WAVE PROPAGATION IN SUNSPOTS

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ABSTRACT. In this paper, wave propagation in sunspot umbrae is analysed. The stratification in a typical umbra is approximated by a model atmosphere, extending vertically from a depth of a few thousand kilometres below the photosphere, to the transition region. No boundaries are imposed in the horizontal direction. It is assumed that this atmosphere is embedded in a uniform vertical field. Using a Rayleigh-Ritz variational technique, the normal mode frequencies of the umbra are calculated for different values of the horizontal wave number k. A theoretical diagnostic diagram is generated. An interesting feature of the solutions is the existence of 'avoided crossings', similar to those found in the study of global oscillations. The nature of the wave modes is examined by decomposing the eigenvectors into longitudinal and transverse components. In general, it is found that the character of a mode changes with height. For large k, modes with periods in the 2-3 min range, correspond to low order modes in the present calculation. Above the photosphere, they resemble slow waves.

1. Introduction

Oscillations in the umbrae of sunpots have been widely reported (e.g. Beckers and Schulz,1972; Balthasar and Wiehr,1984; Lites and Thomas,1985; Lites,1986; Abdelatif *et al.*,1986; Thomas et al.,1987 and Gurman,1987). The periods of these oscillations typically lie in the range 2-3 min, although larger periods of some 300-400 s (Bhatnagar et al.,1972; Soltau et al.,1976 and Balthasar et al.,1987) or smaller ones around 100 s (Schröter and Soltau,1976) cannot be ruled out.

A number of theoretical investigations have been carried out to examine wave propagation in sunpot umbrae (e.g., Uchida and Sakurai,1975; Scheuer and Thomas,1981; Thomas and Scheuer, 1982; Leroy and Schwartz,1982 Zhugzhda et al.,1983). Yet a number of uncertainties remain. These are related to the precise values of the oscillation frequencies, their dependence upon the horizontal wave number and the nature of the modes. Scheuer and Thomas (1981) suggested that umbral oscillations are essentially fast waves trapped in a photopheric cavity. On the other hand, Zhugzhda et al. (1983) and Gurman and Leibacher (1984) have argued in favour of a slow mode, trapped in a chromospheric cavity above the temperature minimum. The aim of the present analysis is to: compute the normal mode spectrum for the umbra of a sunspot, generate a diagnostic diagram and classify the modes.

2. Equilibrium Model

In order to compute the wave modes in an umbra, we require an equilibrium model which mimics a real atmosphere in a sunspot. We selected the core umbral model M of Maltby et al. (1985) (kindly provided by T. Abdelatif). Beneath the photosphere, the atmosphere is matched to a convection zone model.

3. Wave Equation

We assume that a uniform vertical magnetic field is embedded in the model atmosphere. The normal

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modes can be determined by linearising the MHD equations, which results in the following wave equation

$$\rho\frac{\partial^2 \xi}{\partial t^2} = -\nabla\delta p + \mathbf{g}\delta\rho + \frac{1}{4\pi}(\nabla\times\delta\mathbf{B})\times\mathbf{B} \tag{1}$$

where $\delta\rho, \delta p, \delta\mathbf{B}$ denote perturbations in density, pressure and magnetic field respectively and ξ is a small displacement of a fluid element. Equation (1) can be recast into an energy integral, which lends itself to a variational treatment (for details see Hasan and Sobouti 1987). Assuming a time dependence of the form $e^{i\omega t}$, the normal frequencies can be calculated using a Rayleigh Ritz variational method.

4. Results

Equation (1) was solved using a Rayleigh Ritz variational method. Rigid boundary conditions were assumed i.e., $\xi_z = \xi_x = 0$ at z=-500 km and $z = 2000$ km. A constant vertical field $B = 2000$ G, corresponding to v_a/c_s =0.84 at $z = 0$ was used, where v_a and c_s denote the Alfvén and sound speeds respectively.

4.1 DIAGNOSTIC DIAGRAM

Figure 1 shows the variation of the wave frequency ν with k (diagnostic) diagram for umbral oscillations, where k is the horizontal wave number. For a fixed value of k, a number of solutions exist satisfying the boundary conditions. These correspond to the normal mode frequencies or harmonics, which form a discrete spectrum. The curves depict the variation of ω with k. The various numbers above each curve denote the order n of the solution. An interesting feature is the absence of accidental degeneracy in the solutions of different orders. It is found that when curves of adjacent orders draw close to one another, an 'avoided crossing 'occurs. The frequency varies from some 2.2 mHz to about 7.5 mHz (corresponding to periods of 450 s and 133 s respectively) for n between 1 and 5.

5. Mode Classification

In order to classify the modes, we decompose the linear displacements, using Helmholtz' theorem, into longitudinal and transverse components and examine the height dependence of the components to see the nature of a mode. It turns out that in the photosphere, the longitudinal and transverse components have comparable magnitude. However, with increase in height, the modes tend to become dominantly longitudinal. In the chromosphere, the character of the mode can be rougly described in terms of a slow wave.

6. Discussion and Conclusions

We now compare the theoretical oscillation spectrum with observations. Assuming an umbral radius of 4200 km we find k= 0.9 Mm^{-1} (i.e., for the lowest order mode which has a vanishing radial displacement at the umbral boundary). Modes with periods around 180 s correspond to n=4 and 5 in the present calculation. The lower order modes with longer periods (around 300 s) probably correspond to the modes observed for example by Balthasar et al. (1987).

The main conclusions of the study are that firstly, the theoretically computed spectrum of umbral oscillations shows the presence of modes with periods in the 2-3 min range. Wave modes with larger periods (around 300s) are also present. Secondly, the diagnostic diagram reveals the presence of 'avoided crossings'. Lastly, the physical nature of the modes is height dependent –

however, in the chromosphere the oscillations resemble slow waves.

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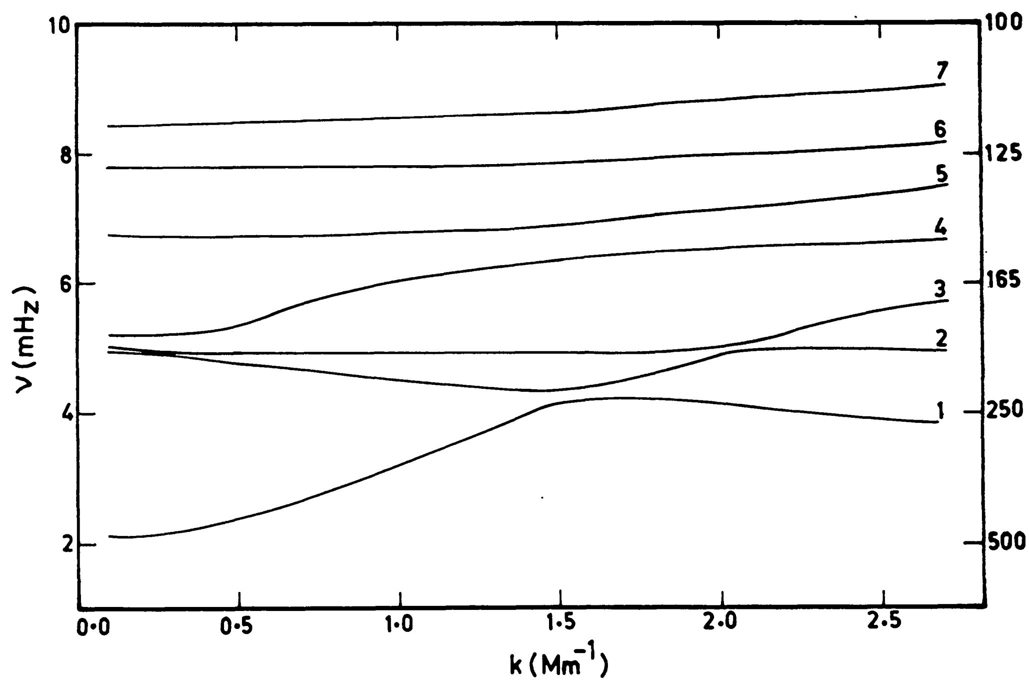

Figure 1. Variation of frequency ν with k in a sunspot umbra, assuming a uniform vertical magnetic field of 2000G.