Bull. Astr. Soc. India (1998) **26**, 173-179

Mechanism of the solar cycle : recent results and new ideas

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Abstract. A recheck of our computations and arguments confirm our recent inferences (Gokhale, 1996) about : (a) existence of variations of ~ 1% in the Sun's spin angular momentum on time scales of planetary orbits, (b) coupling of Sun's internal MHD to the solar system dynamics through the inertial torque caused by the spin variations and (c) the possibility that this coupling maintains, through resonance, torsional MHD oscillations responsible for solar activity.

Some tasks for verification and utilization of the new inferences are listed.

Key words : solar rotation, solar magnetic field, solar activity, solar cycle, solar system dynamics

1. Introduction

It is generally well appreciated that the turbulent dynamo models of the solar cycle are mathematically elegant but raise several difficult questions (cf. e.g., Parker 1995). Legendre-Fourier analysis of the rate of emergence of toroidal magnetic flux from the Sun as inferred from sunspot data (1874-1976), or as seen in magnetograms (1960 onwards), has suggested that the magnetic field and activity on the sun's surface might be resulting from superposition of a set of global oscillation modes (Stenflo & Vogel 1986; Stenflo 1988; Gokhale et al. 1992; Gokhale & Javaraiah 1995). The predictive power of this analysis (Gokhale & Javaraiah 1992) strongly suggests that the Legendre-Fourier terms in the rate of the magnetic flux emergence may be representing real MHD oscillations of the Sun. Presence of common dominant periodicities between the magnetic field and the coefficient of differential rotation (Javaraiah & Gokhale 1995), as well as their respective latitudinal parities, suggest that the global MHD oscillations are in fact torsional MHD oscillations. A model of the 'steady' part of the Sun's poloidal magnetic field, which is consistent with the internal rotation as given by helioseismology, shows that global torsional MHD oscillations might be admitted by the contemporary internal field (Hiremath & Gokhale 1995). The strength of the internal field required for oscillations to have periodicities ~ years or decades is well below the upper limit given by magnetograms, showing that the presence of a steady field admitting the suggested oscillations is not ruled out. The matching of the periodicities in the rate of magnetic flux emergence and in the differential rotation with the conjunction periodicities of pairs of major planets (Javaraiah & Gokhale 1995) hinted the possibility of the presence of coupling between the Sun's internal MHD and solar system dynamics. As described earlier (Gokhale 1996), a preliminary idea about the possible physical reason for such a coupling comes from the variations found in the sum of the orbital angular momenta of the Sun and the nine planets about the barycentre of the solar system as computed from the ephemerides of Bretagnon & Simon (1986). Here I review these recent results and ideas along with some new ideas. In the end I also list some important tasks which will have to be carried out for verifying and utilizing the new inferences.

2. Recent results

2.1 *Temporal variations in the Sun's mean rotation frequency* $<\Omega>$

As reported earlier, (Gokhale, 1996) we have computed the yearly mean values of the z-component of the sum of the angular momenta of the Sun and the planets, $\mathbf{L}_{\odot+p's}$, about the solar system barycentre, during 1700-1894, using the ephermerides of Bretagnon & Simon (1986). From the given upper limits on the errors in the planetary positions we have evaluated in detail the maximum error in the computed values of $L_{z,\odot+p's}$ to be ~ $1.14.10^{46}$ $gm\ cm\ s^{-1}$. On time scales > 5 yr the variations in $Lz,_{\odot}+p's$ are ~ 4.10^{46} $gm\ cms^{-1}$, i.e. 3 times the maximum error. Hence the computed variations imply real variations in $\mathbf{L}_{\odot+p's}$. These in turn imply presence of equal and opposite variations in the Sun's spin angular momentum $\mathbf{S}_{\odot}$ (since, on the relevant time scales the variations in the spins of planets and the smaller bodies will be much smaller). Assuming the variations in the Sun's moment of inertia to be negligible, the inferred variations in $\mathbf{S}_{\odot}$ imply temporal variations ~ 3 nHz in the Sun's mean rotation frequency $<\Omega>$.

2.2 *The 'new term' in the equation of internal MHD and the resulting coupling of internal MHD to SSD*

In any reference frame which is either non-rotating (inertial) or rotating with a constant frequency (e.g. 'heliographic'), each mass element dm of the Sun at a position $\mathbf{r}$ with respect to the sun's centre experiences an inertial force $(-dm)\ (d<\Omega>/\ dt) \times \mathbf{r}$. This implies the necessity of introducing the following *new* 'inertial torque' term in the MHD equation for interaction of rotation and magnetic field inside the Sun :

$$d\mathbf{T}_{inert} = (-dm)\ [\mathbf{r} \times (d<\Omega>/dt) \times \mathbf{r})\ ]. \tag{1}$$

The azimuthal component of the momentum equation (its moment about the rotation axis) takes the form (in self-evident notation) :

$$(d\,/dt)\ [(dm)\mathbf{r} \times d\mathbf{r}\ /dt] = d\mathbf{T}_{em} + d\mathbf{T}_{tidal} + d\mathbf{T}_{inert}, \tag{2}$$

The magnitude of the total inertial torque is :

$$| \mathbf{T}_{inert} | = | \mathbf{T}_{spin} | = | \int d\mathbf{T}_{spin} | \sim 10^{39} \, dyne \; cm, \tag{3}$$

where $\mathbf{T}_{spin}$ is the spin torque ($d\mathbf{S}_{\odot}/dt$), which must satisfy the following constraint for conservation of angular momentum about the barycentre :

$$d\mathbf{S}_{\odot}/dt + d\mathbf{L}_{\odot + p's} / dt = 0. \tag{4}$$

Through the conventional term $d\mathbf{T}_{tidal}$, as well as the 'new' term $d\mathbf{T}_{inert}$ in equation (2), and through equation (4), the Sun's internal MHD is *coupled* to the dynamics of the solar system.

2.3 *Forcing of torsional MHD oscillations by the inertial torque*

If we set aside the two 'coupling' terms $d\mathbf{T}_{tidal}$ and $d\mathbf{T}_{inert}$, the remaining equation (2) yields the Sun's free MHD oscillations, including the 'torsional MHD oscillations'.

A model of the 'steady part' of the Sun's internal magnetic field, which is consistent with the internal rotation given by helioseismology, shows that the 'steady' part of the internal field allows global MHD oscillations with periods in the range of years / decades.

We have found that $\mathbf{T}_{spin}$, and hence the inertial coupling term $d\mathbf{T}_{inert}$, has temporal frequencies in the neighbourhoods of the six conjunction frequencies of the four major planets. This term is also latitude dependent, since the operator $\mathbf{r} \times \mathbf{r} \times$ contains a factor sin^2 (*colatitude*), and its strength (~ 10^{39} *dyne cm*) is much larger than that of $d\mathbf{T}_{em}$.

Hence in equation (2) the new 'coupling' term $d\mathbf{T}_{inert}$ serves as a 'forcing term' for the Sun's global MHD oscillations.

Among the free modes of the torsional MHD oscillations, those whose frequencies are close to the frequencies of $d\mathbf{T}_{inert}$, (i.e. close to the conjunction frequencies of major planets,) can get amplified by resonance.

3. New ideas

3.1 *'Emergence' of toroidal magnetic flux from the radiative core into the convective envelope*

On global scales the Sun's radiative core and convective envelope can be considered as regions of $\eta = 0$ and $\eta = \infty$, respectively, (where η is effective conductivity), separated by the common boundary at $r = r_c$.

Consider a section ' S ' of a meridian plane bounded by $r = 0$, $r = r_c$, and the radii at colatitudes θ and $\theta + d\theta$, which is under a ' torsional ' perturbation such that

$$d\omega / d\theta > 0. \tag{5}$$

The last assumption implies :

$$(\partial / \partial t)\, B\phi\, (r, \theta, t) > 0 \tag{5}$$

everywhere in ' S '

In the Eularian description, the law of magnetic induction requires that the toroidal magnetic flux in ' S ' should tend to be conserved. This implies " emergence of toroidal magnetic flux across $r = r_c$, at a rate :

$$\dot{\Phi} = (\partial / \partial t) \int_S B_\phi (r, \theta, t)\, dS.$$

This is true irrespective of whether the torsional perturbation defined by equation 5 is due to a single mode of global oscillation or a superposition of a spectrum of several global modes.

In Lagrangian description, since $\eta = 0$ upto $r = r_c$, the flux emergence across $r = r_c$ must be associated with a minute radial expansion of the plasma element defined by S into the region $r > r_c$. The convective instability and the high resistivity in $r > r_c$ will remove the emerging flux very fast (e.g. on time scales ~ 10 days : see Howard, 1985). Even the *opposite* phase of any oscillation ($d\omega / d\theta < o$), must also be associated with a radial expansion of the plasma element and emergence of the toroidal flux, but of *opposite* sign.

Thus, a torsional oscillation is inevitably coupled to a g-mode oscillation in the core and evanascence ('buoyancy' mode) in the convective envelope.

Mathematical formulation of these ideas need to be worked out.

3.2 *'Dissipation' of torsional MHD oscillations : Production of activity*

From the foregoing discussion it follows that the toroidal magnetic flux emerging above $r = r_c$ will be in the form of wave packets of various amounts formed by interference, near $r = r_c$, of all the modes of torsional MHD oscillations existing in the spectrum. Above $r = r_c$ these flux packets will be subjected to deformations, shear and strengthening by convection (processes of the conventional dynamos), and rapidly transported into the atmosphere where they ultimately dissipate and producing 'activity' on various scales (e.g., Gokhale & Javaraiah, 1995).

Thus, the emergence of toroidal magnetic flux above $r = r_c$ and its ultimate dissipation near and above $r = r_\odot$ in the form of 'solar activity' provides a process of dissipation of energy of the torsional MHD oscillations in the spectrum.

3.3 *Maintenance of the torsional MHD spectrum*

The resonant forcing of the torsional MHD modes described in section 2.3 and their dissipation described in section 3.3 can together *maintain* the MHD spectrum (and hence the solar activity) at a normal order of magnitude. Quantitatively, the power supplied by the forcing inertial torque of ~ 10^{39} *dyne cm* to the torsional oscillations of magnitude ~ 3 *nH z* will be ~ 10^{30} - 10^{31} *erg* s^{-1}, and hence is adequate to balance the dissipation in the form of solar activity (i.e. the variable excess luminosity) at the observed rate.

If for some reasons the power supply decreases (/ increases), the amplitudes of the oscillations will become lower (/ higher), (as, e.g., during the Maunder minimum) thereby decreasing (/ increasing) the dissipation till the normalcy is restored.

4. The observational verification

The observational verification for the coupling between the Sun's spin and orbital motions comes from the fact that on time scales > 5 yr the temporal variations in the Sun' spin and orbital torques are exactly similar except for the opposite signs and a normalization factor.

Observational verification for the coupling between the solar magnetic cycle and the solar system dynamics comes from the fact that the fourier power of the rate of magnetic flux emergence and the Sun's orbital torque are both concentrated near the conjunction frequencies of the four major planets (with a dominant maximum near the conjunction frequency of Jupiter and Saturn).

5. Conclusions and the future tasks

It seems quite likely that the solar activity and the solar magnetic cycle are maintained by the resonant coupling of Sun's torsional MHD oscillations to the angular momentum exchanges between the Sun and the planets.

However the following tasks need to be performed for quantitative confirmation and utilization of this conclusion in the context of solar physics, stellar physics and physics of gravitation and angular momentum :

A. *Solar Physics :*

1. Continued modeling of the 'steady ' parts of the Sun's internal magnetic field and rotation in more and more details : Use the constraint of iso-rotation along with better analytical forms and newer helioseismic data (e.g. as in Hiremath & Gokhale 1995) as becomes available.

2. Computation of the Sun's global 'torsional MHD modes' : Use normal mode analysis along with the models of the 'steady' field and compare the computed modes with the modes determined from the synoptic data from photo-heliograms, spectro-heliograms, dopplergrams and magnetograms.

3. Mathematical modeling of the 'steady' spectrum of the Sun's torsional MHD oscillations : Determine the steady form of the spectrum introducing phenomenological parameters for (a) the power input by the resonant coupling to the SSD (see Sec. 2.3), and (b) the 'dissipation' in the form of the 'toroidal magnetic flux' that leaves the Sun (Sec.3.1). Compare the computed spectrum with the amplitudes and phases determined from the synoptic data. Also study how the spectrum depends upon the parameters representing the power input and the dissipation.

4. Modeling of the latitudinal asymmetry of solar activity : Model the relation between the latitudinal asymmetry of solar activity and the inclinations of the planetary orbits to the Sun's equator.

5. Modeling the evolution of the solar rotation : Solve the coupled equations of internal MHD and SSD with initial conditions as in the "young" Sun (e.g. with 'initial' magnetic field having a strong non-oscillatory part), and see if the solution settles asymptotically to the 'steady' field and 'steady' differential rotation as 'seen' in the present Sun (e.g. from helioseismology). If not, try to find the initial conditions which lead asymptotically to the present conditions.

B. Solar System Dynamics :

6. Idealized solution of the SSD equations : Formulate and solve the SSD equations in terms of angular momentum components as independent variables, introducing phenomenological parameters for (a) the exchange of angular momentum between the Sun's spin and orbital motions and (b) dissipation of spin energy through the internal MHD modes.

7. Modeling the relation between the amount of solar activity and the solar spin variations : Take the amount of solar activity as the 'dissipation parameter' in '1' and determine the spin variations from the observed variations in the amount of solar activity. Compare the results with the variations in the Sun's spin as mentioned in Section 2.1.

8. An 'Improvement' of the solar system ephemerides : From the variations of the planetary and solar orbital angular momenta determined in '1' above, redetermine the planetary positions as functions of time.

C. Stellar Physics :

9. Relations between a star's activity and its exchange of angular momentum with its gravitational companions : Learn about these relations by applying the results of tasks '1' to '7' to stellar situations wherever possible.

10. Mathematical modeling of the 'loosening' of the gravitational binding of the Sun (/ a star): Set up and solve equations for understanding the relation between (a) the 'loosening' of the gravitational binding of the Sun (/a star) due to the 'buoyant' rising of toroidal magnetic flux packets (Sec. 3.1) and (b) the solar (/stellar) 'mass loss' associated with the 'magnetic flux loss'.

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