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ON ZERO-POINT FLUCTUATIONS, THE COSMOLOGICAL CONSTANT, AND THE GRAVITON MASS

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ABSTRACT

Recently attempts have been made to link vacuum zero-point fields (ZPF), with a nonzero cosmological constant (Λ), which is now treated as a cosmological free variable to be determined by observations. In another recent paper, Λ is related to a graviton mass. This is shown to be incorrect. Flat space propagators for both massless and massive spin-2 particles can be written (independently of gravity) in the context of flat space wave equations. However, they do not correspond to full general relativity with a graviton mass.

Subject headings: cosmology: theory — relativity

1. INTRODUCTION

In recent work, e.g., Rueda, Haisch, & Cole (1995), it has been suggested that the so-called vacuum zero-point fields (ZPF; also known as zero-point fluctuations) may have something to do with cosmological formations such as voids. Moreover, several authors have suggested a connection between the ZPF and the cosmological constant, Λ, which in the Einstein field equations represents the vacuum energy density, $\Lambda g_{\mu\nu}$.

In another recent paper, Michel (1996) has, however, pointed out that the ZPF, not being Lorentz-invariant, cannot represent the Lorentz-invariant energy momentum tensor of the form $\Lambda g^{\mu\nu}$ associated with the cosmological constant. Again, insofar as the Casimir effect is a manifestation of the ZPF, this would imply that Λ was different between the conducting plates (by a huge factor, since we know Λ outside is almost zero), contradicting the fundamental geometrical identity of the Einstein tensor $G^{\mu\nu}$, i.e. $\nabla_\mu G^{\mu\nu} = 0$, which must be shared by anything to which it is equated, such as $\Lambda g_{\mu\nu}$, this in turn implying the strict constancy of Λ in space and time. Although this result appears essentially correct, the other statement made in the paper of Michel (1996), that the inverse length implied by a nonzero Λ corresponds to the graviton mass squared is incorrect and misleading. It may be of interest to note in this connection that the origin of this often-made error goes back to Einstein's initial paper (Einstein 1917), where he introduced the cosmological constant. The fourth section of Einstein's paper "Über ein an den Feldgleichungen der Gravitation anzubringende Zustzglied [About an Additional Term to Be Fixed to the Field Equations of Gravitation]" introduced the cosmological constant Λ. Instead of his field equation $G_{\mu\nu} = k(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$, he now suggested the equation $G_{\mu\nu} - \Lambda g_{\mu\nu} = -k(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$. The motivation for this new constant was that this new extension was completely analogous to extension of the Poisson equation $\nabla^2\phi - \Lambda\phi = 4\pi k\rho$. This modified Poisson equation is nothing but the static Yukawa meson equation with the spherically symmetric solution:

$$\phi = \frac{\text{constant}}{r}\, e^{-r\Lambda^{1/2}}, \quad \Lambda = \left(\frac{m_g c}{\hbar}\right)^2 , \tag{1}$$

m_g being the graviton mass.

Neumann (1896) had indeed proposed just such a modified Poisson equation to introduce an exponential cutoff for the gravitational potential, and Einstein (1917) was apparently unaware of this. While it is true that the Poisson equation modified by a term $-\Lambda\phi$ leads to an exponential cutoff, for the gravitational potential suggesting a graviton mass, Einstein's assertion that the Λ-term in his field equation had the same effect was completely wrong. The original erroneous assertion of Einstein (which was probably his real blunder, rather than just introducing the Λ-term in the field equations!) has continued to lead to misleading statements of the same kind. For instance, Pais (1982) in his Einstein biography writes about the analogy between the Λ-term in Poisson's and Einstein's equations: "he [Einstein] performs the very same transition in general relativity." Thus the point is that the Newtonian weak-field limit of Einstein's equations with a Λ-term is not the modified Poisson equation $\nabla^2\phi + \Lambda\phi = 4\pi K\rho$ but

$$\nabla^2\phi + \Lambda c^2 = 4\pi K\rho . \tag{2}$$

Thus equation (2) does not introduce any exponential cutoff with a graviton mass, but a new repulsive force ($\Lambda > 0$), proportional to mass, pulling away every particle with an acceleration

$$a = c^2\,\frac{\Lambda}{3}\,x , \tag{3}$$

a force derivable from a repulsive oscillator potential

$$\phi = -\Lambda c^2 r^2/6 . \tag{4}$$

Equation (4) rather than equation (1) is the correct solution to the field equations with a Λ-term.

Thus, in the paper of Michel (1996), equation (15), i.e.,

$$\Box h_{\mu\nu} + 2\lambda h_{\mu\nu} = 0 , \tag{5}$$

the linear field equations with a Λ-term cannot be correctly interpreted as implying a graviton mass with its corresponding solution as an exponentially decreasing Yukawa potential. On the contrary, it implies that spatial scales comparable to $\Lambda^{-1/2}$ are not flat, and physically this gives rise to an oscillator-type potential increasing with distance as r^2 as given by equation (4).

Thus it does not lead to an exponential decrease in potential suggesting a graviton mass; rather it gives rise to a confining potential of the simple harmonic type growing with distance as $\phi \simeq \frac{1}{6}\Lambda r^2 c^2$. Moreover, one cannot linearize around a flat spacetime ($g_{\mu\nu} = \eta_{\mu\nu}$) in the presence of a Λ-term; the Minkowski spacetime is replaced by a de Sitter spacetime of constant curvature. One can thus no longer define mass in the wave equations as corresponding to a Lie invariant associated with the Poincaré algebra. So equation (5) is erroneous. The sign of Λ has nothing to do with a quantity like the graviton mass; it just signifies different geometries; a positive Λ corresponds to a de Sitter space corresponding to the SO (4, 1) group, while a negative Λ is an anti–de Sitter space related to the group SO (3, 2).

It has been argued that in the linearized gravitational theory contrary to that of the electromagnetic case, we cannot vary the graviton mass to get the massless limit. (Van Dam & Veltman 1970). This is because, in the lowest order in the gravitational coupling constant G, the effect of the interaction between two particles A and B is given by $\eta_{\mu\nu} = (-1, 1, 1, 1) \sim GT_A^{\mu\nu} D_{\mu\nu\lambda\rho} T_B^{\lambda\rho}$, in which $T_A^{\mu\nu}$ and $T_B^{\mu\nu}$ are conserved energy-momentum tensors of A and B, respectively, and the propagator is given by

$$D_{\mu\nu\lambda\rho} = \frac{1}{2k^2} (\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\nu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}) , \tag{6}$$

k being the graviton momentum. However, when the graviton has a finite mass m, and is described by the Pauli-Fierz Lagrangian, the propagator turns out to be (see, e.g., Mandelstam 1968; Van Dam & Veltman 1970)

$$D_{\mu\nu\lambda\rho} = \frac{1}{2(k^2 + m^2)} \left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \frac{2}{3}\eta_{\mu\nu}\eta_{\lambda\rho} \right) . \tag{7}$$

Equations (6) and (7) are propagators for massless and massive spin-2 particles in flat space, i.e., equations of the form $\Box\phi_{\mu\nu} = 0$ and $\Box\phi_{\mu\nu} + m^2\phi_{\mu\nu} = 0$. To consistently describe gravity with its equivalence principle of universal coupling, one must self-couple the field nonlinearly ad infinitum. For this one has to add the energy momentum tensor for these fields on the right-hand side as source terms, which in turn will modify the field to $\phi'_{\mu\nu}$, which in turn has a corresponding energy momentum tensor which again should be added to the source term, and so on.

In the case of massless spin-2 gravitons, this procedure of adding all self-interactions consistently is known to lead to Einstein's general relatively (GR) (see, e.g., Feynman 1963, 1971).

However, GR entails a massless graviton and therefore an infinite-range gravitational interaction, this being a good description of the force consistent with observations. The only known correction to standard GR is a Λ-term, and, as noted above, this definitely does not introduce a graviton mass. Modifications of standard GR, which include for instance higher derivative curvature terms as contained in an action like the following one:

$$I = \int \sqrt{-g}(\alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2 + \gamma\chi^{-2}R) d^4x ,$$

where $\chi^2 = 16\pi G$, α, β, γ are dimensionless numbers, are known to have static linearized solutions of the field equations which are combinations of Newtonians and Yukawa potentials, m, as shown for instance by Stelle (1978).

The equivalent Poisson equation, would now be of the type $\alpha\nabla^4\phi + \beta\nabla^2\phi = \chi m\delta^3(r)$, which would in general give a finite-range solution given by $\phi \simeq a/r - (b/r) \exp(-\lambda/r)$, corresponding to a graviton mass. But this would be a different theory from standard classical GR, which cannot accommodate such a finite graviton mass or a finite-range force.

Thus in conclusion, the Λ-term in Einstein's equation does not imply a finite range for gravitation (there is no Yukawa-type solution) with a corresponding graviton mass.

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