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Tidal disruption in colliding galaxies

K. S. V. S. Narasimhan *Centre of Advanced Study in Astronomy, Osmania University, Hyderabad 500 007 and Department of Mathematics, The New College, Madras*

Saleh Mohammed Alladin *Centre of Advanced Study in Astronomy, Osmania University, Hyderabad 500 007*

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Abstract. Disruptive effects in colliding galaxies due to their mutual tidal interaction are compard under the impulsive approximation. If M_1 and M_2 are the masses and ρ_1 and ρ_2 the densities of the two galaxies, M_2 suffers greater change in structure than M_1 if $M_1/M_2 > (\rho_2/\rho_1)^{0.5}$. Since the observations suggest an empirical mass-density relation for galaxies of the from $M \propto R^n$ with n between 1 and 2, the more massive and hence the larger galaxies are less dense. It follows that in a hyperbolic encounter between two galaxies, the smaller galaxy suffers greater disruption in spite of its higher density.

Key words : galaxies : collisions—tidal force—stellar dynamics

1. Introduction

When two galaxies collide, the tidal forces exerted by them on the constituent stars generally accelerate the stars with the result that the internal energy (binding energy) U of the galaxies increases at the expense of the external energy (orbital energy) E of the two galaxies. This phenomenon of transfer of energy from the orbital motion of the galaxies to the internal degrees of freedom has been extensively studied. The ratio $\Delta U/|U|$ where ΔU is the increment in the binding energy of the galaxy provides a convenient order-of-magnitude estimate for the disruptive effects of the tidal forces (Alladin 1965). $\Delta U/|U| \geqslant 1$ gives the criterion for the disruption of a galaxy, since according to Jacobi's criterion of stability, a stellar system with positive energy is unstable (Kurth 1957). However, $\Delta U/|U| = 1$ does not necessarily imply the total disruption of a galaxy; a significant portion of its mass may still be bound. Dekel *et al.* (1980) have discussed the dependence of $\Delta U/|U|$ on the collision parameters and have given for two different models of galaxies the relationship between $\Delta U/|U|$ and $\Delta M/M$ where ΔM is the mass of the escaping matter from the galaxy of mass M.

In this paper we study the disruptive effects due to their mutual tidal interactions in colliding galaxies for different mass and density distributions under the impulsive approximation (Spitzer 1958) which is good in the case of fast collisions.

2. Basic equations

Alladin & Parthasarathy (1978) obtained the energy transfer in spherically symmetric, double galaxies under the impulsive approximation. When the relative motion of the two galaxies is a conic of eccentricity e, we have

$$(\Delta U_2)_{sh} = \frac{\pi^2}{(1+e)^2} \frac{G^2 M_1^2 M_2}{p^4 V_p^2} (R_h^2)_2, \qquad \ldots(1)$$

where M_1 and M_2 are the masses of the perturber and the test galaxy respectively, V_p is the relative velocity at minimum separation p of the galaxies and R_h is the half-mass radius of the test galaxy. $(\Delta U)_{sh}$ is the value of ΔU got by considering only those stars that are situated at the median radius. For a polytropic distribution of index $\nu = 4$, $\Delta U / (\Delta U)_{sh} \approx 2$.

The binding energy U of a test galaxy before the encounter is given by

$$U = -\beta \frac{GM^2}{2R_h}, \qquad \ldots(2)$$

where β depends slightly on the chosen model. For a polytropic sphere of index ν varying from 0 to 4, β varies from 0.5 to 0.4.

From equations (1) and (2) we get

$$\mu_2 = \frac{(\Delta U_2)_{sh}}{|U_2|} = \frac{2}{\beta_2} \frac{\pi^2}{(1+e)^2} \frac{G M_1^2}{M_2 p^4 V_p^2} (R_h^3)_2. \qquad \ldots(3)$$

Hence

$$\mu_{21} \equiv \frac{\mu_2}{\mu_1} = \frac{\beta_1}{\beta_2} \frac{M_1^3}{M_2^3} \frac{(R_h^3)_2}{(R_h^3)_1} \qquad \ldots(4)$$

$$= \frac{\beta_1}{\beta_2} \frac{M_1^2}{M_2^2} \frac{\rho_1}{\rho_2} \equiv \beta_{12} M_{12}^2 \rho_{12} \qquad \ldots(5)$$

where ρ is the density within the sphere of radius R_h. If the same model is used for both the galaxies $\beta_{12} = 1$. Sastry (1972) studied the case $\beta_{12} \neq 1$.

Ahmed's (1979) extension of Toomre's (1977) analysis of a head-on collision between galaxies represented by Plummer's model (polytrope $\nu = 5$) leads to

$$\Delta U_2 = \frac{G^2 M_1^2 M_2}{V^2 \alpha_1^2} B(\alpha_{12}) \qquad \ldots(6)$$

and

$$\mu_{21} = M_{12}^3 \alpha_{21}^3 \qquad \ldots(7)$$

where $\alpha_{12} = \alpha_1/\alpha_2$, α_1 and α_2 being the scale lengths of M_1 and M_2 respectively and $B(\alpha_{12})$ is a function of α_{12}. In a Plummer's model, $M = \frac{4}{3}\pi\rho_c\alpha^3$ where ρ_c is the central density and hence

$$\mu_{21} = M_{12}^2 (\rho_{c1})_2 = M_{12}^2 \rho_{12} \quad \ldots(8)$$

where $(\rho_c)_{12} \equiv (\rho_c)_1/(\rho_c)_2$ and ρ_{12} is the ratio of the densities within R_h.

It can be seen from equations (5) and (8) that μ_{21} varies as the product $M_{12}^2\, \rho_{12}$ in the case of non-penetrating as well as head-on collisions.

3. Discussion

It is obvious from equations (5) and (8) that when one galaxy is more massive and more dense than the other, the less massive and less dense galaxy will suffer greater disruption. Generally, we encounter situations in which the more massive one is less dense i.e. if $M_1 > M_2$, then $\rho_2 > \rho_1$. Hence, $M_{12} > \rho_{12}$. It follows from the above equations that the dependence of μ_{21} on the mass ratio is stronger compared to the density ratio. When the galaxies are equally dense, the dependence of μ_{21} on M_{12} is parabolic and the less massive galaxy suffers greater disruption. On the other hand, when the galaxies are equally massive, the dependence of μ_{21} on ρ_{12} is linear and the less dense galaxy is disrupted more. Further, $\mu_{21} \gtrless 1$ according as $M_{12} \gtrless (\rho_{21})^{1/2}$. This implies that the less massive and more dense galaxy will suffer greater disruption when $M_{12} > \sqrt{\rho_{21}}$; when $M_{12} < \sqrt{\rho_{21}}$ the more massive and the less dense galaxy will disrupt more and both the galaxies will be equally disrupted when $M_{12} = (\rho_{21})^{1/2}$. The values of $\Delta U/|U|$ given in Sastry & Alladin (1979) for head-on collisions between spherical polytropic model galaxies of unequal dimensions with relative velocity at closest approach equal to the critical velocity of escape satisfy the above criteria, as their results roughly summarized in table 1 show.

Table 1. Disruptive effects in a head-on collision with relative velocity at closest approach equal to the critical velocity of escape

M_{12}	ρ_{12}	μ_2	μ_1	$\log \mu_{21}$	Remarks
10^6	10^{-3}	1.6×10^{-2}	4×10^{-12}	9.6	$M_{12}>(\rho_{21})^{1/2}$
10^4	10^{-2}	1.2×10^{-1}	4×10^{-8}	6.5	,,
10^2	10^{-1}	9×10^{-1}	4×10^{-4}	3.4	,,
1	1	5.5×10^{-1}	5.5×10^{-1}	0.0	$M_{12}=(\rho_{21})^{1/2}$
10^{-2}	10	4×10^{-4}	9×10^{-1}	-3.4	$M_{12}<(\rho_{21})^{1/2}$
10^{-4}	10^2	4×10^{-8}	1.2×10^{-1}	-6.5	,,
10^{-6}	10^3	4×10^{-12}	1.6×10^{-2}	-9.6	,,

We shall examine these criteria in the light of the observed properties of galaxies. From the empirical statistical relationship between the potential energy and the mass of a bright elliptical galaxy obtained by Fish (1964), it follows that $M \propto R_e^2$ where M is the mass of the galaxy and R_e is the effective radius inside which half the total light is emitted (Freeman 1975). Aarseth & Fall (1980) have pointed out that photometric studies on elliptical galaxies made by Kormendy (1977) and Strom & Strom (1978) lead to mass-radius relations $M \propto R_e^{1.43}$ and $M \propto R_e^{1.67}$ respectively. We consider the relation $M \propto R_e^{1.50}$ which is intermediate between the Fish's law and the others. The implications of equations (5) and (8) under different mass-radius relations

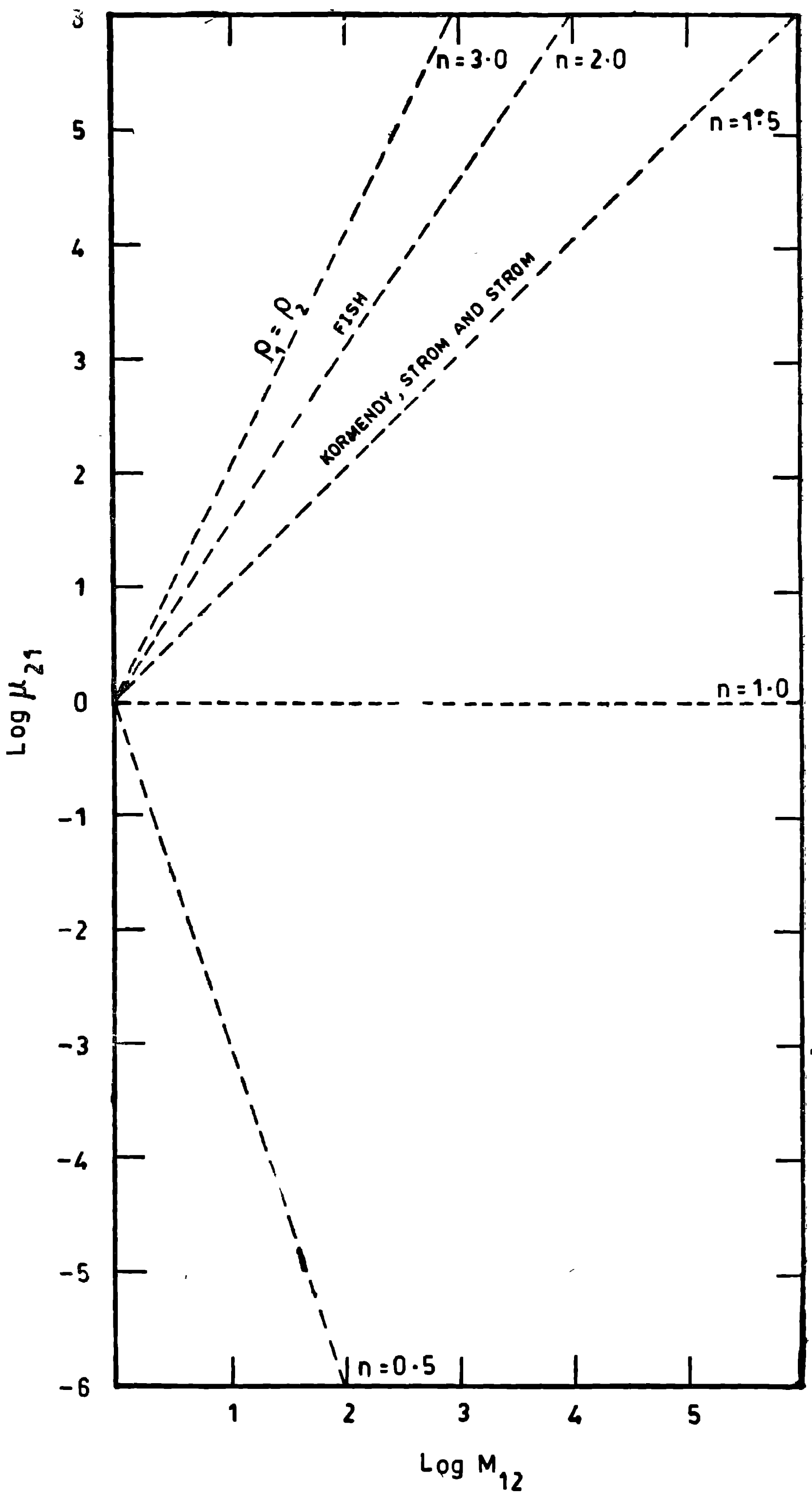

Figure 1. Dependence of μ_{21} on M_{12} for some $M \propto R_h^n$ relations.

are given in table 2 wherein the ratios $(\Delta U)_{21} = \Delta U_2/\Delta U_1$ and $U_{21} \equiv U_2/U_1$ are also incidentally indicated. Figure 1 highlights the dependence of μ_{21} on M_{12} in the case of some of the $M \propto R_h^n$ relations considered.

Table 2. Effects of mutual disruption for various mass-radius relations

M–R relation	M_{12}–ρ_{21} relation	$\mu_{21} = M_{12}^2\,\rho_{12}$	$(\Delta U)_{21} = M_{12}\rho_{12}^2)^{1/3}$	$U_{21} = (M_{12}^{-5}\,\rho_{21})^{1/3}$
$M \propto R_h^{0\cdot5}$	$M_{12}^5 = \rho_{21}$	$M_{12}^{-3\cdot00}$	$M_{12}^{-3\cdot00}$	$M_{12}^{0\cdot00}$
$M \propto R_h^{1\cdot0}$	$M_{12}^2 = \rho_{21}$	$M_{12}^{0\cdot00}$	$M_{12}^{-1\cdot00}$	$M_{12}^{-1\cdot00}$
$M \propto R_h^{1\cdot5}$	$M_{12} = \rho_{21}$	$M_{12}^{1\cdot00}$	$M_{12}^{-0\cdot33}$	$M_{12}^{-1\cdot33}$
$M \propto R_h^{2\cdot0}$	$M_{12} = \rho_{21}^2$	$M_{12}^{1\cdot50}$	$M_{12}^{0\cdot00}$	$M_{12}^{-1\cdot50}$
$M \propto R_h^{2\cdot5}$	$M_{12} = \rho_{21}^5$	$M_{12}^{1\cdot80}$	$M_{12}^{0\cdot20}$	$M_{12}^{-1\cdot60}$
$M \propto R_h^{2\cdot9}$	$M_{12} = \rho_{21}^{29}$	$M_{12}^{1\cdot97}$	$M_{12}^{0\cdot31}$	$M_{12}^{-1\cdot66}$
$M \propto R_h^{3\cdot0}$	ρ = constant	$M_{12}^{2\cdot00}$ $(\rho_1 = \rho_2)$	$M_{12}^{0\cdot33}$	$M_{12}^{-1\cdot67}$

It is clear from the figure that the observed properties of galaxies are such that always the less massive one suffers more disruption in spite of its higher density. In a rich cluster of galaxies the collisions are generally hyperbolic. Therefore the present analysis implies that the smaller galaxies in it undergo greater changes in structure.

It may be noted from table 2 that in the case of the observed mass-radius relations $1 < n < 2$, although the $\Delta U/|U|$ of the less massive galaxy is greater, its ΔU is smaller. Since the change in the orbital energy of the pair is $\Delta E = -(\Delta U_1 + \Delta U_2)$, it follows that the energy gain of the larger galaxy contributes more to the dynamical friction.

As probable examples of systems in which the less massive galaxy is undergoing tidal disruption in a galactic encounter, mention may be made of VV 117 (= Arp 143) and VV 123 (= Arp 141) (Limber 1965, Gallagher *et al.* 1981), LMC and the Galaxy (Tremaine 1976), NGC 3310 (Balick & Heckman 1981) and NGC 1275 (Metik & Pronik 1979).

4. Conclusion

Observations indicate that generally larger galaxies are less dense. The main conclusion of this paper is that in a hyperbolic collision between two unequal galaxies, the smaller one despite its higher density suffers greater change in structure due to the disruptive effects of the tidal forces.

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References

Aarsetoh, S. J. & Fall, S. M. (1980) *Ap. J.* **236**, 43.
Ahmed, F. (1979) *Ap. Sp. Sci.* **60**, 493.
Alladin, S. M. (1965) *Ap. J.* **141**, 768.
Alladin, S. M. & Parthasarathy, M. (1978) *M.N.R.A.S.* **184**, 871; corrigendum : **188**, 303.
Balick, B. & Heckman, T. (1981) *Astr. Ap.* **96**, 271.
Dekel, A., Lecar, M. & Shaham, J. (1980) *Ap. J.* **241**, 946.
Fish, R. A. (1964) *Ap. J.* **139**, 284.
Freeman, K. C. (1975) in *Galaxies and Universe* (eds : A. Sandage, M. Sandage & J. Kristian), University of Chicago Press, p. 429.
Gallagher, J. S., Knapp, G. R. & Faber, S. M. (1981) *Astr. J.* **86**, 1781.
Kormendy, J. (1977) *Ap. J.* **218**, 333.
Kurth, P. (1957) in *Introduction to the Mechanics of Stellar Systems*, Pergaman Press, p. 65.
Limber, D. N. (1965) *Ap. J.* **142**, 1346.
Metik, L. P. & Pronik, I. J. (1979) *Ap.* **15**, 27.
Sastry, K. S. (1972) *Ap. Sp. Sci.* **16**, 284.
Sastry, K. S. & Alladin, S. M. (1979) *Ap. Sp. Sci.* **63**, 203.
Spitzer, L. (1958) *Ap. J.* **127**, 17.
Strom, K. M. & Strom, S. E. (1978) *Astr. J.* **83**, 1293.
Toomre, A. (1977) in *The Evolution of Galaxies and Stellar Populations* (eds : B. M. Tinsley & R. B. Larson) Yale University Observatory, p. 401.
Tremaine, S. D. (1976) *Ap. J.* **203**, 72.