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Structure of small-scale magnetic flux tubes and excitation of sausage mode oscillations

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Abstract. The nature of small-scale magnetic fields in the solar atmosphere is discussed. In the first part of the review, equilibrium models for the structure of magnetic flux tubes are presented. It is shown that the thermodynamic properties of the flux tube atmosphere are sensitive to energy transport by radiation. A comparison of the results with observations shows that the temperature and magnetic field strength are in broad agreement. In the second part, the linear interaction of a flux tube with sound waves or p-modes in the external medium is analysed. The idealized case of an isothermal atmosphere is considered, which allows the mathematical problem to be solved exactly. The detailed behaviour of the flux tube response and its dependence on various parameters is analysed. It is pointed out that in general the interaction is *non-resonant.* An application of the results to the solar atmosphere is discussed.

Key words: MHD - sun: oscillations - sun: magnetic fields

1. Introduction

This paper concerns itself with small-scale magnetic fields outside of sunspots. A large number of observations in the 1970's established that the surface magnetic field on the Sun is structured in the form of intense vertical magnetic elements or flux tubes (for details on the observational properties of tubes see the reviews by Stenflo 1989 and Solanki 1990). These flux tubes are ubiquitous in the solar photosphere and play an important role in the dynamics and energy transport of the atmosphere. Their field strengths are empirically known to be in the range 1-2 kG and their diameters are a few hundred kilometers, which is generally believed to be much less than the horizontal separation between adjacent tubes. These elements occur preferentially at the supergranular boundaries in the network. High resolution spatial observations reveal that the intense flux tubes occur as bright points in the "lanes" between granules.

The aim of the present investigation is to discuss theoretical work on the structure of small-scale magnetic flux tubes and the excitation of oscillations in them through the buffeting

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action of acoustic waves or p-modes in the external atmosphere. The equilibrium structure of a flux tube is first investigated in Sect. 2. The thermodynamic properties of such tubes, computed theoretically, are compared with observations. In Sect. 3 we turn to the interaction of magnetic flux tubes with external p-modes, based upon a simple model treated using a linear analysis. The main conclusions of the work are summarized in Sect. 4.

2. Structure of a thin flux tube

2.1 Model

Let us consider a flux tube extending vertically through the photosphere and convection zone of the Sun. We assume that the atmospheres, inside and outside the tube are in hydrostatic and energy equilibrium. We adopt a cylindrical co-ordinate system and solve the magnetohydrostatic (MHS) equations in the thin flux tube approximation (Defouw 1976; Roberts & Webb 1978). Briefly, this approximation consists of expanding all quantities about the tube axis and assuming that the radial variation is small compared to the vertical variation.

The thermodynamic structure of the tube can be determined by solving the following energy equation:

$$\nabla . \mathbf{F}_c = 4\pi\kappa(J - S), \tag{1}$$

where $\mathbf{F}_c$ denotes the convective energy flux, J is the mean radiation intensity, S is the source function, κ is the Rosseland mean opacity per unit distance. All quantities in equation (1) are evaluated on the tube axis. The above energy equation treats both radiative and convective energy transport. The mean intensity J is determined by solving the radiative transfer equation for 3 angles, assuming grey opacity and local thermodynamic equilibrium. The convective energy flux is calculated using mixing length theory with an additional parameter $\alpha \leq 1$, characterizing the suppression of convection within the flux tube ($\alpha = 1$ in the external atmosphere). The external atmosphere is first constructed by solving the equations of hydrostatic and energy equilibrium for a plane parallel medium, assuming constant vertical energy flux. We then determine the structure of the flux tube by solving the MHS equations. At the interface between the flux tube and the external atmosphere, we assume horizontal pressure balance.

2.2 Results

We now present the results of our model calculations for $\beta = 1.0$, where $\beta = 8\pi p/B^2$, at the top boundary taken at 500 km above $\tau_e = 1$ (p is the pressure, B is the vertical magnetic field strength on the tube axis and τ_e is the continuum optical depth in the external atmosphere). The lower boundary in the computations is taken at a depth of 2000 km below $\tau_e = 1$.

Fig. 1 shows the variation with depth z of the flux tube temperature T_i (full line) and the total vertical energy flux F_{tot} on the tube axis (dashes) for different values of the tube radius $a(0)$. The external temperature T_e is also shown for comparison as a heavy dashed curve. The filled inverted triangles on the solid curves denote the optical depth unity levels within the flux tube. The flux tube is hotter and cooler respectively than the external medium for $z < 0$ and $z > 0$ respectively. We find that in the photospheric layers ($z < 0$), the flux tube is hotter than the ambient medium. The reason for this effect is due to the reduced opacity in the tube,

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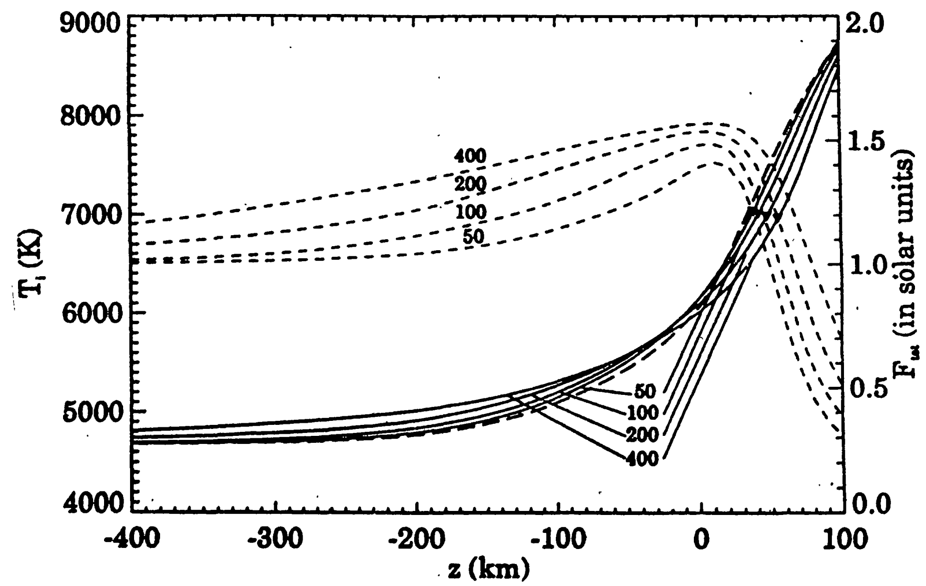

Figure 1. Variation of T_i (solid lines) and F_{tot} with z for different values of $a(0)$ and $\beta = 1.0$. The thick dashed curve corresponds to the temperature distribution in the external atmosphere. The filled inverted triangles denote the levels in the tube where the optical depth is unity.

which results in the vertical optical depth scale in the flux tube being effectively shifted down with respect to the external atmosphere. Consequently, the upper layers (i.e. above $z = 0$) are heated by radiation emanating from the hot layers of the tube. This can be seen by noting that the total vertical energy flux (which is essentially the vertical radiative flux above $\tau = 1$) is greater than the normal solar flux in the upper layers of the tube. Below the photosphere, due to the rapid increase of opacity in the z direction, the vertical radiative flux drops off very rapidly. In these layers, energy conservation implies that the temperature structure is determined by a balance between vertical and horizontal radiative energy flux (the convective flux is still too low at these depths to influence the energy budget). However, at still greater depths (for $z > 400$ km), the opacity becomes sufficiently large that convective energy transport dominates over radiative transport. Energy conservation implies that, in these layers, the vertical convective flux is roughly constant with z (assuming that horizontal convective transport is completely suppressed by the strong magnetic field in the tube). This implies that $T_i < T_e$ at equal depths, at sufficiently large depths.

Table 1 summarizes the properties of flux tubes parameterized by different values of β and tube radius a_0. The magnetic flux is denoted by Φ and z_w denotes the Wilson depression in the tube i.e., the geometric depth corresponding to optical depth unity. The corresponding temperature and magnetic field strength are denoted by T_w and B_w respectively.

Flux tube models corresponding to $\beta = 1.0$ show a reasonable agreement with observations. The models for smaller values of β implying field strengths of around 2000 G which are favoured by some semi-empirical models (Zayer et al. 1990) tend to be too hot. Furthermore, there are theoretical difficulties associated with generating flux tubes with such high field

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Table 1. Various quantities in a flux tube for different values of β and a_0.

β	a_0 (km)	Φ(Mx)	z_w (km)	T_w(K)	B_w(G)
0.1	50	1.4×10^{17}	90	8789	2104
	100	5.5×10^{17}	94	8786	2127
0.5	50	1.2×10^{17}	52	7526	1638
	100	4.8×10^{17}	55	7483	1664
	200	1.9×10^{18}	63	7468	1769
1.0	50	1.0×10^{17}	37	7024	1360
	100	4.1×10^{17}	41	7014	1393
	200	1.6×10^{18}	47	6990	1442
	400	6.6×10^{18}	55	6975	1504

strengths. Convective collapse, which is widely regarded as the process for concentrating the photospheric field, generally produces field strengths around 1300-1400 G at $\tau = 1$ (Hasan 1984, 1985).

Other studies which have modelled the structure of flux tubes are the 2-D models by Deinzer et al. (1984a,b), Grossmann Doerth et al. (1994), Knölker et al. (1991), Steiner, Knölker & Schüssler (1994) and the 3 - D models by Nordlund & Stein (1989, 1990). Magnetostatic models have also been computed amongst others by Ferrari et al. (1985), Kalkofen et al. (1986, 1989), Steiner & Stenflo (1989) and Pizzo, MacGregor & Kunasz (1993).

3. Excitation of flux tube oscillations by by p-modes

We now turn to the interaction of a flux tube with p-modes which are trapped acoustic waves in the external atmosphere. The buffeting action of these p-modes on the magnetic elements can lead to the excitation of flux tube oscillations. Some aspects of this problem have been investigated amongst others by Ryutova and Ryutova (1976), Spruit (1982), Cally (1985), Bogdan (1989), Bogdan et al. (1996), Hasan & Bogdan (1996) and Hasan (1997) see also the reviews by Bogdan and Braun 1995, Spruit 1996 and references therein).

3.1 Mathematical aspects

In the thin flux tube approximation, the linearized MHD equations for a sausage wave that is forced by a external pressure perturbation Π_e with a variation of the form $e^{i(k_x x-\omega t)}$, can be reduced to a second order differential equation for the vertical displacement in the tube (Roberts 1983) of the form

$$\frac{d^2Q}{dz^2} + \left(\frac{\omega^2-\omega_V^2}{c_T^2}\right)Q = e^{-z/4H}\,\frac{1+\beta}{2p_e}\left(\frac{d}{dz} + \frac{g}{c_s^2}\right)\Pi_e\,, \tag{2}$$

where $Q = \xi_z e^{-z/4H}$, ξ_z is the vertical component of the Lagrangian displacement in the tube, z is the height in the atmosphere (positive upwards), γ is the ratio of specific heats, g is the acceleration due to gravity, H is the pressure scale height, c_s is the sound speed, ω is the angular frequency of the perturbation, $\omega_{BV} = g\sqrt{\gamma - 1}/c_s$ is the Brunt- Väisälä frequency, $\omega_V^2 = \omega^2_{BV} + c_T^2/H^2(3/4 - 1/\gamma)^2$ is the cutoff frequency for a sausage wave and c_T is the tube speed defined as $c_T^2 = c_s^2/(1 + \gamma\beta/2)$.

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Knowing Π_e, equation (2) can be solved analytically subject to specified boundary conditions. We assume that the vertical displacement ξ_z vanishes at $z = 0$. For the lower boundary, we assume an outgoing wave solution, so that $\xi_z \sim e^{i(wt+kz)}$, where k denotes the vertical wave number of the tube mode. Thus it can be shown (for details see Hasan 1997) that the vertical displacement, in the limit $z \rightarrow -\infty$, has the form

$$\xi_z \sim e^{z/4H}(1+\beta) \; \frac{(\omega^2-\omega^2_{BV})}{c_s^2} \; \frac{k_e \alpha}{[(k_e^2-k^2+\alpha^2)^2+k^2\alpha^2]} \, , \tag{3}$$

where $k_e \equiv k_{x,e}$ and $\equiv 1/4H$.

From equation (3), we see that the displacement at a particular depth in the tube ξ_z is always finite for all values of k and ω, in contrast to the unstratified case, when a resonance exists for $k=k_e$. This demonstrates that the stratification qualitatively alters the nature of the interaction between a p-mode and thin flux tube. A careful examination of equation (3) (Hasan 1997) reveals that for small k_x, ξ_z is independent of k_x, whereas for large k_x, it drops off as $1/k_x^2$.

3.2 Linear response of a tube

Let us now examine the response Ξ of a tube when it is buffeted by a p-mode. We define the response as the ratio of the total wave energy in the tube to the total wave energy in p-modes in the external medium. The solid curves in Fig. 2 show Ξ/f, where f denotes the magnetic filling factor, as a function of the dimensionless horizontal wavenumber K_x ($K_x \equiv K_{x,e}$) for p-modes in the external atmosphere of different order, assuming β=0.5 and D = 40H, where D is the depth of the lower boundary. The dotted curves denote the frequencies (with reference to the right axis) of the p-modes in the external medium. These modes have a discrete spectrum due to the assumption that the vertical displacement vanishes at the top and bottom boundaries. Clearly, the p-modes have frequencies above the acoustic cutoff frequency, which in our dimensionless units is 0.5.

For a fixed order, corresponding to a specific value of the dimensionless vertical wave number K_e ($K_e \equiv k_{z,e}$) we find that the response is almost flat for $K_x \ll 1$. For low orders, Ξ/f shows a gradual increase with K_x till it reaches a maximum and thereafter it decreases. This maximum value increases with n whereas the value of K_x at which the maximum occurs decreases as n increases. For large enough n, the maximum of Ξ/f is at $K_x = 0$. Furthermore, whereas the response Ξ/f for small K_x increases with mode order, the asymptotic value of Ξ/f (other than for n = 1) appears to be independent of n.

3.2 Application to intense flux tubes on the Sun

Let us consider a pressure scale height such that the acoustic cutoff frequency corresponds to the frequency associated with 5 min. oscillations. This yields a scale height H = 256 km and a sound speed of 10.7 km s^{-1}. Our lower boundary is at $z = -40\,H$ which translates to a depth of about 10^4 km. This is the location of the lower turning point of the acoustic cavity, where ξ_z vanishes. The sound speed at our lower boundary is roughly 37 km s^{-1} (for Υ=5/3). For

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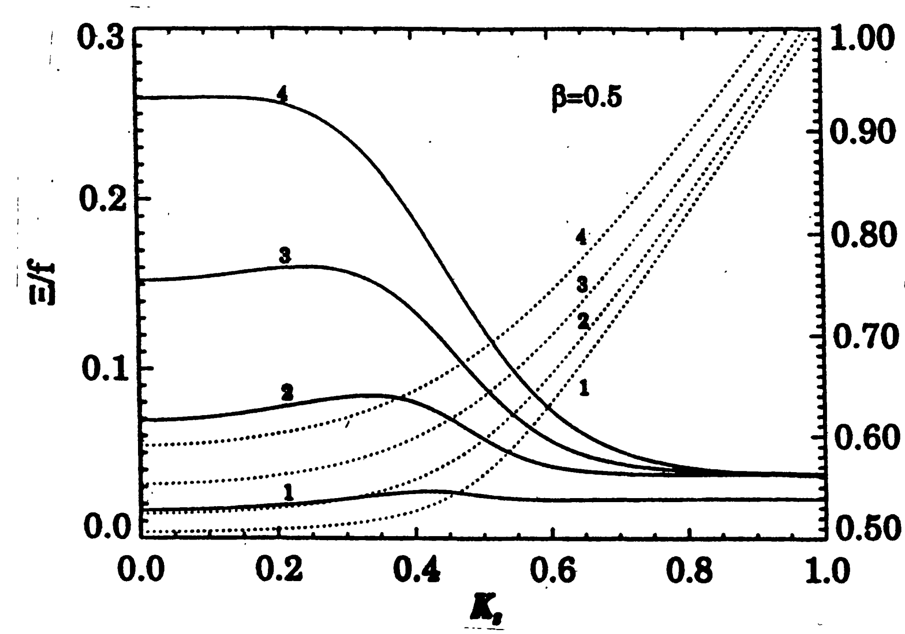

Figure 2. Dependence of the response Ξ /f (solid curves) on the p-mode dimensionless horizontal wave number K_x for different mode orders n, which label the curves, assuming $D = 40\,H$ and $\beta = 0.5$ (henceforth the default parameters). The dotted curves denote the variation of the p-mode dimensionless frequency Ω with K_x for different values of n.

p-modes in the 5-min. range, only modes with $k_x \geq 6.3 \cdot 10^{-9}$ cm^{-1} ($K_x \geq 0.16$) or with degree $l \geq 439$ will be reflected at the lower boundary. This limits the applicability of our analysis to p-modes with degree greater than the above value.

The interaction with a flux tube will lead to a loss of p-mode energy which translates to a finite p-mode line width Γ, which is the ratio of the time-averaged power going down the flux tube to the p-mode energy. Figure 3 shows the variation of Γ/ω as a function of frequency, assuming $\beta = 1.0$ and $H = 256$ km or an acoustic cutoff frequency of 3.3 mHz and $f = 0.01$. The numbers above each curve denote the mode order. These calculations yield a line width which is typically between 10% to 20% of the observed values for the $n = 2$ and $n = 3$ modes respectively. However, as the frequency increases, we find that the line widths decrease.

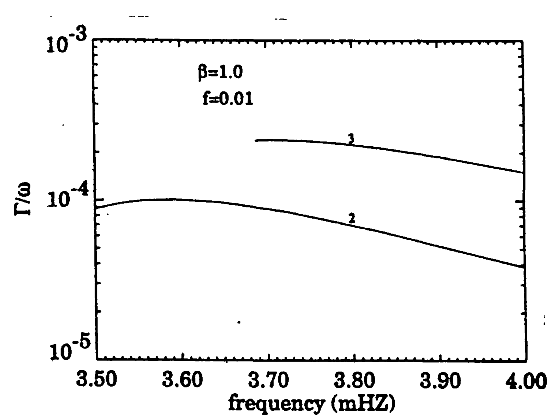

Figure 3. Variation of the line width Γ/ω with p-mode frequency (in mHz) for two values of n, which label the curves, assuming $\beta = 1.0$ and $f = 0.01$.

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4. Conclusions

In the first part of the investigation, equilibrium models for small-scale intense solar flux tubes have been computed. The models show that the thermodynamic structure of the tube is sensitive to radiative energy transport. At equal geometric levels, the tube is generally hotter than its surroundings in the photosphere, but cooler in the convection zone. This temperature difference increases with tube thickness. Our theoretical models for flux tubes are in broad agreement with observations.

The second part of the paper has dealt with the excitation of flux tube oscillations due to the buffeting action of p-modes in the external atmosphere. For mathematical tractability, the idealized case of an isothermal atmosphere has been considered. Although this assumption is not realistic for the atmosphere below the photosphere, it has the advantage that it enables us to solve the equations analytically and thereby understand in detail the factors affecting the response of a tube to buffeting by p-modes. Current work in progress considers in detail the solution of the time dependent equations, which involve formulating the interaction as an initial value problem.

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