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Telescopes : Their design and maintenance

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Abstract. We discuss the properties of the two-mirror telescope family, bringing out their aberration characteristics and their sensitivity to the misalignment and wrong spacing of the primary and secondary mirrors.

We also stress that the realization of the telescope according to the correct design is only the very first step, and that equally important is their maintenance. To this end, we discuss the Shack-Hartmann system we have developed for testing telescopes, and give examples of common defects found in telescopes and how they can be corrected.

1. Introduction

The design of telescopes has a long and fascinating history and goes back to the time of Galileo and Newton, with each step forward going hand-in-hand with the developments in technology.

The currently most popular type of telescope for professional astronomers, the Ritchey-Chretien telescope, with its hyperbolic primary and secondary mirrors, did not come into common use till the 1980s, mainly due to difficulties in manufacturing of hyperbolic mirrors.

In the next section we discuss the various members of the two-telescope family and their characteristics. We will follow the notation of Schroeder (1987). The reader is also referred to the detailed article by Wetherell and Rimmer (1972), as well as the recent book of Wilson (1997). It must be noted that different authors follow different sign conventions, and this should be kept in mind when intercomparing results.

The two main types of two-mirror telescopes are:

- Cassegrain : Concave primary mirror, convex secondary mirror.
- Gregorian : Concave primary and secondary mirrors.

Their characteristics depend on the various parameters chosen for the telescopes.

While it is important to start with a good telescope design, equally important is their maintenance, as the performance of a good telescope can easily be degraded due to the aberrations caused by the misalignment of mirrors, as well as by support problems. This can lead to loss of observing efficiency (as the light from the object being observed is distributed over more pixels), as well as cause problems in data reduction in crowded fields (e.g. the aberration coma gives rise to asymmetric images, which are very difficult to account for by software).

For this a wavefront sensor is needed to anaylze the performance of the telescope.

2. Which parameters to use for designing a telescope?

One needs at least 4 parameters to define a telescope.

Two parameters are related to distance: 1) The diameter of the entrance pupil, which normally is defined by the primary mirror (though in some cases it can also be defined by the secondary mirror, especially for IR telescopes).
2) The back focal distance β.
The other two parameters related to the power of the telescope are
3) The focal ratio of the telescope, which gives the resolution of the telescope at the focal plane.
4) The magnification m of the telescope.

These 4 parameters, plus a 5th, the field angle θ, can be used to completely characterize a telescope. Once these four parameters are fixed, the telescope is completely designed.

Clearly the choice of the parameters depends on a number of factors: e.g. the value of the back focal distance β is determined by the instrumentation which is planned for a telescope. The choice of the magnification *m* involves a tradeoff between how fast the primary mirror can be (to keep the telescope tube short to have a smaller dome to save on costs as well as to minimize air-seeing effects), the final telescope ratio, and the sensitivity of the telescope to aberrations.

3. The Parameters used to describe a telescope

The parameters used to describe the telescope are (see Schroeder 1987):

- $k=Y_2/Y_1$; $\rho = R_2/R_1$, where R_1 and R_2 are the vertex radii of curvature.
- m (magnification)$=-S_2'/S_2=F/F_1=f/f_1$.
- β = Back focal distance in units of f_1
- $F_1 = f_1/D$ (focal ratio of primary mirror); $F=f/D$ (focal ratio of telescope).

4. Equations for field aberrations

The Seidel aberrations of a mirror can be obtained from Fermat's principle, and have been the subject of much theoretical study (see, e.g. the treatment of Schroeder 1987, as well as the references he cites). The Seidel theory for single mirrors can be extended to two-mirror telescopes. Here we give the expressions for the three major field aberrations : spherical aberration (SA), tangential coma (TC) and astigmatism (AST) for two-mirror telescopes. We also discuss the curvature of the image plane.

4.1 Spherical aberration SA (at best focus)

$$\frac{m^3}{64F^3}\left\{K_1+1-\frac{k^4}{\rho^3}\left[K_2+\left(\frac{m+1}{m-1}\right)^2\right]\right\}$$

This is the fundamental aberration equation of a two-mirror telescope, as it determines the choice of the conic coefficients K_1 and K_2, given the telescope parameters. Indeed, once the conic coefficient of one of the mirrors is fixed, the conic coefficient of the other mirror can be obtained from the above equation. For example, in a Classical Cassegrain telescope, a parabolic primary is chosen (K_1= -1), while for a Dall-Kirkham telescope, the secondary is spherical ($K_2 = 0$).

The conic coefficients in terms of the telescope parameters

	Classical	Ritchey-Chretien	Dall-Kirkham
K_1	-1 (Parabolic)	$-1-\frac{2(1+\beta)}{m^2(m-\beta)}$	$-1+\frac{(m-1)^3(1+\beta)}{m^3(m+1)}\left(\frac{m+1}{m-1}\right)^2$
K_2	$-\left(\frac{m+1}{m-1}\right)^2$	$-\left(\frac{m+1}{m-1}\right)^2-\frac{2m(m+1)}{(m-\beta)(m-1)^3}$	0 (Spherical)

Once the conic coefficients are fixed such that SA = 0, the other aberrations are obtained:

4.2 Tangential coma (TC)

$$3\frac{\theta}{16F^2}\left[1+\frac{m^2(m-\beta)}{2(1+\beta)}(K_1+1)\right].$$

It can be seen that TC depends on the conic coefficient K_1. Thus, if we choose K_1 in such a way that TC = 0, we can obtain K_2 from the equation for SA to get zero spherical aberration. This is the so-called aplanatic telescope, commonly called the Ritchey-Chretien telescope.

Evidently, both K_1 and K_2 have to be free parameters for this to happen: thus for the Classical and Dall-Kirkham telescopes, where one of the conic coefficients is fixed *a priori*, coma is present. TC varies linearly as the field angle θ, and changes sign with it.

4.3 Astigmatism AS (at best focus)

$$\frac{\theta^2}{2F}\left[\frac{m^2+\beta}{m(1+\beta)}-\frac{m(m-\beta)^2}{4(1+\beta)^2}(K_1+1)\right]$$

Since K_1 has already been chosen to make TC = 0, it can be seen from the above equation that astigmatism will be present for all the two-mirror telescopes we have discussed. Indeed, for the R - C telescope, AS (along with curvature of field) defines the useful field. AS depends on the square of the field angle θ, and thus does not change sign with it. In the above equation, AS is the diameter of the image at best focus, i.e. between the two astigmatic lines.

4.4 Curvature of field

The last major aberration is curvature of field, which along with AS, limits the usefulness of the field of an aplanatic telescope.

$$\mathcal{K}_m = \frac{2}{mR_1}\left[\frac{(m^2-2)(m-\beta) + m(m+1)}{m(1+\beta)} - \frac{m(m-\beta)^2}{2(1+\beta)^2}(K_1+1)\right].$$

Since m is positive for a Cassegrain telescope and negative for a Gregorian, it follows that the surface for best images is concave (negative $\mathcal{K}$m) and convex (positive $\mathcal{K}$m) respectively as seen from M_2.

Properties of Cassegrain and Gregorian telescopes.

Cassegrain

Name	Property	K_1	K_2
Classical	Zero SA, but has both field coma and astigmatism	– 1 (Parabolic)	< – 1 (Hyperbolic)
Aplanatic (Ritchey-Chretien)	Zero SA and field coma, but has field astigmatism	<1 (Hyperbolic)	< – 1 (Hyperbolic)
Dall-Kirkham	Zero SA, but has both field coma and astigmatism	$-1<K_1<0$ (Oblate ellipsoid)	0 (Spherical)

Gregorian

Name	Property	K_1	K_2
Classical	Zero SA, but has both field coma and astigmatism	– 1 (Parabolic)	$-1<K_2<0$ (Oblate ellipsoid)
Aplanatic	Zero SA and field coma, but has field astigmatism	$-1<K_1<0$ (Oblate ellipsoid)	$-1<K_2<0$ (Oblate ellipsoid)

5. Some comments on the above equations

Some general conclusions can be drawn from the equations given above:

- The aberrations are a direct function of the magnification m, which limits how fast the primary mirror can be, as $F = mF_1$.
- They are inversely proportional to the telescope focal ratio, i.e. they are smaller for slower telescopes.

It is the task of the telescope designer to balance the two effects.

6. Some examples

We now give the aberrations for different types of telescopes, using the following 4 parameters: 1) Telescope focal length: 35000 mm. 2) Telescopes diameter: 3500 mm. 3) Focal length of primary mirror: 7000 mm. 4) Back focal distance : 1200 mm.

From these data we see that we are dealing with an F/11 telescope with a moderately fast M_1 (F/2), and a moderate magnification (m = 5).

Fig. 1 shows the variation of these aberrations with field angle θ. It is seen that coma limits the useful field for the classical and Dall-Kirkham telescopes, with the latter having a coma of 14.2" at a field angle of 10'. Field coma is zero, of course, for a R-C telescope.

As far as astigmatism and field curvature is concerned, there is little difference between the Classical and R-C telescopes, while the D-K has a much lower value for both these aberrations. However, the use of the D-K telescope is ruled out because of its field coma, as noted above.

Here we have represented the field curvature in terms of the defocus disk with respect to a flat field. Thus, it seen that for an R-C telescope, it is the curvature of field which sets the limit to the useful field, with astigmatism also playing an important role.

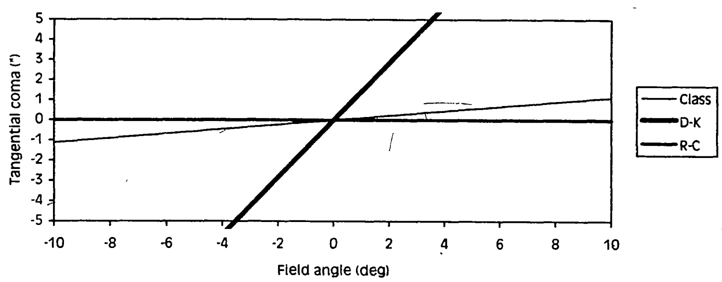

Figure 1. The variation of TC as a function of field angle for a classical, R-C and D-K telescope with the parameters given in Section 6. Note that for the R-C telescope TC=O.

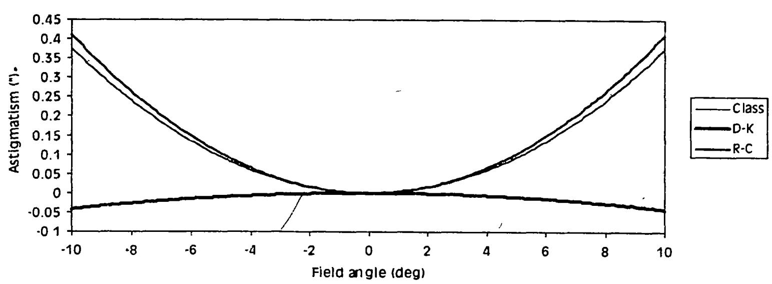

Figure 2. The variation of astigmatism as a function of field angle for a classical, R-C and D-K telescope with the parameters given in Section 6. There is little difference between a classical and R-C telescope.

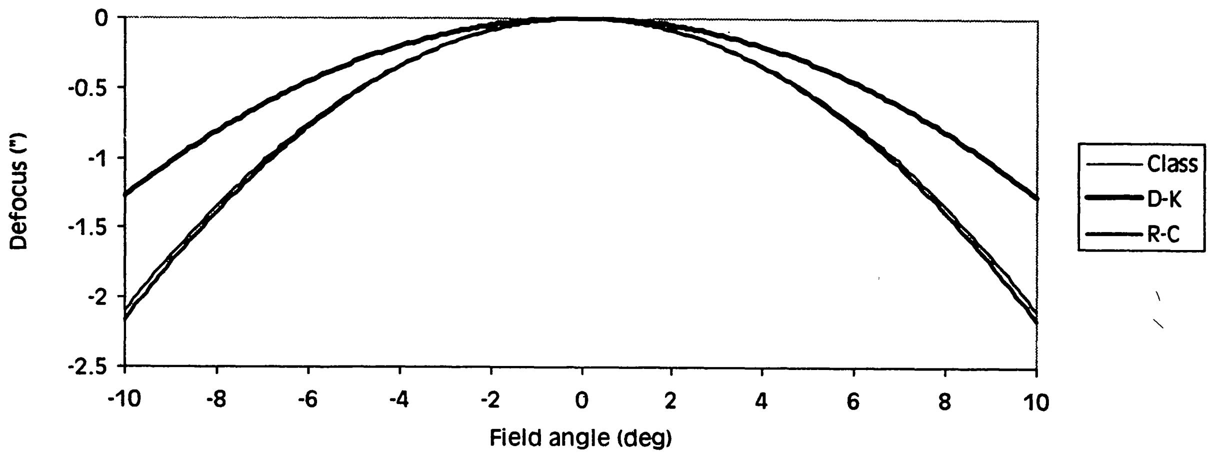

Figure 3. The variation of curvature of the image surface as a function of field angle for a classical, R-C and D-K telescope with the parameters given in Section 6. The curvature of field is expressed in terms of the defocus with respect to a flat field. Like for astigmatism, there is little difference between a classical and R-C telescope.

7. Aberrations due to despace error of M_2

Aberrations are generally thought to arise due to the inability of optical systems to correctly image off-axis objects. However, aberrations can also arise for on-axis objects, due to misalignment of M_1 and M_2, as well as due to the wrong spacing (compared to the nominal design values) between them. The former cause coma due to tilt and decentering, while the latter cause SA. Astigmatism due to misalignment and despace is negligible. It should be emphasized that the aberrations due to despace error are constant over the field, that is they do not depend on the field angle. The equations governing these aberrations are:

7.1 Tangential coma due to decentering :

$$\frac{l}{f} \; \frac{3\,(m-1)^3}{32F^2}\left[K_2\left(\frac{m+1}{m-1}\right)\right]$$

The displacement / is positive in the +y direction.

7.2 Tangential coma due to tilt

$$\alpha\,\frac{3(m-1)\,(1+\beta)}{16F^2}$$

The angle α is negative when M_2 is rotated in the counterclockwise direction.

7.3 Spherical aberration due to wrong positioning of the focal plane

$$-\frac{1}{16F^3}\left[m(m^2-1)-(m-1)^3\left[K_2+\left(\frac{m+1}{m-1}\right)^2\right]\right]\frac{ds_2}{f_1}$$

Here ds_2 is the displacement of M_2, and is negative when M_2 is moved towards M_1, and gives positive SA.

7.4 Examples

Fig. 4 shows the above run of TC due to decentering and SA with the error in positioning of M_2 for the telescope with parameters given in Section 6. It is seen that the tolerances are fairly strict: even a 1 mm decentering gives rise to a large amount of coma. Fig. 5 is the same as Fig. 4, except that here we assume that M_1 is faster : F/1.5 instead of F/2 (or m = 6.67 instead of 5): there is a marked increase in the aberrations, underscoring the sensitivity of telescope aberrations to m. Also note that the D-K is relatively insensitive to decentering and despace, due to its spherical M_2.

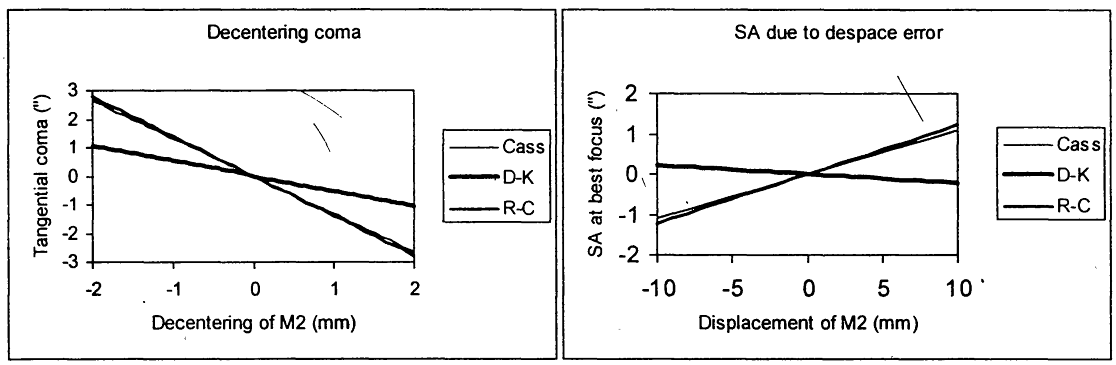

Figure 4. The change of decentering coma and SA due to misalignment (left) and despace (right) for a telescope with parameters given in Section 6. Note the particularly tight tolerance for the centering of M_2 with respect to the optical axis. Again, there is little difference between the tolerances for a classical and R-C telescope.

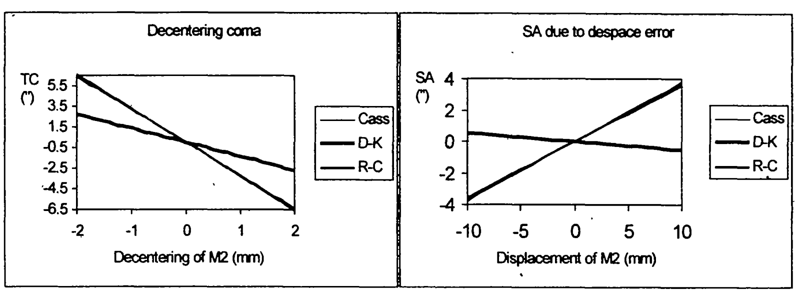

Figure 5. Same as Figure 4, for an F/10 telescope, but with a faster M1 of F/1.5 instead of F/2, i.e. with m=6.67. Note the strong increase in the aberrations.

8. Measuring the aberrations using the SH wavefront sensor

Till now we have discussed the theoretical aspects of aberrations. Now we discuss how we can actually measure them using a wavefront sensor.

Among the aberrations listed above, the field aberrations are fixed once the telescope design has been decided, and nothing can be done about them, except by designing auxiliary optics like field correctors.

However, the other aberrations due to despace error can be minimized by proper maintenance of the telescope.

As Figures 4 and 5 show, the maintenance tolerances are fairly strict. Thus a method is needed to measure these aberrations, and then to correct them. In the following sections we discuss the Shack-Hartmann system, which is becoming increasingly popular all over the world for analyzing telescopes. Using practical examples, we will show that a SH system it is possible not only to have an accurate estimate of coma, astigmatism and spherical aberration, but also to identify mirror support errors.

8.1 Deflection in the focal plane : The equations of SH analysis

The angular deflection θ in the focal plane due to an aberrated wavefront W is given by:

$$\theta = \frac{1}{R_m} \frac{dW(r, \phi)}{dv}$$

where v (r, φ) is a normalized variable, and Rm is the radius of the mirror.

For a telescope of focal length f, the corresponding linear shift δ is given by :

$$\delta = f\theta$$

or

$$\delta = 2\, F \frac{dW\ (r, \phi)}{dv}$$

where F is the focal ratio of the telescope.

The deviation is mapped on to the CCD detector as :

$$\delta_{CCD} = 2F \frac{f_{lenslet}}{f_{coll}} \frac{dW(r, \phi)}{dv}$$

Here f_{coll} is the focal length of the collimating lens which images the telescope pupil on the

SH lenslet array, each of whose lenslets has focal length $f_{lenslet}$

The deflection in the x-direction, or the x-derivative at any point (r, φ), can be written as,

$$\delta_x = \frac{\partial W(r,\varphi)}{\partial x} = \frac{\partial W}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial W}{\partial \varphi} \frac{\partial \varphi}{\partial x}$$

or

$$\delta_x = \frac{\partial W}{\partial_r} \cos \varphi - \frac{\partial W}{\partial \varphi} \frac{\sin \varphi}{r}$$

Similarly, in the y-direction

$$\delta_y = \frac{\partial W(r,\varphi)}{\partial y} = \frac{\partial W}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial W}{\partial \varphi} \frac{\partial \varphi}{\delta y}$$

or

$$\delta_y = \frac{\partial W}{\partial r} \sin \varphi + \frac{\partial W}{\partial \varphi} \frac{\cos \varphi}{r}$$

For any optical element, δ_x and δ_r are the measured deviations on the CCD, and the parameters $F_{flenslet}$ and f_{coll} are known. If we fix the form of $W(r, \phi)$, then the coefficients of this aberration function can be computed using a least-squares fit. It is usual to express the wavefront aberration in terms of the Zernike polynomials (Bhatia, Ciani and Rafanelli 1994).

It is important to emphasize that δ_x and δ_y contain all the information regarding the telescope, i.e. not only the effects of low-order aberrations, but also other effects, e.g. due to air currents, mirror deformation etc. Once the dominant effects of lower order aberrations are removed, it is possible to see the other effects.

9. The SH wavefront sensor

The general purpose wavefront sensor, which can test telescopes from F/5 to F/35 is shown above. The instrument is in use at a number of observatories: the Telescopio Nazionale Galileo, the European Southern Observatory, Munich, the Institute of Astrofisica, Tenerife, and the Observatory the Capodimonte, Naples.

A PC controls the instrument. The analysis software is run from a graphical user interface under Windows 95/NT, and also controls all the functions of the instrument (stepper motor, reference source intensity, CCD etc).

The analysis section of the software has the following functions:

Output from analysis based on classical Zernike polynomials : Focus, coma, spherical aberration, astigmatism, and other higher order aberrations (up to 43 terms); 80 % EE, wavefront and Strehl ratio: MTF from spot diagrams.

Graphical output : Spot diagrams, EE profile and distribution of residuals over the pupil; 3D and contour plot of wavefront.

Diagnostics: Given the telescope parameters, the software suggests the changes required to correct defocus, coma and spherical aberration. In addition, mirror support problems can be identified from the contour plots of the wavefront after subtracting out the lower-order aberration terms.

Reducing noise due to air effects : Both at the telescope and in the laboratory, air effects can limit the accuracy of the analysis: the software has the facility for averaging coefficients from multiple frames to improve the accuracy.

MTF, EE and PSF from diffraction theory : It is useful to be able to compare the results obtained from SH analysis with theoretical ones obtained from diffraction analysis. For this, the software has the following options to obtain MTF, EE and PSF: (i) for a perfect mirror, aberrated mirror, ripple and micro-ripple on the mirror surface. (ii) for pixel, seeing, pointing error of the telescope. (iii) Any combination of the above.

Design of telescope using analytical theory: It is often useful to have a quick look at a particular telescope design, without having to do ray-tracing. Given 4 parameters, the program prints out the characteristics of a telescope (Classical, Ritchey-Chretien, Gregorian, Dall-Kirkham and inverted Cassegrain) in terms of : (i) Sensitivity of the telescope due to misalignment (decentering and tilt) of the secondary mirror. (ii) Sensitivity of telescope to despace error between primary and secondary mirrors. (iii) Field aberrations.

10. Some examples of the results obtained from a SH sensor

We have used this instrument to analyze the 2.5m Nordic Optic Telescope (NOT) in the Canary islands. We present some results below.

10.1 The actual wavefront of the telescope and the contour plot: Evidence for decentering coma

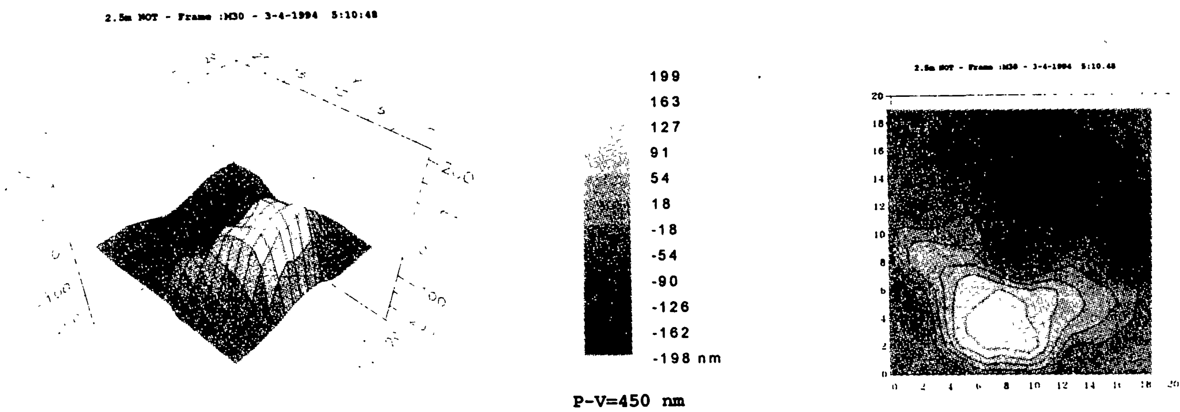

Figure 6. The figures above show that M_1 and M_2 were not properly aligned, giving rise to a wavefront form, which is typical of coma. The aberration can be corrected by properly aligning the two mirrors. The software suggests how the corrections can be achieved.

10.2 The wavefront after mathematical subtraction of 7 low-frequency aberration terms: evidence for the presence of support problems

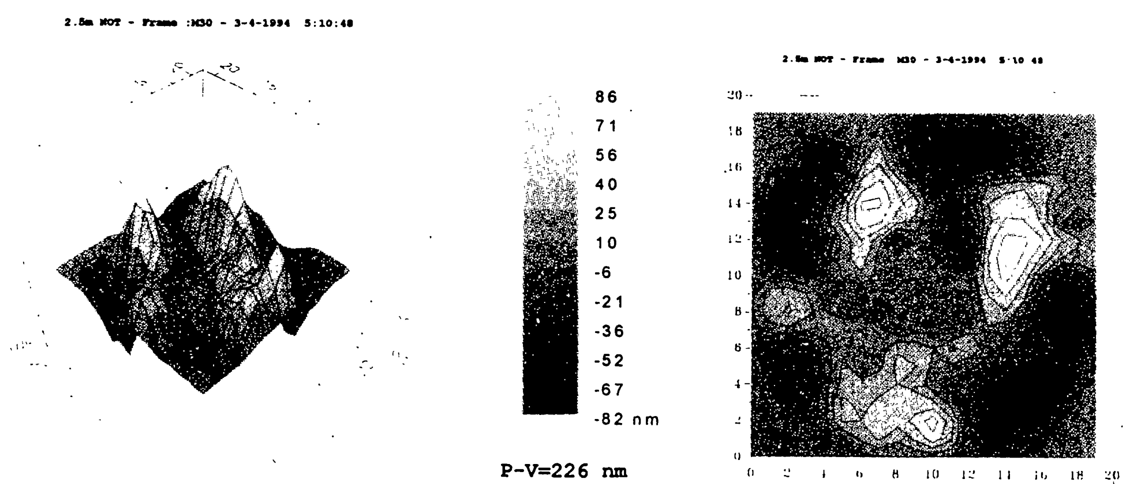

Figure 7. The above wavefronts were obtained from those of Fig.6 after mathematically subtracting out the 7 lower order Zernike aberrations (defocus, tilt, coma, spherical aberration, triangular coma and quadratic astigmatism): the presence of support problems is clearly seen: the wavefront should otherwise have had very little structure. Using these plots the mirror supports can be adjusted.

Support problems can give rise to astigmatism, triangular coma and quadratic astigmatism. Triangular coma, for example, can give triangular images, and is caused by the underloading or overloading of the 3 fixed points of the support system of M_1.

10.3 The Zernike coefficients

Term	Coeff (NM)	Error	Coeff (Arsec)	Error	Aangle (DEG)	Error
Def	683.0	12.2	1.72	0.03	–	–
Coma	91.8	10.3	0.38	0.04	33.7	0.1
SA	2.8	4.9	0.02	0.03	–	–
AST	42.5	18.1	0.07	0.03	-42.1	10.1
Tri. coma	13.3	15.8	0.04	0.04	-11.5	4.2
Quad. AST	49.0	10.9	0.20	0.04	-17.7	1.5 10.3

The above table shows the output from the SH analysis for the NOT telescope. Coma and quadratic astigmatism have values (0.38 and 0.20 arcsecs respectively) which show that the telescope had decentering coma and support problems. This is confirmed by the wavefronts shown in Fig.6 and Fig.7.

10.4 The encircled energy profile

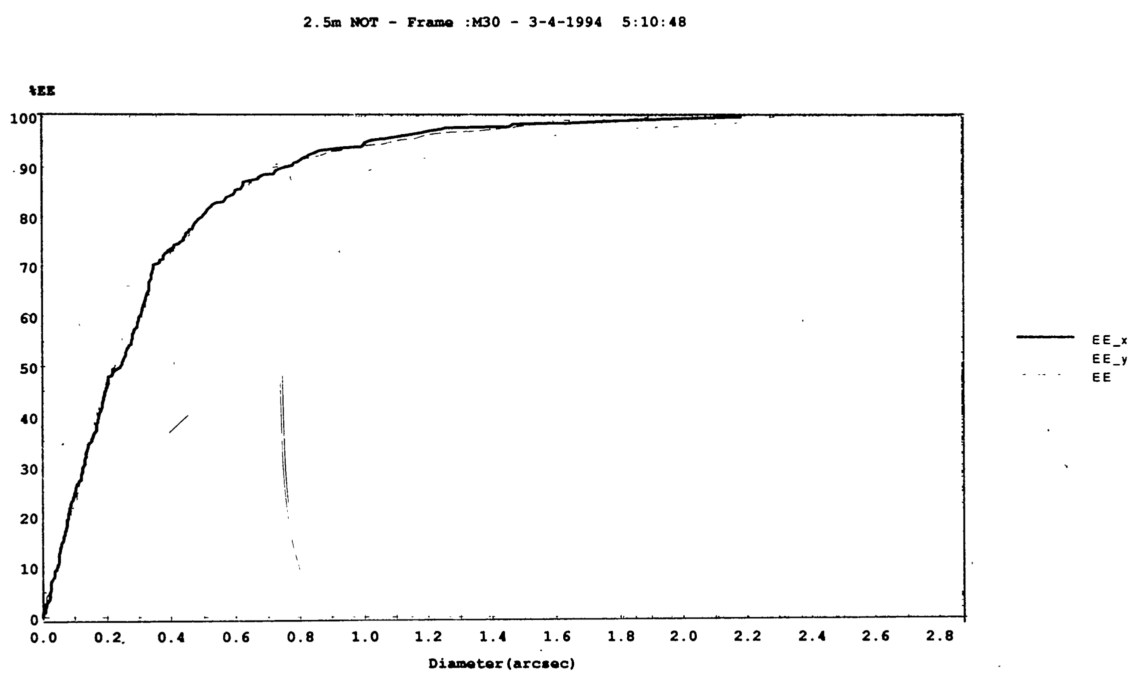

Figure 8. The above figure shows the observed Encircled Energy profile of the telescope after the mathematical subtraction of tilt and defocus: the 80% Encircled Energy is within 0.7 arcseconds. Air effects worsened the actual quality of the telescope.

10.5 The spot diagram

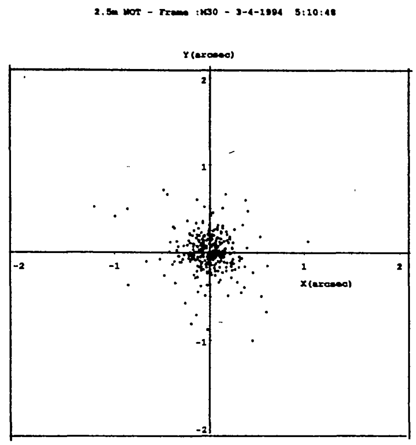

Figure 9. The above figure shows the observed spot diagram of the telescope after the mathematical subtraction of tilt and defocus. Air effects worsened the actual quality of the telescope.

11. Conclusions

In this paper we have discussed the aberration characteristics of different types of two-mirror telescopes, and brought out the characteristics of the Classical, Ritchey-Chretien and Dall-Kirkham telescopes. It is seen that a Ritchey-Chretien telescope is the best choice for a modern telescope, as it has two of the aberrations corrected: spherical aberration and coma. However, such a telescope still needs auxiliary optics to compensate for the effects of astigmatism and field curvature if it has to be used for wide-field imaging.

Theoretical computations show that image quality can be easily degraded by the misalignment of the primary and secondary mirrors, and that it is crucial to maintain the alignment.

While a good telescope design is necessary, equally important is the maintenance of these telescopes. To this end we have shown that it is necessary to have a wavefront sensor to understand in detail the problems of the telescope: e.g. misalignment, wrong spacing between the mirrors, and support problems. Once these problems have been identified, corrective action can be taken. In effect, the telescope can consistently give image quality for which it was designed.

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