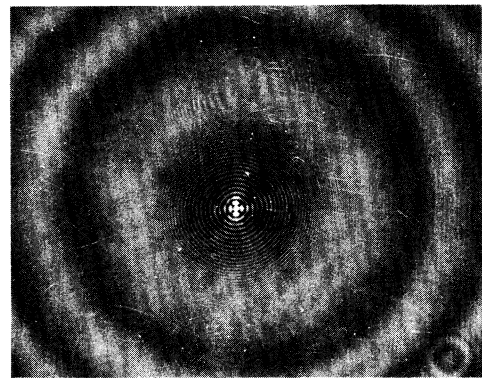
Pyramidal error p can be measured using either Eq. (5) or Eq. (8). This provides a means to counter check the error.

4. ILLUSTRATION

A right angle prism with two equal sides and a hypotenuse face of length 26.66 mm has been tested for the above mentioned errors using the Fizeau interferometer and by the method described in this paper.



(a)



(b)

Fig. 7. Interferogram of the test prism obtained using (a) the Fizeau interferometer and (b) the method described in Sec. 3.1.1.

4.1. Measurements using the Fizeau interferometer

4.1.1. Right angle error

The interferogram of the test prism is shown in Fig. 7(a). The angle between the two wavefronts reflected by a right angle prism is given by⁵

$$\phi = \frac{2\lambda D}{Ld} \sin \frac{\theta}{2} \text{ rad} \tag{9}$$

and the right angle error ϵ is given by

$$\epsilon = \frac{\phi}{2N} \tag{10}$$

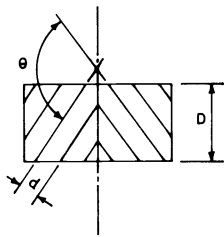


Fig. 8. Determination of right angle error using Fizeau interferogram.

where λ is the wavelength of light used, d is the fringe spacing, L is the physical part size, N is the refractive index of the prism material, and θ , d , and D are as shown in Fig. 8. For the test prism, $\lambda = 0.6328 \times 10^{-3}$ mm, $d = 4$ mm, $D = 11$ mm, $L = 26.66$ mm, and $\theta = 4.5^\circ$ for the first fringe from the bottom. Therefore,

$$\varphi = \frac{2 \times 0.6328 \times 10^{-3} \times 11}{26.66 \times 4} \sin \frac{4.5}{2}$$

$$= 0.0000051 \text{ rad} = 1.057 \text{ arcsec} , \quad (11)$$

$$\varepsilon = \frac{1.05}{2 \times 1.51} = 0.35 \text{ arcsec} . \quad (12)$$

4.1.2. OPD error

The interferogram of the test prism is shown in Fig. 9(a). The angle between the wavefronts is given by²

$$\alpha = \frac{\lambda}{2Na} , \quad (13)$$

where a is the fringe spacing. For the test prism

$$\alpha = \frac{0.6328 \times 10^{-3}}{2 \times 1.51 \times 18.85} = 2.29 \text{ arcsec} . \quad (14)$$

4.2. Measurements using the present method

4.2.1. Right angle error

The interferogram of the test prism is shown in Fig. 9(b). The values of dx and dy when the prism was tested in the manner shown in Fig. 2 were $dx = 1.5$ mm and $dy = 0.75$ mm, and when tested in the manner shown in Fig. 5 were $dx = 1.5$ mm and $dy = 0.5$ mm.

From Eq. (4), the right angle error ε is given by

$$\varepsilon = \frac{1.5 \times 26.66}{8 \times 1.51^2 \times 1138^2} = 0.349 \text{ arcsec} , \quad (15)$$

which is in close agreement with the value obtained in Eq. (12).

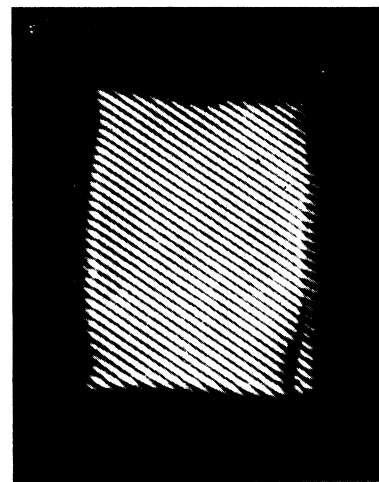
4.2.2. Pyramidal error

From Eq. (5), the pyramidal error of the prism is given by

$$p = \frac{0.75 \times 26.66}{8 \times 1.51^2 \times 1138^2} = 0.174 \text{ arcsec} . \quad (16)$$

4.2.3. Base angle error

From Eq. (7), the base angle error δ of the test prism is given by



(a)



(b)

Fig. 9. (a) OPD record of the test prism using the Fizeau interferometer. (b) Interferogram of the test prism obtained using the method described in Sec. 3.1.2.

$$\delta = \frac{1.5 \times 18.85}{8 \times 1.51^2 \times 661^2} - \frac{\varepsilon}{2} = 0.557 \text{ arcsec} . \quad (17)$$

4.2.4. OPD error

The angle between the wavefronts causing the OPD error [Fig. 9(b)] is given by

$$\alpha = N(2\varepsilon - 4\delta) = 2.3103 \text{ arcsec} . \quad (18)$$

The values from Eqs. (14) and (18) are in close agreement within experimental errors.

5. DISCUSSION

It is apparent from the illustration given in Sec. 4 that the values of the pyramidal, base angle, and right angle errors as calculated by the present method are in close agreement with that of the measurements obtained using the Fizeau interferometer. The present method has advantages over the Fizeau method, particularly in situations where the prism has OPD errors and the fringes obtained by the Fizeau method are not straight but curved. Another advantage comes in terms of recognizing the sign of the right angle error. In the case of the Fizeau method, the sign of the right angle error is determined by the direction of motion of the fringes when the prism is given a slight forward tilt. In the present method, it is more straightforward, defined by the

position of the center of the fringe system with respect to the reference diffraction mark. In addition, the proposed method provides a direct means of estimating the pyramidal and base angle errors, which is quite difficult with the Fizeau interferometer.

6. REMARKS

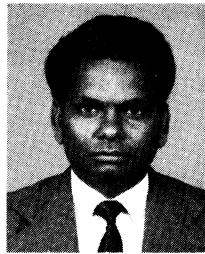
The present method of testing a right angle prism is capable of giving subarcsecond accuracy. It is simple to use and quite straightforward in interpretation. This method can be used for testing components having right angles like cube corners, dove prisms, etc. The major advantage of this method is that it has no aperture restriction since the test is done using a divergent beam of light. The test setup can be easily arranged and adopted without much cost. This method has been used successfully for the evaluation of the errors of a solid coherence interferometer during its final stages of fabrication.

7. ACKNOWLEDGMENTS

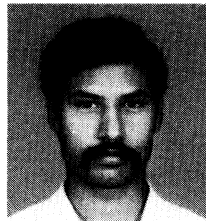
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A. K. Saxena obtained his M.Sc. and Ph.D. degrees in physics from Lucknow University, India, in 1968 and 1977, respectively. His early research activities were mainly directed toward developing new polarization techniques for the study of refractive index fields. From 1974 to 1977 he worked on instrumentation for high resolution spectroscopy observation of the sun and stars and during the February 1980 total solar eclipse performed a high resolution spectroscopic study of the solar corona using multislit spectrography. He has developed a new polarization interferometer technique using a simple device (Babinet compensator) for the precise quantitative evaluation of optical surfaces. Dr. Saxena is presently the head of the Optics Division of the Indian Institute of Astrophysics. He is a member of the International Astronomical Union, SPIE, the Astronomical Society of India, and the Indian Vacuum Society and is a fellow of the Optical Society of India. He was selected for inclusion in the first edition of *Marquis's International Who's Who in Optical Science and Engineering*.



L. Yeswanth was born in 1954 in Kolar Gold Fields, India. He received his master's degree in mathematics with a specialization in theoretical astrophysics and theory of relativity from Bangalore University. Presently, he is working in the Optics Division of the Indian Institute of Astrophysics as a senior research assistant. He is involved in laboratory testing of optical components and test data evaluation.