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AN INTERPRETATION OF THE DIFFERENCES IN THE SOLAR DIFFERENTIAL ROTATION DURING EVEN AND ODD SUNSPOT CYCLES

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ABSTRACT

Using the data on sunspot groups during the period 1879–2004, we have found that the solar equatorial rotation rate during the odd-numbered sunspot cycles is well correlated with the equatorial rotation rate of the *preceding* even-numbered sunspot cycles, which is similar to the well-known Gnevyshev & Ohl rule (G-O rule) in sunspot activity. This indicates that a 22 yr cycle in the equatorial rotation rate begins in an even-numbered cycle and ends in the *following* odd-numbered cycle, the same as a solar magnetic cycle (Hale cycle), as inferred from the G-O rule. On the other hand, the latitudinal gradient of the solar rotation during the even-numbered cycles is found to be well correlated with that of the *preceding* odd-numbered cycles. This result indicates that a 22 yr cycle in the latitudinal gradient begins in an odd-numbered cycle and ends in the *following* even-numbered cycle. That is, the phase of the beginning of a 22 yr cycle in the latitudinal gradient is different by about 180° relative to the beginning of a 22 yr magnetic cycle.

Subject headings: Sun: activity — Sun: magnetic fields — Sun: rotation — sunspots

1. INTRODUCTION

Hale & Nicholson (1925) discovered the change of magnetic polarity of the bipolar sunspot groups at the beginning of each 11 yr sunspot cycle. This proved the existence of the 22 yr solar magnetic cycle (Hale's cycle). Gnevyshev & Ohl (1948) found an empirical relation between the sums of the sunspots in Waldmeir odd-numbered sunspot cycles and even-numbered cycles. The well-known Gnevyshev-Ohl rule, or G-O rule, states that the sum of sunspot numbers over an odd-numbered cycle exceeds that of its preceding even-numbered cycle. These authors also found that an even-numbered cycle and its following odd-numbered cycle are highly correlated (91%) and that the correlation between an odd-numbered cycle and its following even-numbered cycle is weak (50%). They interpreted the aforesaid correlations as the following: each 22 yr magnetic cycle (Hale cycle) begins with the beginning of an even-numbered cycle and ends with the end of the following odd-numbered cycle. Subsequently, many authors have confirmed this empirical relation from several solar indices and have claimed that the true physical cycle of the Sun is a 22 yr magnetic cycle that consists of an even-numbered activity cycle and the next odd-numbered activity cycle (see Obridko 1995; Komitov & Bonev 2001 and references therein). However, the physics behind the G-O rule is not yet fully understood. Durney (2000) proposed that the G-O rule may be a consequence of nonlinear interactions of velocity and magnetic field that provide the stabilizing mechanism.

Arguments have been advanced by Gough & McIntyre (1998) that the Sun's radiative interior must be stabilized against differential rotation by the presence of a primordial poloidal magnetic field. While there are serious timescale concerns discussed by Gilman (2000) about how such a magnetic field could influence the solar cycle, Gough (2000) points out that a reversing dynamo field could be produced by the effects of motions in the solar tachocline even though the source of the field starts with the nonvarying primordial field. One would reasonably expect that a solar cycle produced in this way would retain a signature of the polar orientation of the deep primordial field; perhaps each 11 yr half-cycle would be stronger or weaker, depending on whether the surface poloidal field is aligned with or opposed to the primordial interior field. This potential cause of the G-O rule adds impetus to our desire to better understand this rule's statistical significance.

It is generally agreed that the Sun's differential rotation drives the solar dynamo for generating all solar activity (Babcock 1961); however, the role of the differential rotation as a participant in the cycle of magnetic activity variation is not yet clear. Hence, the study of variations in the solar differential rotation is important for understanding the Sun's internal dynamics and the variations in the solar magnetic activity, as well as for finding the cause of the variations in the differential rotation. Many such experiments have been made by a number of scientists using different data and different methods (see Javaraiah & Komm 2002; Komm & Javaraiah 2002; Antia 2002).

Recently, Javaraiah (2000, 2001, 2003a) found the existence of significant differences in the differential rotation during even-numbered cycles (12, 14, 16, 18, 20, and 22) and odd-numbered cycles (13, 15, 17, 19, and 21), using sunspot data during the period 1879–2002. In addition, Kirov et al. (2002) and Georgieva et al. (2005) found significant differences in the differential rotation during positive and negative polarity cycles (intervals between the changes in the polar magnetic field). Therefore, it is worthwhile to check whether the inference drawn by Gnevyshev & Ohl (1948) from amplitudes of sunspot cycles can also be drawn from the differences in the differential rotation during even- and odd-numbered cycles. If this is possible, then it may be very useful for understanding the physics behind the G-O rule and the 22 yr magnetic cycle. In view of this, we have revisited the cycle-to-cycle modulations in the equatorial rotation rate and the rotational latitudinal gradient determined by Javaraiah (2003a) and studied these modulations in detail.

The determination of solar rotation using sunspots is made difficult by several factors of solar and instrumental origin. The

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TABLE 1

Sidereal Differential Rotation Coefficients

Cycle	Whole Disk		Southern Disk		Northern Disk	
	A	B	A	B	A	B
12 (1879–1889)	2.943 ± 0.003	−0.540 ± 0.036	2.935 ± 0.004	−0.442 ± 0.048	2.953 ± 0.005	−0.669 ± 0.056
13 (1890–1901)	2.945 ± 0.003	−0.502 ± 0.031	2.941 ± 0.004	−0.504 ± 0.040	2.949 ± 0.004	−0.492 ± 0.048
14 (1902–1912)	2.928 ± 0.003	−0.458 ± 0.043	2.933 ± 0.005	−0.462 ± 0.060	2.924 ± 0.004	−0.465 ± 0.063
15 (1913–1922)	2.932 ± 0.003	−0.537 ± 0.031	2.937 ± 0.004	−0.507 ± 0.046	2.929 ± 0.004	−0.589 ± 0.043
16 (1923–1933)	2.930 ± 0.003	−0.493 ± 0.027	2.929 ± 0.004	−0.476 ± 0.042	2.931 ± 0.004	−0.504 ± 0.036
17 (1934–1943)	2.934 ± 0.002	−0.594 ± 0.022	2.938 ± 0.003	−0.613 ± 0.030	2.930 ± 0.003	−0.574 ± 0.033
18 (1944–1953)	2.926 ± 0.002	−0.520 ± 0.021	2.927 ± 0.003	−0.497 ± 0.028	2.926 ± 0.003	−0.555 ± 0.032
19 (1954–1964)	2.926 ± 0.002	−0.541 ± 0.016	2.930 ± 0.003	−0.572 ± 0.027	2.924 ± 0.003	−0.524 ± 0.020
20 (1965–1975)	2.928 ± 0.002	−0.479 ± 0.023	2.929 ± 0.003	−0.464 ± 0.037	2.926 ± 0.003	−0.483 ± 0.030
21 (1976–1986)	2.926 ± 0.003	−0.420 ± 0.022	2.928 ± 0.004	−0.454 ± 0.031	2.923 ± 0.004	−0.382 ± 0.032
22 (1987–1996)	2.910 ± 0.003	−0.416 ± 0.024	2.907 ± 0.004	−0.428 ± 0.033	2.913 ± 0.004	−0.406 ± 0.034
23 (1997–2004)[a]	2.922 ± 0.004	−0.509 ± 0.030	2.918 ± 0.005	−0.477 ± 0.042	2.925 ± 0.005	−0.535 ± 0.044

Notes.—The values for the coefficients have been obtained from the formula $\omega(\phi) = A + B\sin^2\phi$ deduced using sunspot group data of individual sunspot cycles (see Javaraiah 2003a), where ϕ is the latitude and the values of A and B are in units of μrad s^{-1}.

[a] Cycle 23 is currently incomplete.

measured rotation rate depends very much on the characteristics of the sunspots or sunspot groups used (for a recent review, see Javaraiah & Komm [2002]). Instrumental effects such as local distortion of the image scale due to atmospheric seeing or telescopic aberration, inaccuracy of positioning the solar axis on solar images, etc., cause data sets from one observatory to be hard to compare with observations from other observatories (e.g., Howard et al. 1999). We compare the aforesaid cycle-to-cycle modulations deduced from Greenwich data with corresponding modulations deduced using the sunspot positions measured from Kodaikanal Observatory, Mount Wilson Observatory, Kanzelhöhe Solar Observatory, and the National Astronomical Observatory of Japan (NAOJ). The discrepancies between the results found from Greenwich data and the other data sets are discussed.

2. DATA AND ANALYSIS

The solar differential rotation can be determined accurately from the heliographic positions and the epochs of the observations of large number of sunspots or sunspot groups using the standard form: $\omega(\phi) = A + B\sin^2\phi$, where $\omega(\phi)$ is the solar sidereal angular velocity at latitude ϕ and the coefficients A and B represent the equatorial rotation rate and the latitudinal gradient of the rotation, respectively. We have used Greenwich data on sunspot groups during the period 1879 January 1–1976 December 31 and spot group data from the Solar Optical Observing Network (SOON) during the period 1977 January 1–2004 August 10. Details on the data reduction and determinations of the values of the differential rotation coefficients A and B, listed in Table 1, were described by Javaraiah (2003a). Cycle 23 is included in the present analysis, since more than 75% of it is completed, and we expect that the values of A and B will not change significantly with the addition of the data from the remainder of the cycle. Figure 1 shows the cycle-to-cycle modulations of A and B as deduced from the whole solar disk. Figure 2 shows the cycle-to-cycle modulations of A and B in the northern and the southern hemispheres. To get information on the reliability of these variations in Figure 1, we also show the cycle-to-cycle modulations of A and B determined from the measurements of Kodaikanal Observatory, Mount Wilson Observatory, Kanzelhöhe Solar Observatory, and NAOJ. We analyzed the daily data of sunspots and sunspot groups measured at Kodaikanal Observatory

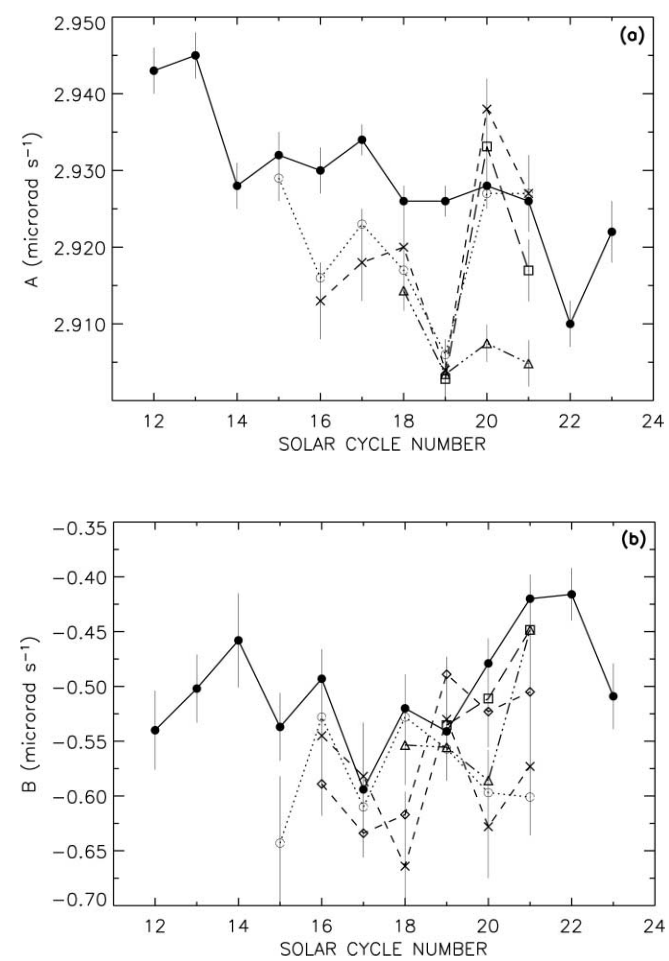

Fig. 1.—Cycle-to-cycle variations of A and B (whole-disk data; note that cycle 23 is not yet complete), determined using the following databases: Greenwich and SOON (*filled circles and solid curve*), Kodaikanal Observatory (*open circles and dotted curve*), Mount Wilson Observatory (from groups: *crosses and short-dashed curve*; from spots, in panel *b*: *small squares and short-dashed curve*), Kanzelhöhe Solar Observatory (*triangles and triple-dot–dashed curve*), and NAOJ (*large squares and long-dashed curve*).

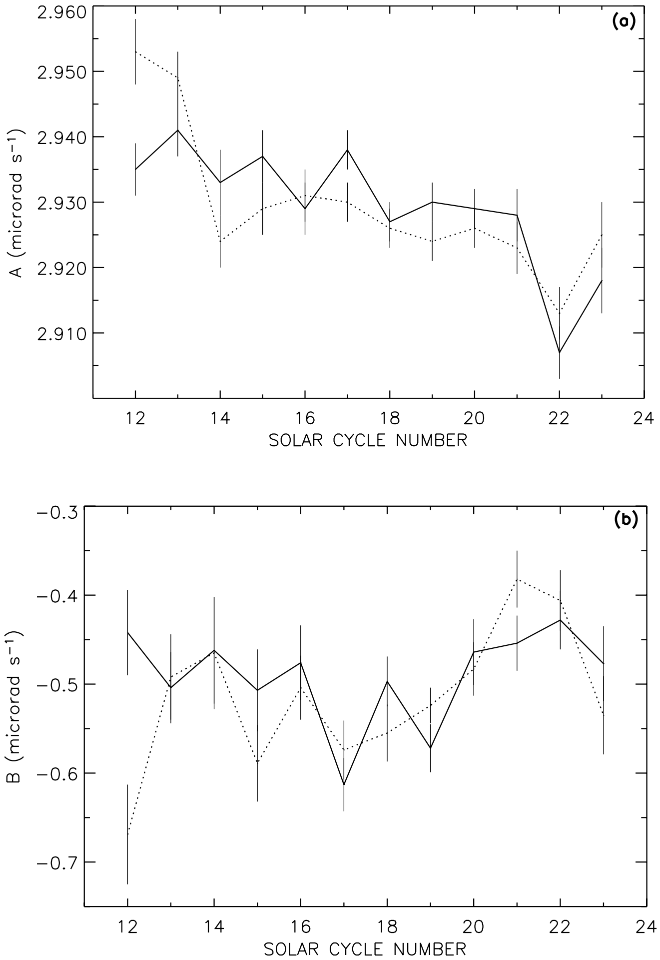

FIG. 2.—Cycle-to-cycle variations of A and B in the northern (*dotted curve*) and southern (*solid curve*) hemispheres, determined using the Greenwich and SOON data.

(1906–1987) and at Mount Wilson Observatory (1917–1985) by taking them from the Web site of the National Geophysical Data Center (NGDC).[2] We have used the values of A and B determined by Hanslmeier & Lustig (1986) and Kambry & Nishikawa (1990) from the sunspot drawings of Kanzelhöhe Solar Observatory (for cycles 18–21) and NAOJ (for cycles 19–21), respectively. The modulations of both A and B derived from the Kodaikanal sunspot data are found to be similar to those derived from the spot group data shown in Figures 1*a* and 1*b*. In the case of the Mount Wilson data, the modulation of A derived from sunspot data is also found to be similar to that derived from the spot group data, but we find considerable differences between the modulations of B derived from the sunspot data and the spot group data, as shown in Figure 1*b*.

The modulation of B deduced from Greenwich and SOON data shown in Figure 1*b* indicates that, without the data point of cycle 12, the absolute value of B during an even-numbered cycle is somewhat less than that of its preceding odd cycle. This systematic behavior suggests the existence of a $\sim$22 yr cycle in B (see also Javaraiah & Gokhale 1995; Javaraiah & Komm 1999; Javaraiah 2000, 2001, 2003a). A similar systematic behavior can also be seen in Figure 1*a* for values of A deduced from Greenwich and SOON data, but it is not as well defined as that of B (see also Javaraiah 2000, 2001, 2003a; Georgieva et al. 2005). From Figure 2 it can be seen that the aforesaid long-term variations of A and B exist in each hemisphere. However, there are some notable differences in the amplitudes of the variations of A and B in the northern and the southern hemispheres (Javaraiah 2003a).

Figure 1 shows that there are considerable differences in the modulations of A and B derived from Greenwich data and those from the other data sets. The variations of A determined from the data other than the Greenwich data are substantially larger than those from Greenwich data. This is particularly evident around cycle 19. The moderate drop in A from cycle 17 to cycle 18, which was found from the Greenwich data (Javaraiah 2003a), seems to be extended to cycle 19, and the drop is also larger in all the other data sets. In the case of B, the Mount Wilson sunspot data and the Greenwich data resemble each other, but Mount Wilson spot group data differ from all the other data sets. From cycle 19 to cycle 21 the modulation of B determined from the NAOJ data is similar to that of the Greenwich data, but those determined from Mount Wilson, Kodaikanal, and Kanzelhöhe spot group data are considerably different from that determined from the Greenwich data.

It should be noted here that the aforesaid different data sets have considerable differences in the observations, measurements, definitions of spot groups, treatment of the data in the analysis, etc., which can cause considerable discrepancies in the derived results (Howard et al. 1984, 1999; Hanslmeier & Lustig 1986; Balthasar & Fangmeier 1988; Kambry & Nishikawa 1990). The Greenwich data have been compiled from the majority of the white-light photographs that were secured at the Royal Greenwich Observatory and at the Royal Observatory at the Cape of Good Hope. The gaps in their observations were filled with photographs from other observatories, viz., Kodaikanal Observatory (India), the Hale Observatory (California), and the Heliophysical Observatory (Debrecen, Hungary), etc. The SOON data were compiled in a similar way to that of the Greenwich data (Hathaway et al. 2003). The measurements from Mount Wilson Observatory (1917–1985) and Kodaikanal Observatory (1906–1987) were made using the same technique (Howard et al. 1999). The main difference between the Mount Wilson and Kodaikanal and the Greenwich spot group data is that the former consist of measurements of individual umbrae of sunspots and the positions of sunspot groups are area weighted, while the latter is a collection of measurements of total areas (umbrae and penumbrae) of groups of sunspots and the positions are geometric positions of centers of groups. Lustig & Wöhl (1995) showed the existence of a relatively larger latitude dependence in the evolution of sunspot groups from Mount Wilson data compared to those from the Greenwich data. This may have a role in the aforesaid different behavior of variations in B determined from Mount Wilson data. Yoshimura & Kambry (1993) analyzed corrected NAOJ data and found a monotonic increase in A from cycle 18 to cycle 20 and a drop from cycle 20 to cycle 21. In addition, the increase in the values of A from cycle 19 to cycle 20 is small in the Kanzelhöhe data. Thus, the big jump in the value of A from cycle 19 to cycle 20 determined from the Mount Wilson and Kodaikanal data may also be not real. On the other hand, the value of the synodic rotation period during cycle 19, as determined by Obridko & Shelting (2001) from the solar global magnetic field data, is less than the value during cycle 18 (this behavior is similar to the behavior of A, as determined from Greenwich data). In view of all these results, we believe that the variations in A and B determined from Greenwich and SOON data are reasonably reliable.

[2] Daily sunspot data available at ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_REGIONS/SUNSPOT_REGION_TILT.

3. ANALYSIS OF THE DIFFERENCES IN A AND B DURING ODD AND EVEN CYCLES

Table 1 provides sequences of A and B coefficients that span 12 solar cycles. From this table one can extract different time series and study in particular the differences in A and B during odd and even cycles. From a statistical point of view, any two time series extracted from Table 1 define a bivariate population (x_i, y_i). Because we are mainly interested in studying the possibility of a linear relationship between the two variables of the bivariate population, we use Pearson's linear correlation coefficient r to measure the strength of the observed correlation, defined as follows:

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}},$$

where $\bar{x}$ and $\bar{y}$ are the means of the x_i and y_i, respectively. It is well known that the correlation coefficient is in general a rather poor statistic for deciding whether an observed correlation is statistically significant. The reason is that the individual distributions of x and y are ignored in the calculation of r, so there is no unique way to compute its distribution in the case of the null hypothesis. However, if the null hypothesis is that x and y are uncorrelated ($r = 0$), it can be shown that under some generally acceptable assumptions the statistic

$$t = r\sqrt{\frac{N-2}{1-r^2}}$$

is distributed like a Student's t-test with $\nu = N - 2$ degrees of freedom (e.g., Alder 1977). In order to decide whether a given correlation coefficient indicates a linear relationship for a sample of size N, we test the hypothesis that the sample is chosen from a population for which $r = 0$ and, therefore, determine the probability that from such a population a sample of size N is taken for which the correlation coefficient equals or exceeds the absolute value of r calculated for the given sample. Here we use a two-tailed test of significance. If the probability p is less than 5%, we reject the hypothesis that the sample is taken from a population in which there is no linear relationship. Hereafter, the even-numbered cycle time series, the *preceding* odd-numbered cycle time series, and the *following* odd-numbered cycle time series of A and B are denoted by A and B with subscripts $2n$, $2n - 1$, and $2n + 1$, respectively.

On the basis of the above criterion, we found that A_{2n+1} is well correlated with A_{2n} ($r = 0.896$, $t = 4.032$, and $p = 1.6\%$). This is shown in Figure 3a. The correlation between A_{2n-1} and A_{2n} is found to be statistically insignificant ($r = 0.450$, $t = 0.874$, and $p = 44.7\%$). It is interesting to note that these correlations between the A time series are consistent with the correlations between the corresponding time series of sunspot activity found by Gnevyshev & Ohl (1948). As shown in Figure 3b, the B-coefficient time series have an opposite behavior, since B_{2n} is found to be well correlated with B_{2n-1} ($r = 0.986$, $t = 10.284$, and $p = 0.2\%$), while the correlation between B_{2n} and B_{2n+1} is found to be negligible ($r = 0.096$, $t = 0.193$, and $p = 85.7\%$).

Once the correlation has been established, the next step is to determine the coefficients of the linear regression. The measurement errors in both the x_i and y_i variables makes our case considerably harder than that of the classical technique, which fits a straight-line model to the data using the linear least-squares method. In this paper we used the approach described in Jefferys

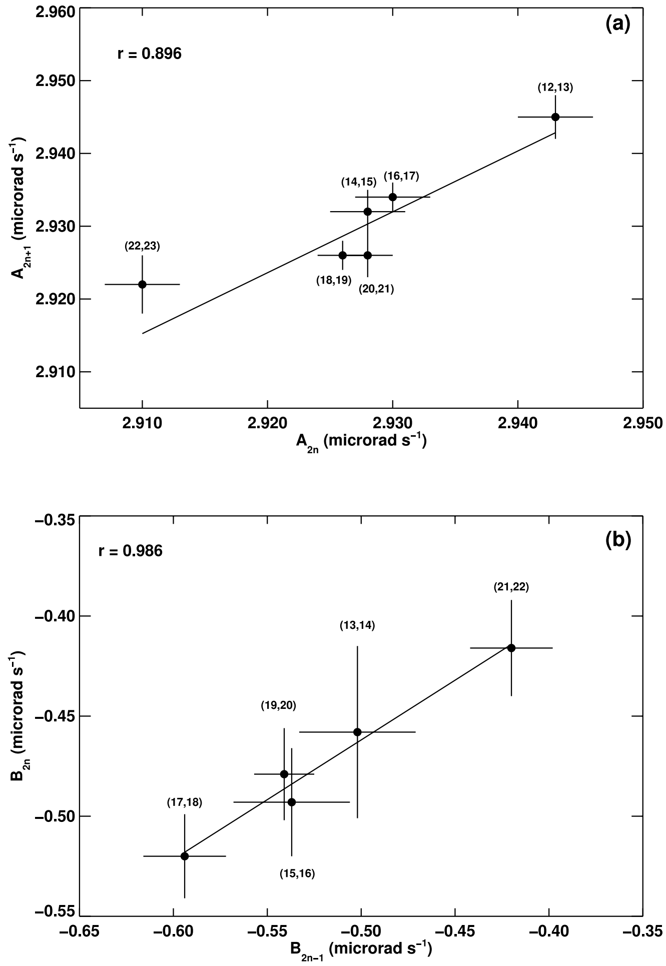

FIG. 3.—Plots of the correlations (a) between A_{2n+1} and A_{2n}, where subscripts represent Waldmeir cycle numbers with $5 < n \leq 11$, and (b) between B_{2n} and B_{2n-1}, where $6 < n < 12$. The solid lines represent the linear relationships between A_{2n+1} and A_{2n} (eq. [1]) and B_{2n} and B_{2n-1} (eq. [2]). The value of the correlation coefficient (r) is also given. Near each data point, the corresponding pair of odd and even cycles is shown.

(1980, 1981), which provides a more general method for fitting models to observed data by least squares without most of the assumptions required by the standard method.

We obtained the following linear regression fits (Fig. 3, *solid lines*) between A_{2n+1} and A_{2n} and between B_{2n} and B_{2n-1}:

$$A_{2n+1} = (0.84 \pm 0.18)A_{2n} + (0.48 \pm 0.54)\ \mu\text{rad s}^{-1}, \quad (1)$$

$$B_{2n} = (0.60 \pm 0.21)B_{2n-1} - (0.16 \pm 0.11)\ \mu\text{rad s}^{-1}, \quad (2)$$

where the uncertainties in the coefficients are the formal 1 σ errors from the fit. The goodness of fit was calculated by comparing the value of the χ^2 merit function to the χ^2 probability distribution for $N - 2$ degrees of freedom. This test shows that both relationships are well described by a linear model, in agreement with the results of our analysis on r. In the case involving the A coefficients, it must be pointed out that most of the regression is dominated by the two *outlying* points (12, 13) and (22, 23), which makes the argument for A slightly weaker. However, the positions of the data points are consistent with the long-term variations in A described by Javaraiah (2003a), and the position of the data point of the cycle pair (24, 25) is expected to be near the position of the data point of the cycle pair (22, 23) (Javaraiah 2003b). These relations can be used to predict the amplitude of A during an odd-numbered cycle from the known amplitude of A during the *preceding* even-numbered cycle and the amplitude of B during an even-numbered cycle from the known amplitude of B during the *preceding* odd-numbered cycle.

In each hemisphere the correlations between the even- and odd-numbered cycle time series of A and also between the corresponding time series of B are found to be similar to the correlations between the corresponding time series for the whole disk found above. In the southern hemisphere, the correlations between A_{2n+1} and A_{2n} and between B_{2n} and B_{2n-1} are found to be weaker than the correlations between the corresponding time series of A, and between those of B in the northern hemisphere. (In the southern hemisphere, the correlation between A_{2n-1} and A_{2n} is found to be almost the same as the correlation between the A_{2n+1} and A_{2n}.)

4. SUMMARY, DISCUSSION, AND CONCLUSION

The differences in the solar differential rotation during the even- and odd-numbered sunspot cycles deduced from Greenwich and SOON data on sunspot groups during the period 1879–2004 suggest the following:

1. The equatorial rotation rate during odd-numbered sunspot cycles seems to be well correlated with that of the preceding even ones but not with that of the following ones.
2. The latitudinal gradient during the even-numbered cycles is found to be well correlated with that of the preceding odd ones but not with that of the following ones.

The above results indicate that the sunspot cycles are not independent and are connected by long-term variations in the equatorial rotation rate and the latitudinal gradient. In particular, result 1 suggests that a 22 yr cycle in the equatorial rotation rate begins during an even-numbered cycle and ends during the following odd-numbered cycle; i.e., the epochs of the beginning and the ending of a 22 yr cycle in the equatorial rotation rate seem to be similar to the corresponding epochs of a solar magnetic cycle, as inferred by Gnevyshev & Ohl (1948). On the other hand, result 2 indicates that a 22 yr cycle in the latitudinal gradient begins during an odd-numbered cycle and ends during the following even-numbered cycle.

The above interpretations essentially mean that the phases of the 22 yr cycles in the equatorial rotation rate and the latitude gradient measured relative to the beginning of a solar magnetic cycle may be approximately 0° and 180°, respectively. To examine this further, we calculated the cross-correlations between A, B, and the amount of sunspot activity (S). We have taken the values of S, i.e., the sum of monthly sunspot numbers over a sunspot cycle, for sunspot cycles 12–21 from Wilson (1988). The monthly sunspot numbers for cycle 22 were also taken from the NGDC Web site.[3] Figure 4 shows the cross-correlation functions (CCFs) of the pairs (A, B), (A, S), and (B, S). A positive value of lag indicates that the first time series leads the second one. In this figure CCF(A, B) and CCF(A, S) have maximum negative correlation values at lag = 0, whereas the maximum (positive) correlation between B and S is at lag = −3. These results are consistent with the interpretation of the results and the phase relationships discussed above. A possible reason for the hemispheric differences in the correlations between the time series of A and between the time series of B (§ 3) may be considerable hemispheric differences in the periodicities of A and B (Georgieva & Kirov 2003; Georgieva et al. 2005).

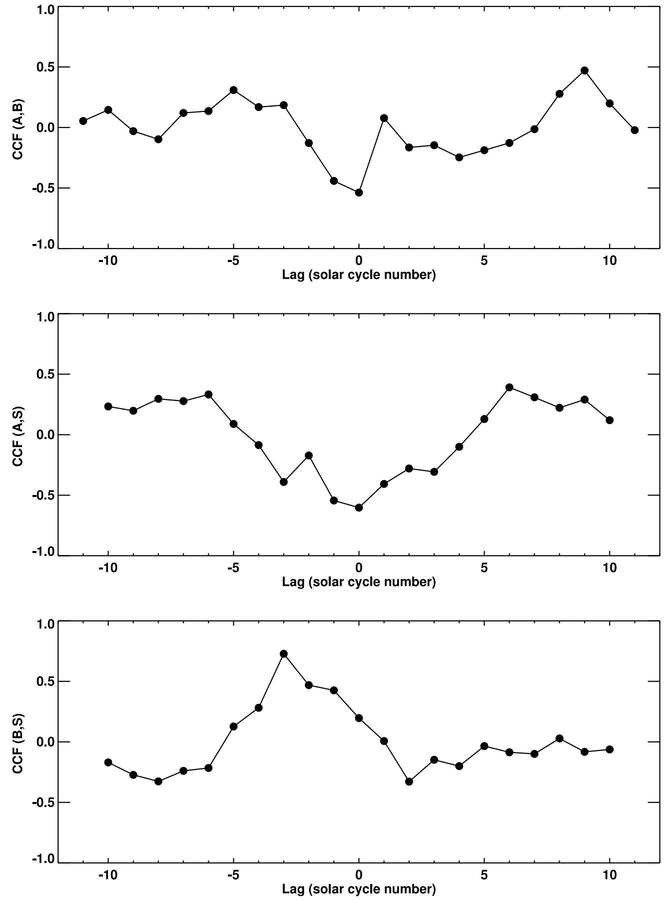

FIG. 4.—CCFs of the pairs (A, B), (A, S), and (B, S). A positive value of lag indicates that the first time series leads the second one.

The reliability of the results obtained in this paper critically depends on the length and homogeneity of the time series investigated. We have used the longest set of rotational data available today, the Greenwich and SOON measurements, and the correlations we derived are statistically significant. Different data sets, however, have been shown to provide different conclusions about the variation of the solar rotation. For example, Obridko & Shelting (2001) showed considerable variation in the rotation period determined from the solar global magnetic field. On the other hand, no significant variations are found in the equatorial rotation rate deduced from solar surface Doppler velocity measurements (Ulrich & Bertello 1996) and from the variation of the Sun's mean magnetic field (Haneychuk et al. 2003). In addition, as discussed in § 2, there are some inconsistencies among the rotational results obtained from the sunspot data measured at different observatories that need to be further investigated.

The interpretation drawn by Javaraiah (2003a) from the correlations found between the ratios of B and the amount of sunspot activity and the one drawn in this paper from the correlations found between the time series of B alone contradict each other. The relatively short lengths of the time series used in that paper may be the reason for the discrepancy.

Our results provide a strong support to the idea that the Sun's basic physical cycle is the 22 yr magnetic cycle rather than the 11 yr activity cycle. In particular, the results support the idea

[3] Monthly sunspot data available at ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/MONTHLY.PLT.

that the 22 yr magnetic cycle may be a torsional magnetohydrodynamic oscillation, first considered by Walén (1944) as a mechanism for the solar cycle (see also Layzer et al. 1979; Gokhale & Javaraiah 1995; Hiremath & Gokhale 1995). However, the source of perturbations needed for such oscillations and details about the Sun's internal magnetic field are not yet known.

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