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Radiative Transfer with Compton Scattering in Spherically Symmetric Shells

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Abstract

We have solved the equation of radiative transfer with Compton scattering in spherically symmetric shells. The specific intensity was expanded by a Taylor series and the first three terms were retained in solving the transfer equation. We assumed optical depths of 2 and 5 in a spherical shell whose outer radii (B) and inner radii (A) are in the ratio of 2 and 5. Multiple Compton scattering redistributes the initial energy over a range of 1 to 10 Compton wavelengths. A good fraction of the incident radiation is transferred across the shell, the radiation being redistributed in wavelength, the percentage of which depends on the optical thickness of the medium.

Key words: Compton scattering; Radiative transfer; Spherical symmetry.

1. Introduction

Compton scattering produces a change in the energy of a photon when it is scattered by an electron. The free electrons that exist in the outer layers of stars produce the broadening of spectral lines. The study of Compton scattering is necessary since this physical process is operative in many objects, such as X-ray binaries and accretion discs.

There are several aspects of studying Compton scattering. Comptonization in a magnetized plasma was studied by Nagel (1981), Ipser and Price (1983), Ochelkov and Usov (1983) and others. The Compton cross section is of considerable importance and has been studied by Barbosa (1982), Gould (1984), Xia et al. (1985), Daugherty and Harding (1986) etc. Compton scattering in the accretion discs was investigated by Ipser and Price (1983), Sunyaev and Titarchuk (1985). Line transfer with Compton scattering is an important problem. It is interesting to compute the lines and to compare these with those observed. This problem was treated by Missana and Piana (1976), Missana (1982). Radiative-transfer problems have been dealt with by several authors: Chandrasekhar (1960), Code (1967), Viik (1968a, b), Pomraning and Froehlich (1969), Langer (1979), Bloemen (1985), Fukue et al. (1985), Guilbert (1986), Kirk (1986), Mészáros and Bussard (1986), Nishimura et al. (1986), Peraiah (1986), Kirk et al. (1986) and others.

The purpose of this study was to determine how γ-rays or X-rays change their energies and how they become reflected and transmitted due to Compton scattering in the spherical shells of the outer layers of stars.

We used the following assumptions:

(1) The medium is divided into spherical shells, each of equal optical thickness.

(2) Either the motion of an electron is neglected, or the electron kinetic energy $\ll$ photon energy.

(3) Isotropic scattering.

(4) Neglect the photon-energy dependence on the scattering cross section.

The scattered radiation is a function of the wavelength and, therefore, we must consider the specific intensities in different wavelengths. For this purpose, the specific intensity was expanded in a Taylor series, as was done by Chandrasekhar (1960) and Code (1967). However, they only considered the first derivative, $\partial I/\partial\lambda$, of the specific intensity; and we also included the second derivative, $\partial^2 I/\partial\lambda^2$ for a better approximation.

2. Calculations

The equation of transfer with Compton scattering in spherical symmetry is written as (Code1967)

$$\mu\frac{\partial U(r,\ \mu,\ \lambda)}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial U(r,\ \mu,\ \lambda)}{\partial \mu}$$
$$=-KU(r,\ \mu\ ,\ \lambda)+K[(1-\omega)B$$
$$+\frac{\omega}{4\pi}\int_{-1}^{+1}\int_{0}^{2\pi}U(r,\ \mu',\ \lambda-\delta\lambda)d\varphi' d\mu'], \qquad (1)$$

where

$$U(r,\ \mu,\ \lambda)=4\pi r^2 I(r,\ \mu,\ \lambda). \qquad (2)$$

$I(r,\ \mu,\ \lambda)$ is the specific intensity of a ray making an angle of $\cos^{-1}\mu$ with the radius vector, r, with wavelength λ. Also,

$$\delta\lambda=\gamma(1-\cos\Theta), \qquad \gamma=\frac{h}{m_e c}=0.024\ \text{Å}, \qquad (3)$$

and B is the Planck function. Though we have included this term for the sake of completeness, we do not take account of it in the numerical calculations presented here. K is the coefficient of absorption and the scattering angle Θ is given by

$$\cos\Theta=\mu\mu'+(1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos\varphi'. \qquad (4)$$

Here, ω = albedo for single scattering.

We must consider the contribution of $U(Z,\ \mu',\ \lambda-\delta\lambda)$ to the radiation field after scattering. We expand $U(Z,\ \mu',\ \lambda-\delta\lambda)$ by a Taylor series as

$$\begin{aligned} U(Z,\ \mu',\ \lambda-\delta\lambda) &= U(Z,\ \mu',\ \lambda) - \frac{\delta\lambda}{1!}\frac{\partial U(Z,\ \mu',\ \lambda)}{\partial\lambda} + \\ &\quad \frac{(\delta\lambda)^2}{2!}\frac{\partial^2 U(Z,\ \mu',\ \lambda)}{\partial\lambda^2} + \cdots . \end{aligned} \tag{5}$$

After substituting equation (5) into (1) and integrating over φ' we obtain

$$\begin{aligned} &\mu\frac{\partial U(Z,\ \mu,\ \lambda)}{\partial Z} + \frac{1-\mu^2}{r}\frac{\partial U(r,\ \mu,\ \lambda)}{\partial\mu} = -KU(Z,\ \mu,\ \lambda) \\ &\quad +K\left[\!\!\left[(1-\omega)B + \frac{\omega}{2}\int\left\{U(Z,\ \mu',\ \lambda) - \gamma(1-\mu\mu')\frac{\partial U(Z,\ \mu',\ \lambda)}{\partial\lambda}\right.\right.\right. \\ &\quad \left.\left.\left. +\frac{1}{2}\gamma^2\left[(1-\mu\mu')^2 + \frac{1}{2}(1-\mu^2)(1-\mu'^2)\right]\frac{\partial^2 U(Z,\ \mu',\ \lambda)}{\partial^2\lambda}\right\}d\mu'\right]\!\!\right] . \end{aligned} \tag{6}$$

We retain the two terms containing $\partial U/\partial\lambda$ and $\partial^2 U/\partial\lambda^2$ and choose the points for wavelength in steps of the Compton wavelength. If we choose n discrete points of λ, we then have n equations, each similar to that given in (6). Therefore, we have n terms of $\partial U_i/\partial\lambda_i$ and $\partial^2 U_i/\partial\lambda_i^2$ in n equations. The term $\partial U_i/\partial\lambda_i$ is replaced by its difference,

$$\frac{\partial U_i}{\partial\lambda_i} = \frac{U_i - U_{i-1}}{\lambda_i - \lambda_{i-1}}.$$

When all of the n equations are considered, the quantities $\partial U_i/\partial\lambda_i$, $\partial U_{i+1}/\partial\lambda_{i+1}$, $\partial U_{i+2}/\partial\lambda_{i+2}$ etc. form a vector. Consequently, the terms $\partial U/\partial\lambda$ in the n equations when combined form $\boldsymbol{D}_1\boldsymbol{U}$, where

$$\boldsymbol{D}_1 = \begin{bmatrix} -\frac{1}{\lambda_2-\lambda_1} & \frac{1}{\lambda_2-\lambda_1} & & & \\ -\frac{1}{\lambda_2-\lambda_1} & 0 & \frac{1}{\lambda_3-\lambda_1} & & \\ & -\frac{1}{\lambda_4-\lambda_2} & 0 & \frac{1}{\lambda_4-\lambda_2} & \\ & & \ddots & \ddots & \\ & & & & -\frac{1}{\lambda_n-\lambda_{n-1}} \end{bmatrix}, \tag{7}$$

and

$$\boldsymbol{U} = [U_1, U_2, \ldots, U_n]^{\mathrm{T}}. \tag{8}$$

Here, the λ's are the quadrature points (trapezoidal points here) and the U_1, U_2 etc. are the corresponding intensities. Similarly, the $\partial^2 U/\partial\lambda^2$ term can be written as $\boldsymbol{D}_2\boldsymbol{U}$.

Here,

$$D_2 = \begin{bmatrix} \frac{1}{(\Delta\lambda)^2} & -\frac{2}{(\Delta\lambda)^2} & \frac{1}{(\Delta\lambda)^2} & & \\ 0 & \frac{1}{(\Delta\lambda)^2} & -\frac{2}{(\Delta\lambda)^2} & \frac{1}{(\Delta\lambda)^2} & \\ & & & \ddots & \\ & & & \ddots & \frac{1}{(\Delta\lambda)^2} \\ & & & & -\frac{1}{(\Delta\lambda)^2} \end{bmatrix}, \tag{9}$$

and $\boldsymbol{U}$ is the corresponding intensity vector. We choose m angle points on the Gauss-Legendre quadrature.

The transfer equation in (6) is integrated on an angle-wavelength mesh, as described in Peraiah and Grant (1973) (hereafter referred to as PG). The procedure is exactly the same and we need not repeat it here. After integration, we obtain

$$\begin{aligned} \boldsymbol{M}\left[\boldsymbol{U}^+_{n+1} - \boldsymbol{U}^+_n\right] + \tau_{n+1/2}\boldsymbol{U}^+_{n+1/2} = \tau_{n+1/2}\Big[(1-\omega)\boldsymbol{B}^+_{n+1/2} \\ + \left(\frac{1}{2}\omega P^{++}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^+}{\tau_{n+1/2}} - \gamma\boldsymbol{P}^{++}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \right. \\ \left. +\frac{1}{2}\gamma^2\boldsymbol{P}^{++}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}\right)\boldsymbol{U}^+_{n+1/2} \\ + \left(\frac{1}{2}\omega\boldsymbol{P}^{+-}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^-}{\tau_{n+1/2}} - \gamma\boldsymbol{P}^{+-}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \right. \\ \left. +\frac{1}{2}\gamma^2\boldsymbol{P}^{+-}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}\right)\boldsymbol{U}^-_{n+1/2}\Big]. \end{aligned} \tag{10}$$

For the oppositely directed beam

$$\begin{aligned} \boldsymbol{M}\left[\boldsymbol{U}^-_{n+1} - \boldsymbol{U}^-_n\right] + \tau_{n+1/2}\boldsymbol{U}^-_{n+1/2} = \tau_{n+1/2}\Big[(1-\omega)\boldsymbol{B}^-_{n+1/2} \\ + \left(\frac{1}{2}\omega P^{-+}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^-}{\tau_{n+1/2}} - \gamma\boldsymbol{P}^{-+}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \right. \\ \left. +\frac{1}{2}\gamma^2\boldsymbol{P}^{-+}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}\right)\boldsymbol{U}^+_{n+1/2} \\ + \left(\frac{1}{2}\omega\boldsymbol{P}^{--}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^+}{\tau_{n+1/2}} - \gamma\boldsymbol{P}^{--}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \right. \\ \left. +\frac{1}{2}\gamma^2\boldsymbol{P}^{--}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}\right)\boldsymbol{U}^-_{n+1/2}\Big]. \end{aligned} \tag{11}$$

The quantities $\boldsymbol{Q}^{++}$, $\boldsymbol{Q}^{--}$ etc. in PG will become

$$\begin{aligned} \boldsymbol{Q}^{++}_{n+1/2} &= \frac{1}{2}\omega_{n+1/2}\boldsymbol{P}^{++}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^+}{\tau_{n+1/2}} \\ &- \gamma\boldsymbol{P}^{++}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_1 + \frac{1}{2}\gamma^2\boldsymbol{P}^{++}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{12}$$

$$\begin{aligned} \boldsymbol{Q}^{--}_{n+1/2} &= \frac{1}{2}\omega_{n+1/2}\boldsymbol{P}^{--}_{n+1/2}\boldsymbol{C} + \frac{\rho\boldsymbol{\Lambda}^{+}}{\tau_{n+1/2}} \\ &- \gamma\boldsymbol{P}^{--}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_1 + \frac{1}{2}\gamma^2\boldsymbol{P}^{--}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{13}$$

$$\begin{aligned} \boldsymbol{Q}^{-+}_{n+1/2} &= \frac{1}{2}\omega_{n+1/2}\boldsymbol{P}^{-+}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^{-}}{\tau_{n+1/2}} \\ &- \gamma\boldsymbol{P}^{-+}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_1 + \frac{1}{2}\gamma^2\boldsymbol{P}^{-+}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{14}$$

$$\begin{aligned} \boldsymbol{Q}^{+-}_{n+1/2} &= \frac{1}{2}\omega_{n+1/2}\boldsymbol{P}^{+-}_{n+1/2}\boldsymbol{C} - \frac{\rho\boldsymbol{\Lambda}^{-}}{\tau_{n+1/2}} \\ &- \gamma\boldsymbol{P}^{+-}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_1 + \frac{1}{2}\gamma^2\boldsymbol{P}^{+-}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{15}$$

where

$$\boldsymbol{P}^{++} = \begin{bmatrix} \boldsymbol{p}^{++} & & \\ & \boldsymbol{p}^{++} & \\ & & \ddots \end{bmatrix} = \boldsymbol{P}^{+-} = \boldsymbol{P}^{-+} = \boldsymbol{P}^{--} \tag{16}$$

(for the definitions of $\boldsymbol{\Lambda}^{+}$ and $\boldsymbol{\Lambda}^{-}$ see PG). Further,

$$\boldsymbol{p}^{++}(\mu_j,\ \mu_k) = 1, \tag{17}$$

$$\boldsymbol{P}_1^{++} = \begin{bmatrix} \boldsymbol{p}_1^{++} & & \\ & \boldsymbol{p}_1^{++} & \\ & & \ddots \end{bmatrix}, \tag{18}$$

where

$$\boldsymbol{p}_1^{++}(j,\ k) = [1 - (+\mu_j)(+\mu_k)]\,, \tag{19}$$

and

$$\boldsymbol{P}_1^{+-} = \begin{bmatrix} \boldsymbol{p}_1^{+-} & & \\ & \boldsymbol{p}_1^{+-} & \\ & & \ddots \end{bmatrix}, \tag{20}$$

where

$$\boldsymbol{p}_1^{+-}(j,\ k) = [1 - (+\mu_j)(-\mu_k)]\,. \tag{21}$$

$\boldsymbol{p}_1^{--}$ and $\boldsymbol{p}_1^{-+}$ are similarly defined. We can also write

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{m} & & \\ & \boldsymbol{m} & \\ & & \ddots \end{bmatrix}, \quad \boldsymbol{m} = [\mu_{jk}, \delta_{jk}] \tag{22}$$

and

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c} & & \\ & \boldsymbol{c} & \\ & & \ddots \end{bmatrix}, \quad \boldsymbol{c} = [c_{jk}, \delta_{jk}]. \tag{23}$$

The quantities $\boldsymbol{S}^{++}$, $\boldsymbol{S}^{--}$, $\boldsymbol{S}^{+-}$, $\boldsymbol{S}^{-+}$ in Peraiah and Grant (1973) become

$$\begin{aligned} \boldsymbol{S}^{++}_{n+1/2} = \boldsymbol{M} - \frac{1}{2}\tau_{n+1/2}\boldsymbol{E} + \frac{1}{4}\tau_{n+1/2}\omega_{n+1/2}\boldsymbol{P}^{++}_{n+1/2}\boldsymbol{C} - \frac{1}{2}\rho\boldsymbol{\Lambda}^{++} \\ -\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{++}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} + \frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{++}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{24}$$

$$\begin{aligned} \boldsymbol{S}^{--}_{n+1/2} = \boldsymbol{M} - \frac{1}{2}\tau_{n+1/2}\boldsymbol{E} + \frac{1}{4}\tau_{n+1/2}\gamma^2\omega_{n+1/2}\boldsymbol{P}^{--}_{n+1/2}\boldsymbol{C} + \frac{1}{2}\rho\boldsymbol{\Lambda}^{++} \\ -\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{--}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} + \frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{--}_{2,n+1/2}\gamma^2\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \end{aligned} \tag{25}$$

$$\begin{aligned} \boldsymbol{S}^{-+}_{n+1/2} = \frac{1}{4}\tau_{n+1/2}\omega_{n+1/2}\boldsymbol{P}^{-+}_{n+1/2}\boldsymbol{C} + \frac{1}{2}\rho\boldsymbol{\Lambda}^{-}\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{-+}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \\ -\frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{-+}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}, \quad \text{and} \end{aligned} \tag{26}$$

$$\begin{aligned} \boldsymbol{S}^{+-}_{n+1/2} = \frac{1}{4}\tau_{n+1/2}\omega_{n+1/2}\boldsymbol{P}^{+-}_{n+1/2}\boldsymbol{C} - \frac{1}{2}\rho\boldsymbol{\Lambda}^{-}\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{+-}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} \\ -\frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{-+}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}. \end{aligned} \tag{27}$$

Further,

$$\begin{aligned} (\boldsymbol{\Delta}^{+})^{-1}_{n+1/2} = \boldsymbol{M} + \frac{1}{2}\tau_{n+1/2}\boldsymbol{E} - \frac{1}{4}\tau_{n+1/2}\omega_{n+1/2}\boldsymbol{P}^{++}_{n+1/2}\boldsymbol{C} + \frac{1}{2}\rho\boldsymbol{\Lambda}^{+} \\ +\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{++}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} - \frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{++}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2} \end{aligned} \tag{28}$$

and

$$\begin{aligned} (\boldsymbol{\Delta}^{-})^{-1}_{n+1/2} = \boldsymbol{M} + \frac{1}{2}\tau_{n+1/2}\boldsymbol{E} - \frac{1}{4}\tau_{n+1/2}\omega_{n+1/2}\boldsymbol{P}^{--}_{n+1/2}\boldsymbol{C} + \frac{1}{2}\rho\boldsymbol{\Lambda}^{+} \\ +\frac{1}{2}\tau_{n+1/2}\gamma\boldsymbol{P}^{--}_{1,n+1/2}\boldsymbol{C}\boldsymbol{d}_{1,n+1/2} - \frac{1}{4}\tau_{n+1/2}\gamma^2\boldsymbol{P}^{--}_{2,n+1/2}\boldsymbol{C}\boldsymbol{d}_{2,n+1/2}. \end{aligned} \tag{29}$$

Here, $\tau_{n+1/2}$ is the mean optical depth of the n-th layer and $\boldsymbol{E}$ is the unit matrix.

The transmission and reflection operators follow those given in PG.

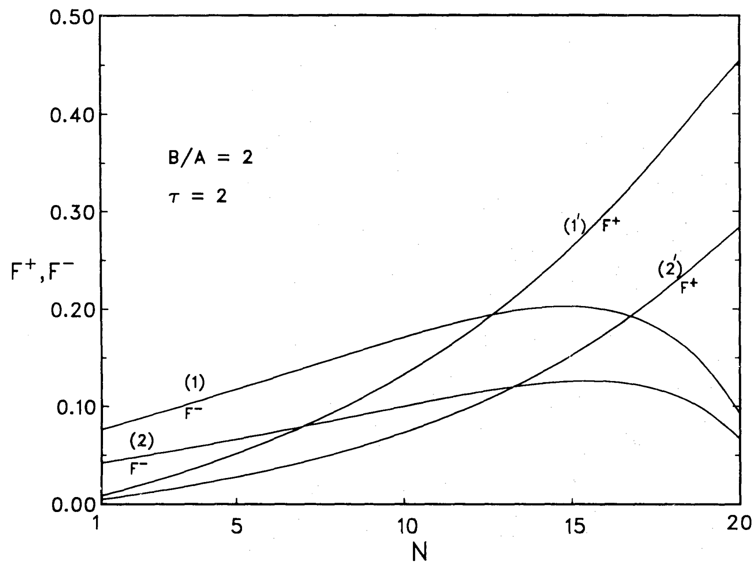

Fig. 1. The fluxes defined as $F^{+} = \int_0^1 I^{+}(\mu)\mu d\mu$ and $F^{-} = \int_0^1 I^{-}(\mu)\mu d\mu$ are plotted against the shell numbers, curves (1) and (1′) represent the fluxes without the $\partial^2 U/\partial\lambda^2$ terms and curves (2) and (2′) represent the curves calculated by including the $\partial^2 U/\partial\lambda^2$ term.

3. Results and Discussions

The radiation field is calculated according to a procedure described in PG. We have considered two different cases of total optical depths, $T = 2$ and 5. We treated the two cases $B/A = 2$ and 5, where B and A are the outer and inner radii of the spherical shell. The medium is divided into 20 spherically symmetric shells, each of equal radial optical thickness. We have considered 4 angle points and 10 frequency points. The wavelength points were chosen in units of Compton wavelength, γ, in the form of trapezoidal points taken as γ, 2γ, ..., 10γ, so that the step length is one Compton wavelength. The roots and weights of the Gauss-Legendre quadrature on $\mu\varepsilon(0,1)$ were chosen as the angle points. We chose 4 angle points (μ_1, μ_2, μ_3, and μ_4) on $\mu\varepsilon(0,1)$ with the corresponding weights, C_1, C_2, C_3, and C_4 (Abramowitz and Stegun 1972) given by

$$\mu_1 = 0.06943, \qquad C_1 = 0.17393;$$
$$\mu_2 = 0.33001, \qquad C_2 = 0.32607;$$
$$\mu_3 = 0.66999, \qquad C_3 = 0.32607;$$
$$\mu_4 = 0.93057, \qquad C_4 = 0.17393.$$

We have given incident radiation at $\tau_{\max} = \tau$; no radiation is incident at $\tau = 0$, as follows:

$$\boldsymbol{U}^{-}(\mu_j,\ X(1),\ \tau_{\max} = \tau) = 1, \tag{30}$$

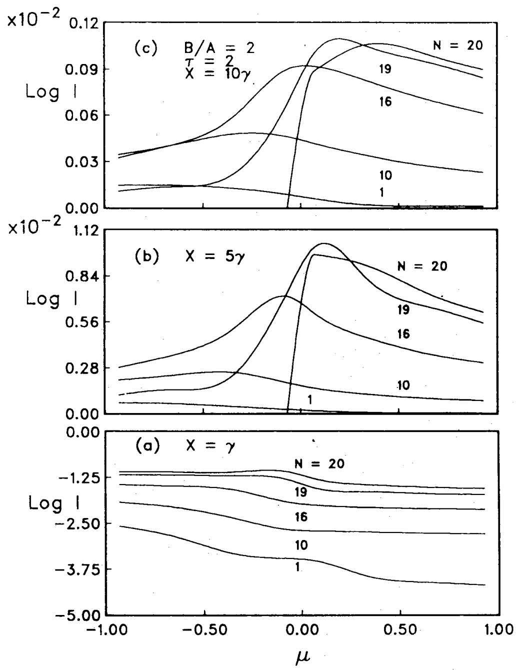

Fig. 2. Angular distribution of specific intensities for $B/A = 2$, $\tau = 2$, (a) $X = \gamma$, (b) $X = 5\gamma$, (c) $X = 10\gamma$.

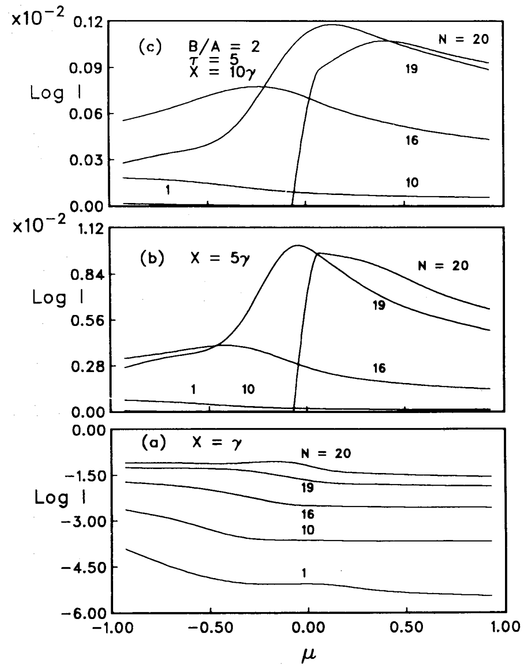

Fig. 3. Angular distribution of intensities for $B/A = 2$, $\tau = 5$, (a) $X = \gamma$, (b) $X = 5\gamma$, (c) $X = 10\gamma$.

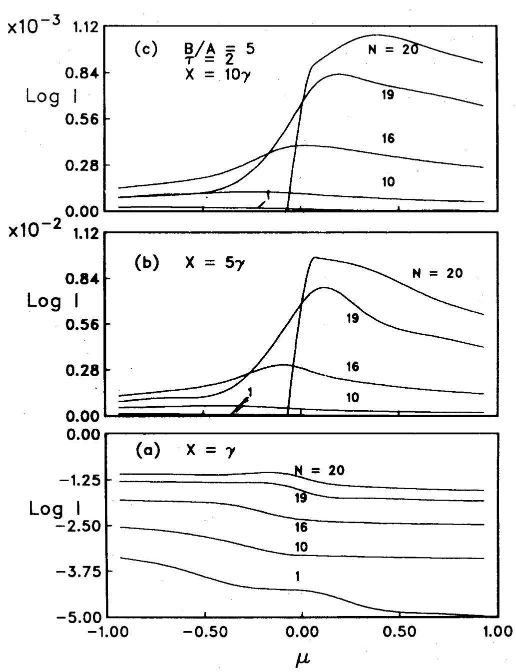

Fig. 4. Angular distribution of intensities for $B/A = 5$, $\tau = 2$, (a) $X = \gamma$, (b) $X = 5\gamma$, (c) $X = 10\gamma$.

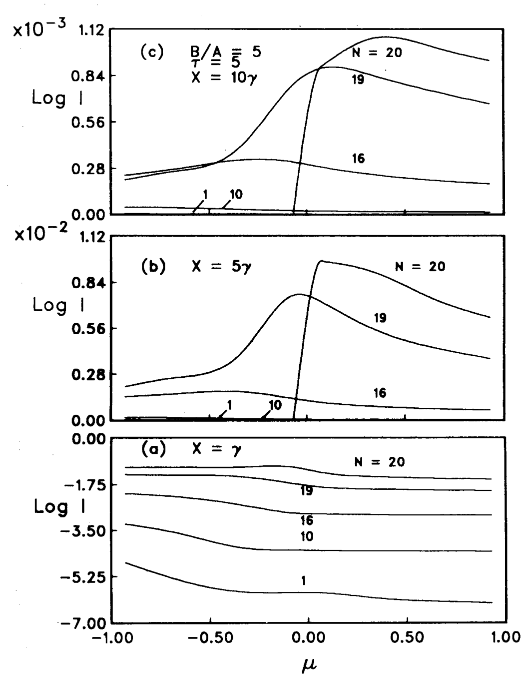

Fig. 5. Angular distribution of intensities for $B/A = 5$, $\tau = 5$, (a) $X = \gamma$, (b) $X = 5\gamma$, (c) $X = 10\gamma$.

$$\boldsymbol{U}^{-}(\mu_j,\ X(2,\ldots,10),\ \tau_{\max}=\tau)=0, \tag{31}$$

and

$$\boldsymbol{U}^{+}(\mu_j,\ X_i,\ \tau=0)=0. \tag{32}$$

We would like to know if any differences arise if we neglect the third term on the R.H.S. of equation (5). For this purpose we plotted the quantities

$$F^{\pm}=\int_0^1 I^{\pm}(\mu)\mu d\mu, \tag{33}$$

where F^{+} and F^{-} are the back-scattered and forward-scattered fluxes and

$$I^{\pm}(\mu)=U^{\pm}(r,\ \mu)/(4\pi r^2) \tag{34}$$

across the medium. In figure 1 curves (1) and (1′) represent curves without the $\partial^2 U/\partial\lambda^2$ term and (2) and (2′) represent curves calculated by including the $\partial^2 U/\partial\lambda^2$ term. At $N=20$, F^{+} (the back-scattered radiation) show maximum differences, while F^{-} show considerable change at $N=1$. The quantities F^{+} show no change at $N=1$ but the outwardly directed F^{-} show substantial amount of change throughout the medium. We also notice that the quantities F^{-} at $N=1$ for the two cases are not equal because of the contribution from diffuse radiation generated when the $\partial^2 U/\partial\lambda^2$ term is included. It is therefore necessary to include the term $\partial^2 U/\partial\lambda^2$ in the calculations.

In figure 2 we have shown the angular distribution of the intensities at the boundaries of the shells $N=20$, 19, 16, 10, and 1 for $X=\gamma$, 5γ, and 10γ and for the values of B/A and τ shown in the figure. There is a gradual reduction in the intensities from the shell $N=20$ ($\tau_{\max}=2$) to the shell $N=1$ ($\tau=0$) for one Compton wavelength. We also notice that more radiation is directed towards the emergent side than towards the incident side [$\tau=\tau_{\max}$, see equations (30) to (32)]. The radiation at $N=1$ is about three orders of magnitude less than that at $N=20$.

The intensities corresponding to $X=5\gamma$ and 10γ behave in a different manner. We see here that though magnitudes of the intensities are reduced they tend to have maxima around $-0.4<\mu<+0.4$, except for those at $N=1$. It is interesting to note how the energy is redistributed in wavelengths up to $X=10\gamma$ when we have a given incident radiation corresponding to only the wavelength at $X=\gamma$.

In figure 3 we described the angular distribution of intensities for $\tau=5$ for wavelengths corresponding to $X=\gamma$, 5γ, and 10γ. The same tendency of variation as can be seen in figure 2 is repeated here. The angular distribution of intensities is shown in figure 4 corresponding to $B/A=5$ and $\tau=2$ and $X=\gamma$, 5γ, and 10γ, respectively. Similarly, the results in figure 5 are for $B/A=5$, $\tau=5$ and $X=\gamma$, 5γ, and 10γ. The variation of the intensities is similar in all these situations, except that the magnitude of the intensities is reduced considerably.

In figure 6 we plotted F^{+} and F^{-} at $N=1$ and 20 for $\tau=2$, versus $X=\gamma$ to 10γ. This shows how the incident radiation at $X=\gamma$ is redistributed along $X=2\gamma$ to 10γ. Here, we showed the results for $B/A=2$ and 5. In the upper panel ($N=20$) the differences between the values of F's for $B/A=2$ and 5 are so small that they are not graphically resolvable. We see that there is a gradual reduction in the fluxes

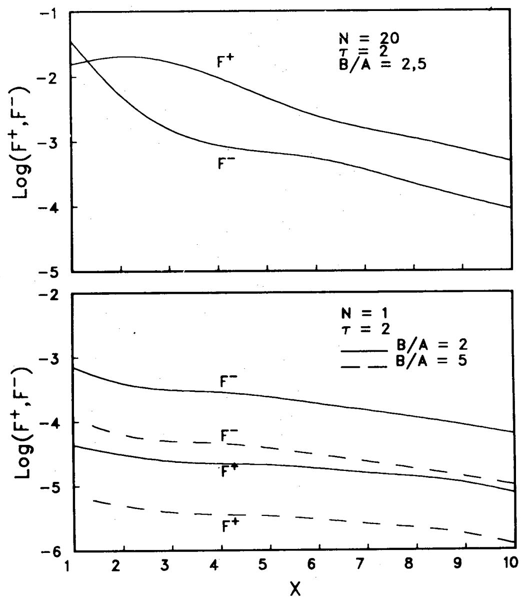

Fig. 6. F^+, F^- are plotted against X for $N = 20$ (upper panel, the differences between the values of F's for $B/A = 2$ and 5 are very small and graphically unresolvable and therefore only single curves are shown) and $N = 1$ (lower panel), $\tau = 2$, $B/A = 2,\ 5$.

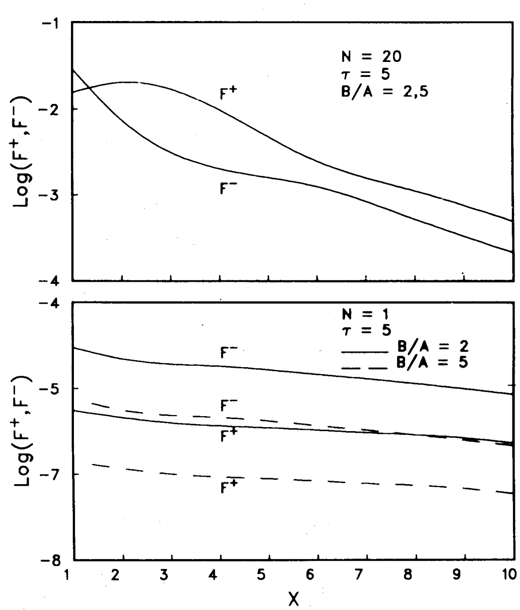

Fig. 7. Same as those given in figure 6 for $N = 1$ and 20.

from $X = \gamma$ to $X = 10\gamma$ in both the cases of F^+ and F^-, the reduction being a factor of approximately 10^{-3}. It is noteworthy that a good amount of radiation at $X = \gamma$ is scattered into far away wavelengths. In figure 7 we plotted F^+ and F^- at $N = 1$ and 20 for $\tau = 5$ versus X. These results are similar to those given in figure 6. The variation of the F's in the lower part of the figures 6 and 7 represent the emergent side of the medium. The differences in the fluxes at $X = \gamma$ and at $X = 10\gamma$ are not as striking as at $N = 20$. This is an important result: that at the emergent side the radiation is almost evenly redistributed all along $X = \gamma$ to 10γ. This is true for $B/A = 2$ and 5 and $\tau = 2$ and 5.

4. Conclusion

We have shown that inclusion of $\partial^2 U/\partial\lambda^2$ term changes the results of Compton scattering substantially. It is also shown that Compton scattering is capable of redistributing energy over a large range of wavelengths and across an optically thick medium.

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