SELF-ORGANIZATION PROCESSES ON THE SUN : THE HELIOSYNERGETICS

V.Krishan
Indian Institute of Astrophysics, Bangalore 560034, India

and

E.I.Mogilevskij
IZMIRAN 142092, TROISTSK, Moscow Region, Moscow, USSR

Abstract Non-linear interactions between small fluid elements, magnetized or otherwise, in an energetically open nonlinear system facilitate the formation of large coherent stable structures. This is knwon as self-organization. We interpret solar granulation on all scales and the formation and evolution of some structures in solar active regions to be the result of self-organization processes occur ing in a turbulent medium.

Introduction

In an isolated turbulent medium, decay of large eddies into small eddies is a common occurrence. The converse i.e. small eddies coalescing into large ones can be accomplished in an energetically open system with very special properties. Generally, such a system must be dissipative and describable by a nonlinear partial differential equation. In the absence of dissipation, the equation should have two or more quadratic or higher order conversed quantities called invariants. On the inclusion of dissipation, the invariants decay differentially. The nature of the nonlinear interaction between the fluid elements is such that the slow decaying invariant (say B) cascades towards large spatial scale and the fast decaying invariant (say A) cascades towards small spatial scales. The intial random field of velocities and magnetic fields can then be described through a variational principle in which the invariant A is minimized keeping the invariant B constant i.e.

$$\delta A - \lambda \delta B = 0$$

where $\delta A, \delta B$ are small variations and λ is the Lagrange multiplier Hasegawa (1985). Kraichnan (1967) found that in two-dimensional hydrodynamic turbulence, the energy invariant cascades towards large spatial scales and the enstrophy invariant towards small spatial scales where it suffers heavy dissipation. It is this property of selective decay that facilitates the formation of large structures. But can one use the results of 2-D turbulence theory to explain phenomena on a real atmosphere which is a 3-D system? What one needs is the process of inverse cascade to occur in a 3-D system and Levich and Coworkers (Levich and Tzvetkov 1985)

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have shown such a possibility. We will use the ideas developed in this work to construct a model of solar granulation in its entirety from granules to giant cells.

One can also study the dynamics of a turbulent magnetoactive medium consisting of small scale strongly magnetized elements embedded in a large spatial scale weakly magnetized ambient medium. It can be shown that a stimulating disturbance of the ambient magnetic field enhances the nonlinear interaction between the small elements and the complete system is describable by a Schrodinger type equation that gives a full spectrum of discrete structures depending on the available free energy. This approach will be used to search sources of solar flares.

Modeling Solar Granulation

The cellular velocity patterns observed on the solar surface are believed to be manifestations of convective phenomenon occuring in the sub-photospheric layers. (Antia et al (1984) Bogart et al (1980) Brandt et al (1988) Schwarzschild (1975) and Simon and Leighton (1964). Here, we explore if the excitation of random small scale motions can lead to large organized structures which are observed in the form of granules, mesogranules, supergranules and giant cells. In this picture, large helicity fluctuations present in a turbulent medium play an essential role in the inverse cascading process. The helicity density γ, a measure of the knottedness of the vorticity field is defined as $\gamma = \vec{V}\cdot(\vec{\nabla}\times\vec{V})$. It is found that the quantity I defined as $I = \int \langle \gamma(x)\, \gamma(x+r) \rangle d^3r$ is also an invariant of an ideal 3-D hydrodynamic system in addition to the total energy. Assuming a quasinormal distribution of helicities, the invariant I can be expressed as:

$$I = C \int [E(k)]^2\, dk \tag{1}$$

where C is a constant and $E = \int E(k)\, dk$ is the total energy density. Using Kolmogorovic arguments one finds inertial range for energy invariant to be:

$$E(k) \propto k^{-5/3} \quad \text{and} \quad E \propto L^{2/3} \tag{2}$$

and for 'I' invariant to be:

$$E(k) \propto k^{-1} \quad \text{and} \quad E \propto \log \frac{L(t)}{\ell} \tag{3}$$

where L(t) is the largest length scale excited at time t Levich and Tzvetkov (1985). In analogy to the 2-D case (Hasegawa 1985) one expects that in 3-D the invarant 'I' would cascade towards large spatial scales and the energy towards small spatial scales. The cascading of 'I' towards large spatial scales essentially enhances the correlation length of helicity fluctuations, without much increase in energy associated with it, eq.(3). We propose that this regime of turbulence favours the formation of solar granulation at the smallest scales. One can ask if there is an upper limit to the size of granules, the lower limit of course is determined by dissipation. We recall that all atmospheres are restricted in the vertical direction due to gravity. The largest dimension of fully 3-D structures is given by the ratio $I/E = L = L_Z$ where L_Z is the characteristic vertical scale. We identify this scale with the size of the region with superadiabatic temperature gradient, since this is the region that provides energy in the vertical velocity field which then drives the horizontal flow. This limit (Nelson and Musman 1978) of 1000 km puts the 3-D granules right at the top of the convection zone. When the correlation length of helcity fluctuations reaches the limit L_Z, it can only grow in the horizontal plane. Another consequence of the growth of correlation length is that the velocity and vorticity get aligned which reduces the nonlinear term $(\vec{V}\cdot\vec{\nabla})\vec{V} = \vec{\nabla}\times(\vec{V}\times\vec{\omega})$ of the Navier-Stokes equation and thus retards the flow of energy to small spatial scales. With the growth of correlation length only in the horizontal plane, the system becomes more and more anisotropic. Under these circumstances, the vertical component of velocity V_Z becomes independent of (x,y,z) and the horizontal components V_x and V_y independent of Z, leading to $W_{x,y} = (\vec{\nabla}\times\vec{V})_{x,y} = 0$. The invariant I becomes

$$I = \int \langle (V_z \omega_z)^2 \rangle \, dx\, dy\, dz$$
$$\simeq L_z \langle V_z^2 \rangle k^2 V_k^2 k^{-2} \propto V_k^2 = k E(k) \propto L^{2/3} \quad (4)$$

and from $I = \int I(k) dk$, one finds $I(k) \propto k^{-5/3}$ (5)

Here L is the largest length scale in the horizontal plane. Thus I(k) spectrum coincides with the energy spectrum of 2-D turbulence $E(k) \propto k^{-5/3}$ corresponding to the inverse cascade. One expects that an increasing fraction of energy is transferred to large spatial scales as the anisotropy in the system increases. We propose this part of the turbulence spectrum to be conducive to the formation of supergranules which appear predominantly on large horizontal scales with energy spectrum given by eq.(4). The intermediate region where a 3-D system is developing anisotropy, the energy spectrum being given by eq.(3) can be identified with mesogranulation or the gap between granulation and supergranulation. One can again ask if there is a limit to the size of supergranules. The growth of large structures in a highly anisotropic turbulence can again be interrupted as a result of symmetry breaking caused by the Coriolis force. The length scale L_c where the nonlinear term of Navier-Stokes equation becomes comparable to the coriolis force, can be determined from

$$(\vec{V}\cdot\vec{\nabla})\vec{V} = 2(\vec{V}\times\vec{\Omega}) - \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

or

$$L_c \simeq V/\Omega \quad (6)$$

where $\vec{\Omega}$ is the angular velocity. Given sufficient energy, structures of size L_c must form. At these large spatial scales, the system simulates 2-D behaviour and enstrophy conservation begins to play its role. One may consider scales $L \gtrsim L_c$ as a source of vorticity injection into the system. The enstrophy then cascades towards small scales with a power law spectrum given by

$$E(k) \propto K^{-3} \quad \text{and} \quad E \propto L^2 \tag{7}$$

Thus there is a break in the energy spectrum as energy must cascade to larger spatial scales as $L^{2/3}$ and to small scales as L^2. Therefore the energy must accumulate at $L \sim L_c$ and eventually pass on to the highest possible scales of the general circulation of the atmosphere. It is tempting to identify this region of turbulence with the excitation of giant cells. This perhaps is the complete story of the energy spectrum, Figure (1) in the turbulent medium of the solar atmosphere.

Energetics

According to the picture (Levich and Tzvetkov 1985) presented here, the energy in the larger structures has been inverse cascaded from the smaller structures, if so, then the energy density per unit gram E(L) in the large scale L should not exceed that in the small scale (1). From the energy spectrum $E \propto L^{2/3}$, it follows that

$$E(L) \leq \left(\frac{E_o(\ell)}{\tau}\right)^{2/3} L^{2/3} \simeq E_o(\ell) \tag{8}$$

where τ is thetime for which energy injection must occur. If we take τ to be the lifetime of the larger structure then for $\tau \sim 20$ hrs for supergranules and $E_o(\ell) \sim (0.5)^2$ km^2/Sec^2 for energy density/gm in the granules, one gets $L \sim L_{SG} \sim 36000$ Km which is the typical size of a supergranule. The size of a giant cell can be determined from eq.(6) for $\Omega = (2\pi/27)$ day^{-1} and assuming $V \sim 0.3$ Km/Sec for supergranular velocity (since they provide the stirring force for turbulence that organizes itself into giant cells) one gets $L_c = L_{GC} \sim 1.17 \times 10^5$ Km. Again the energy content of giant cells should not exceed that of supergranules. Using equation (7) for the energy spectrum in this region, one gets:

$$E_{GC} \leq \left(E(L_{SG})/L_{SG}^2\,\tau\right)^{2/3} L_{GC}^2 \lesssim E(L_{SG}) \tag{9}$$

where $(E(L_{SG})/L_{SG}^2\tau)$ is the enstrophy injection rate. For $L_{GC} \sim 10^5$ Km, τ = 30 days = lifetime of giant cell, $L_{SG} \sim 30{,}000$ Km, and $E_{SG} \sim (0.3)^2$ Km^2/Sec^2. One finds that eq.(9) can be barely satisfied. Furthermore, in the presence of coriolis force, the pressure balance condition becomes:

$$\frac{1}{\rho}|\vec{\nabla} P| \simeq V^2/L_c + F_c \simeq 2V^2/L_c \tag{10}$$

in contrast to the case with no coriolis force where $\frac{1}{\rho}|\vec{\nabla}P| \simeq V^2/L_c$. Thus one concludes that larger energy density is required to maintain structures at scale L_c. This may be the reason for their rare observability. The appearance of structures at Lc must be accompanied by a corresponding increase in the convective flux and therefore probably of total solar flux. Total solar luminosity changes of 1% have been observed. If we attribute all of this 1% to increase in the convective flux, equation (10) can be satisfied and structures of size Lc can be excited. The differential rotation of the sun favours the formation of larger structures at the polar regions in comparison to the equatorial regions. This is further substantiated by the fact that the dominantly open magnetic fields in polar regions do not inhibit flow of convective flux. Thus one may look for probable correlation between polar phenomena and solar luminosity enhancements with the appearance of giant cells. A very steep spectrum eq.(7) practically forbids further organization of turbulence into structures larger than Lc. The simultaneous organization of the velocity and magnetic field is discussed elsewhere in these proceedings.

Energy injection in solar active regions

The observations show that magnetic field variations in the flaring active regions either are of the same order as without flares, or have a reversible character. Therefore the currents associated with the evolutionary changes in the active regions though play an active role (like accelerating the particles) in the flare process are not the primary sources of flare energy. It is proposed here that the current changes that are induced by the external stirring provided by the active region subphotospheric zone in the form of non-linear impulses, are responsible for the flaring of the plasma.

The solar convection zone can be regarded as a kinetic active medium where turbulent dynamo generates most probably, small scale (cross section 10^7 cm; length 10^8 cm) magnetoplasma elements with a lifetime of 10^3 sec a magnetic field strength of 2-3 KG (Parker 1979). If the energy supply is sufficient, the concentration of these magnetic elements reaches a threshold (and then exceeds) value at which pairwise non-linear interactions become effective. This is because each magnetic element absorbs energy from the surrounding turbulent medium of the convection zone and emits sound waves in the nearby zone, which primarily results in a close coupling between magnetic elements. Such a process of non-linear wave interaction repeated more than once in element pairs and other combinations leads to the formation of large scale structures such as pores, sunspots, arches, etc. This is an equivalent description of the inverse cascade of energy where it tends to accumulate at the largest scales. A dispersive disturbance propagating in such a non-linear medium can develop into a soliton which retains its characteristics over long distances of propagation. We propose that this is the form in which flare energy is transported from subphotospheric layers to the flaring region, Mogilevskij (1984).

The common occurrence of helical structures in a flare active region is evidence of the presence of helicity in the magnetic and velocity fields at the photospheric and subphotospheric layers. As discussed in Sec. III, the formation of large structures is favoured in a medium possessing large correlated helicity fluctuations. The helicity density h of magnetic field B is defined as $h = \vec{A}\cdot\vec{B} = \vec{A}\cdot(\vec{\nabla}\times\vec{A})$ where $\vec{A}$ is the vector potential such that $\vec{B} = \vec{\nabla}\times\vec{A}$. The inverse cascade of energy to large scales follows the increase in the correlation length of the helicity fluctuations, which happens in a Gromeko-Beltramy flow given by $\vec{\nabla}\times\vec{V} = \alpha\vec{V}$. Analogous to this situation, the magnetic field is also expected to attain this form since the increase of magnetic helicity leads to the alignment of the magnetic field $\vec{B}$ and the vector potential $\vec{A}$ such that $\vec{\nabla}\times\vec{A} = \alpha\vec{A}$ and $\vec{\nabla}\times\vec{B} = \alpha\vec{B}$ where α is a constant. If we take the model of convection zone suggested by Spruit (1978), then for subphotospheric layers $\beta = (E_{kin}/E_{mag}) \approx 1$. In this case, the kinematics of magnetoplasma in the active region will be combined with Gromeko-Beltramy motion i.e. $\vec{\nabla}\times\vec{V} = \alpha\vec{V}$, $\vec{\nabla}\cdot\vec{V} = 0$, $\vec{V} < V_s$ where V_s is the sound speed. Under these circumstances the magnetic and velocity fields are transported such that

$$\vec{\nabla}\times(\vec{B}\cdot\vec{\nabla})\vec{V} = \alpha(\vec{B}\cdot\vec{\nabla})\vec{V}$$

and

$$\vec{\nabla}\times(\vec{V}\cdot\vec{\nabla})\vec{B} = \alpha(\vec{V}\cdot\vec{\nabla})\vec{B} \tag{11}$$

The term $(\vec{B}\cdot\vec{\nabla})\vec{V}$ describes velocity transport at the magnetic field gradient and the term $(\vec{V}\cdot\vec{\nabla})\vec{B}$ describes magnetic field transport at the velocity gradient. Then operator rotor over these convection terms will define the currents and Eqs.(11) will represent the force free character of these currents. This is the energy state to which an active region will evolve under the absence of external disturbances. We shall consider the problem of a large wavelength disturbance ($\lambda \gtrsim 10^7$ cm) propagating vertically across the magnetic field near the neutral line (at B = 0) at the photospheric level. The origin of this disturbance could be traced to the restricted zones of enhanced energy release (e.g. due to He^3 abundance) at the core convection zone boundary. This energy could be transported towards the photosphere efficiently in the form of long non-linear waves in a medium of decreasing pressure and density. These upward propagating waves in a dispersive medium like the convection zone may develop into solitons. The non-linear Eqs.(11) in a dispersive medium can be shown to attain the form

$$\frac{\partial U}{\partial t} + (U+1)\frac{\partial U}{\partial z} + \frac{(U+1)}{\alpha^2}\frac{\partial^3 U}{\partial z^3} = 0 \tag{12}$$

where U represents the velocity or magnetic field disturbance. If propagation takes place in a non-linear medium with a dispersion $(\omega/U+1) = k + \epsilon k^3$ where $\epsilon = 1/\alpha^2$ is the dispersion length, the intial disturbance $U = U_o e^{i(k\cdot z - \omega t)}$ is transformed in accordance with Eq.(12) into an MHD soliton of the type

$$U = U_o \,\mathrm{Sech}^2 \xi \;, \quad \xi = \left(\frac{U_o \alpha^2}{12}\right)^{1/2}\left(z - \frac{U_o}{3}\right) \tag{13}$$

For such a disturbance, the energy conservation law may be written as

$$W\lambda = \text{Constant} \;, \quad \lambda \simeq z \leq \delta$$

where $W \simeq U_o^{3/2}$ is the energy in a unit wavelength of the disturbance λ. The total energy that reaches the photosphere in one MHD-soliton propagating at speed $V \gtrsim V_A$, the Alfven speed can be estimated as

$$E_m \simeq \epsilon_m V_A \ell^2 \Delta t \tag{14}$$

where $\epsilon_m = b^2/8\pi$ is the average magnetic energy density in the soliton, t is the lifetime of the soliton at the photospheric level. If we assume $b \sim 4 \times 10^2$ Gauss, $V_A \sim 6 \times 10^6$ cm/sec, $\ell = 7 \times 10^8$ cm and $t = 6 \times 10^2$ sec, then $E_m \sim 10^{31}$ ergs. Thus either a series of successive solitons or a distribution of solitons in the vicinity of the neutral line may account for the total energy of a large flare. In fact it can be shown that a single large soliton can be formed from the many interacting solitons through self-organization processes, Hasegawa (1985). Dissipationless KDV equation has an infinite number of conserved quantities; its solution permits n solitons and dispersive wave for arbitrary initial conditions. In the presence of dissipation, the differential dissipation of the conserved quantities leads to an inverse cascade in the wave number spectrum and the soliton gas is expected to condense into one big soliton, Hasegawa et al. (1981). Since the conserved quantities of KDV equations are of order higher than quandratic, the organized state obtained from variational principle is non-linear localized state.

Conclusion

Formation of large coherent structures in an otherwise turbulent medium occurs if it is injected with enough energy. A Navier-Stokes system has been investigated to explore the possibility of organizing energy associated with solar granules into supergranules and giant cells. The suggested correlation between the solar luminosity increases and the appearance of the giant cells should be looked for. In addition, with the availability of extremely high resolution observations of the photospheric velocity fields (Brandt et al. 1988) it should be possible to estimate the quantity 'I' at various heights on the photosphere. The general overall spectrum except at the breaks is the well-known Kolmogorov law $K^{-5/3}$.

In the second example trying to understand the nature of the initial source of flare energy,it is found that a magnetoactive medium possessing helicity will submit itself to the formation of solitons which themselves could be organized state of a non-linear medium describable by KDV equation. A gas composed of many solitons can get further organized into one big soliton which would carry energy enough for a big flare. Observationally, this phenomenon could be identified with magnetic transients, multiple flares and multiple electron beams accelerated during the collision of this soliton with the pre-existing current carrying plasma. In fact, one could equally well give a 'soliton interpretation' for the supergranular and the giant cells. We hope to accomplish this in the near future.

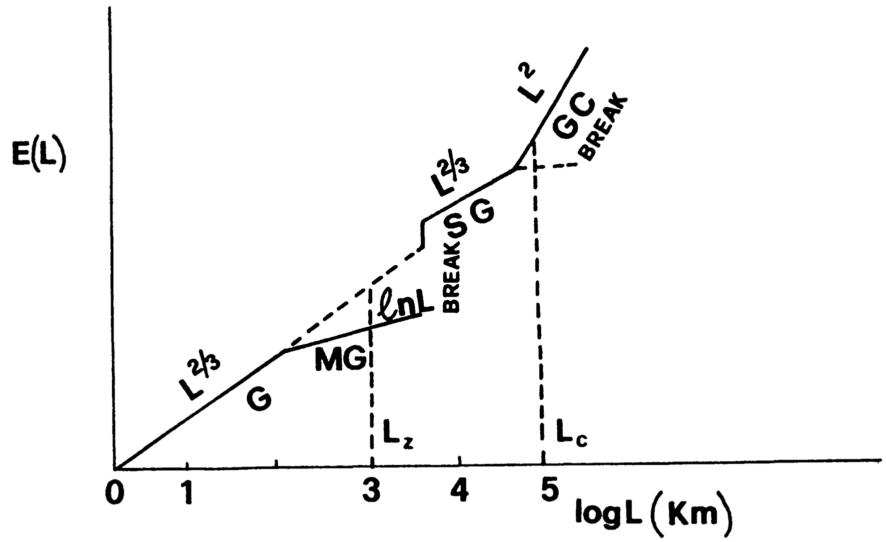

Fig. 1. Turbulent energy spectrum. L_Z = scale of the first break due to anisotropy, L_C = scale of the second break due to the Coriolis force, G = granule, MG = mesogranule, SG = supergranule and GC = giant cell.

References

Antia,H.M., Chitre,S.M. and Narasimha,D. 1984, Ap. J. 282, 574-583.
Bogart,R.S., Gierasch,P.J. and MacAuslan,J.M. 1980, Ap. J. 236, 285-293.
Brandt,P.N. et al. 1988, Nature 335, 238-240.
Hasegawa,A. 1985, Advances in Physics 34, 1-42.
Hasegawa,A., Kodama,Y. and Watanabe,K. 1981, Phys. Rev. Lett. 47,1525.
Kraichnan,R.H. 1967, Phys. Fluids 10, 1417-1423.
Levich,E. and Tzvetkov,E. 1985, Physics Reports 128, No.1, 1-37.
Mogilevskij, E.I. 1984 Preprint No.43a (517).
Nelson,G.D. and Musman,S. 1978, Ap.J. 222, L69.
Parker,E.N. 1979, Cosmical Magnetic Fields, Clarendon Press, Oxford.
Schwarzchild,M. 1975, Ap. J. 195, 137-144.
Simon,G.W. and Leighton,R..B. 1965, Ap. J. 140, 1120-1147.
Spruit,H. 1978, in Magnetic Flux Tubes and Transport of Heat in the Convection Zone of the Sun, Utrecht, Holland.

DISCUSSION

CHITRE: I am just curious to know how the energy content in granulation compares with that in super-granulation in your inverse-cascade picture.

KRISHAN: Since the energy in the larger structures has been inverse-cascaded from smaller structures, the energy density per unit gram E(L) in the large scale (L) should not exceed that in the small scale (ℓ). This consideration determines the sizes of super-granules and giant cells if the energy injection at small scales takes place at least as long as the lifetime of the larger structure.

WEISS: Recent high-resolution observations at La Palma and the Pic du Midi suggest that granules emerge at the centres of meso-granules and move outwards. Some large granules explode, generating rapid outward motion and it is not clear how much of the meso-granular flow is produced by these exploding granules. Is this mechanistic description compatible with your statistical model?

KRISHAN: In my statistical model, meso- and super-granules originate from a reorganization of the energy contained in granules. In this sense, it may be compatible with the observations that link meso- and super-granules with the agglomeration and explosion of granules. More can be said only after the dynamics of these structures has built up.