Mon. Not. R. Astron. Soc. **371,** 303–311 (2006) doi:10.1111/j.1365-2966.2006.10651.x

Theoretical study of partial frequency redistribution function in irradiated, moving atmospheres of close binary components

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Accepted 2006 June 2. Received 2006 May 22; in original form 2005 November 19

ABSTRACT

We have studied the effects of partial frequency redistribution function with angle-averaged $R_{\mathrm{II-A}}$ in irradiated and moving atmospheres of close binary components. We have considered the atmospheric extension of the primary component to be twice the radius of the primary component in a close binary system. We have considered two cases: (i) when the atmosphere is at rest and (ii) when the atmosphere is moving. In both the cases, we have computed the line profiles along the line of sight for a given optical depth. The irradiation from the secondary component is assumed to be one, five and 10 times the self-radiation. The line fluxes in the line of sight are calculated by using the total source functions due to self-radiation of the primary component and due to the irradiation from the secondary component. We have noted double-peaked emission lines in the case of a static medium and a reduction of emission peaks in the case of velocity field.

Key words: radiative transfer – binaries: close.

1 INTRODUCTION

It is a well-known fact that in stellar atmospheres, photon redistribution occurs due to scattering during the formation of spectral lines (Hummer 1962, 1969; Shine, Milkey & Mihalas 1975; Vardavas 1976; Mihalas 1978; Peraiah 1978). Complete redistribution at a given frequency point in the line rarely occurs in a stellar atmosphere. Redistribution can occur not only in the frequency but also in the angle. The problem becomes more complicated when radial motions of the gases are taken into account. It is the scattering integral which occurs in the source function that introduces major changes in the photon redistribution in frequency and angle. One must take into account each photon that is scattered on emission and absorption. This is possible only when we consider partial redistribution of the photons scattered into other frequencies and angles. In a spherically symmetric moving medium, the frequency redistribution strongly depends on the angle. This angle dependence of the frequency occurs irrespective of the fact whether we consider angle-dependent or angle-averaged redistribution functions in a moving medium.

In recent years, Peraiah & Srinivasa Rao (1998) and Srinivasa Rao & Peraiah (2000) studied the effects of irradiation on the line formation in the expanding atmospheres of the components of close binary system in dust-free and dusty atmospheres, respectively. They considered two-level atom approximation in non-LTE (local thermodynamic equilibrium) situation and noted that irradiation enhances the emission in the lines and obtained P cygni type profiles with red emission and blue absorption. Peraiah & Srinivasa Rao (2002) extended this work in the case of distortion and expanding atmospheres. In this case, distortion is measured in terms of the ratio of angular velocities at the equator and pole (X), mass ratio of the two components (m_2/m_1), the ratio of centrifugal force to that of gravity at the equator of the primary (f) and ratio of the equatorial radius of the primary to the distance between the centres of gravity of the two components (r_e/R). A seventh degree equation is obtained to describe the distorted surface in a uniform rotation in terms of the above-mentioned parameters. The equation of line transfer is solved in the comoving frame of the expanding atmosphere of the primary using complete redistribution in the line. They used a linear law of velocity of expansion so that the density varies as r^{-3} where r is the radius of the star, satisfying the law of conservation of mass. They found that rotation broadens the line profile, and we also obtained P cygni type of line profiles. Similar calculations were also done in the case of dusty atmosphere by Srinivasa Rao (2003). Srinivasa Rao (2005) extended the work in a case of non-uniform rotation. The work mentioned above was done in the case of complete redistribution of photon. Now, we are interested in studying the effects of partial frequency redistribution (PRD) in close binaries.

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In Section 2, we have calculated the self-radiation of the primary component. Section 3 deals with the method of obtaining the irradiation from the secondary component. Sections 4 and 5 show the calculation of total source function and their fluxes, respectively. In Sections 6 and 7, we discuss the results and the conclusions, respectively.

2 CALCULATION OF SELF-RADIATION (S_s) OF THE PRIMARY COMPONENT

The equation of line transfer is solved in the (x, μ, r) system using the discrete theory of radiative transfer (Peraiah 1984). The frequency range in the observer's frame is from $(-x - \mu V)$ to $(+x + \mu V)$ where V is the velocity of the moving gas in units of the mean thermal velocity, $x = (\nu - \nu_0)/\Delta\nu$, ν is the frequency, ν_o is the line centre frequency and $\Delta\nu$ is the Doppler width. We will present a general formalism to include complete and partial redistribution functions in spherical symmetry. The equation of transfer for a non-LTE, two-level atom in spherically symmetric atmosphere is given by Peraiah (1978)

$$\mu\frac{\partial I(x,\mu,r)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I(x,\mu,r)}{\partial\mu} = k_{\mathrm{L}}[\beta + \phi(x,\mu,r)][S(x,\mu,r) - I(x,\mu,r)] \quad [\mu \in (0,1)] \tag{1}$$

and for oppositely directed beam

$$-\mu\frac{\partial I(x,-\mu,r)}{\partial r} - \frac{1-\mu^2}{r}\frac{\partial I(x,-\mu,r)}{\partial\mu} = k_{\mathrm{L}}[\beta + \phi(x,-\mu,r)][S(x,-\mu,r) - I(x,-\mu,r)] \quad [\mu \in (0,1)], \tag{2}$$

where all the symbols have their usual meaning like $I(x, \pm\mu, r)$ is the specific intensity of the ray at an angle $\cos^{-1}\mu$ [$\mu \epsilon (0, 1)$] with the radius vector at the radial point r with frequency x. The source function $S_s(x, \pm\mu, r)$ is given by

$$S_s(x,\pm\mu,r) = \frac{\phi(x,\pm\mu,r)S_{\mathrm{L}}(x,\pm\mu,r) + \beta S_{\mathrm{C}}(r)}{\phi(x,\pm\mu,r)+\beta}, \tag{3}$$

where β is the ratio of $K_{\mathrm{c}}/K_{\mathrm{L}}$ of opacity due to continuous absorption per unit interval of x to that of line centre andϕ $(x, \pm\,\mu, r)$ is the line profile function and $S_{\mathrm{C}}(r)$, $S_{\mathrm{L}}(x, \pm\,\mu, r)$ are the continuum and line source functions, respectively. In the following equation, continuum scattering due to electrons is neglected

$$S_{\mathrm{C}} = \rho(r)B[\nu_0, T_e(r)], \tag{4}$$

where ρ is an arbitrary factor which is unity and $B[\nu_0, T_e(r)]$ is the Planck function for frequency ν_0 at temperature T_e. In equation (3), we have retained the functional dependence of $\pm\mu$ in S_{L} as this is needed for angle-dependent redistribution functions given as

$$S_{\mathrm{L}}(x,\pm\mu,r) = \frac{1-\epsilon}{\phi(x,\pm\mu,r)}\int_{-\infty}^{+\infty}\mathrm{d}x'\int_{-1}^{+1}R(x,\pm\mu;x',\mu)I(x',\mu,r)\,\mathrm{d}\mu' + \epsilon B(r). \tag{5}$$

The quantity ϵ is the probability per scattering that a photon is thermalized by collisional de-excitation of the excited states, and is given by

$$\epsilon = \frac{C_{21}}{\left\{C_{21} + A_{21}\left[1-\exp(-h\nu_0/kT)\right]^{-1}\right\}}, \tag{6}$$

where C_{21} is the collisional transition rate from level 2 to 1 and A_{21} is the Einstein spontaneous emission rate for transition from level 2 to 1. The quantities h and k are the Planck function and Boltzmann constant, respectively. Now let us consider angle-dependent PRD function, and the profile function $\phi(x, \mu)$ is given by

$$\phi(x,\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}x'\int_{-1}^{+1}R(x,\mu;x',\mu')\,\mathrm{d}\mu'. \tag{7}$$

To solve equation (1), we will apply discrete space theory of radiative transfer in spherical symmetry. In this method, we represent integration by summation and derivatives by differences. For the sake of completeness, some of the important steps of the derivation for obtaining the solution of equation (1) are given in Appendix A. Now, let us consider the angle-averaged redistribution function $R_{\mathrm{II-A}}$ given by Hummer (1962)

$$R_{\mathrm{II-A}}(x,x') = \frac{1}{\pi^{\frac{3}{2}}}\int_{\frac{1}{2}|\,\overline{x}-\underline{x}\,|}^{+\infty}\mathrm{e}^{-u^2}\left[\tan^{-1}\left(\frac{\overline{x}+u}{a}\right) - \tan^{-1}\left(\frac{\underline{x}-u}{a}\right)\right]\mathrm{d}u, \tag{8}$$

where $\underline{x}$ and $\bar{x}$ are the minimum and maximum of x and x', and a is the damping constant (1.0×10^{-3}) corresponding to pure radiation damping. In a moving medium, we need to compute the four redistribution functions, namely, $R(x + \mu V, x' + \mu' V)$, $R(x + \mu V, x' - \mu' V)$, $R(x - \mu V, x' + \mu' V)$ and $R(x - \mu V, x' - \mu' V)$. For the ϕ profile, we can integrate R over x', but we used Voigt profile. As the medium becomes inhomogeneous in a moving medium and the line centre optical depth is greater than the critical step size τ_{crit} [in principle τ_{crit} depends on the number of angles, the number of frequencies, the velocity gradients, the local velocities, the curvature factors, the line profile functions and the thermalization parameter ϵ; for details see Periah (2002, chapter 8, p. 232)], we need to divide the medium into shells and subshells to satisfy the conditions of stability and non-negativity of the solution. If the shell is halved p times, then the optical depth and the curvature factors in each shell are given by

$$\tau_{SS} = \mathrm{e}^{-p}\tau_s, \tag{9}$$

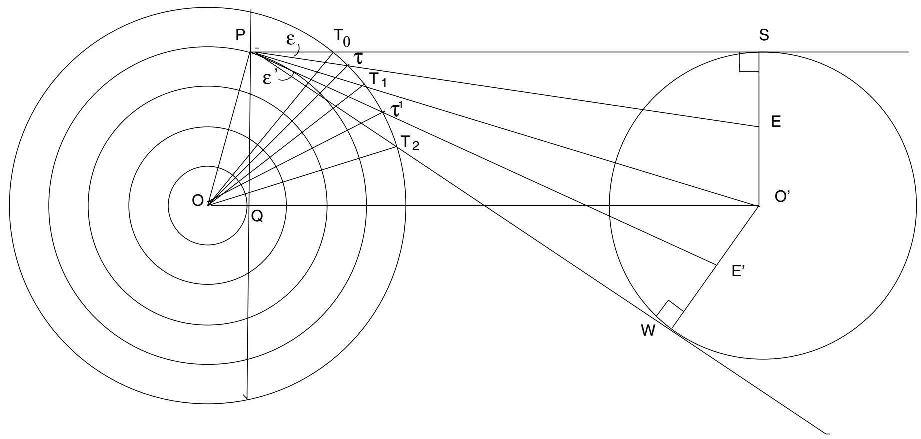

Figure 1. Schematic diagram of binary components with incident radiation from the surface of the secondary.

and

$$\rho_{SS} = \frac{\rho_S 2^{-p}}{[1 - \rho_S(2^{-1} - 2^{-p})]}, \tag{10}$$

where τ_s and ρ_s are the optical depth and curvature factor in each shell, respectively.

3 CALCULATION OF IRRADIATION (S_I) FROM THE SECONDARY COMPONENT

The shapes of the components of the system are assumed to be spherical. For the sake of completeness, the geometry of the model is shown in Fig. 1. Let O and O' be the centres of the primary and the secondary, respectively. The atmosphere of the primary is assumed to be spherical and divided into several discrete shells. We calculate the source functions of the radiation field emerging from the secondary component (see Fig. 1) and incident on the atmosphere of the primary component. We consider the set of rays, such as ST_0P, $E\tau P$, $O'T_1P$, $E'\tau'P$, WT_2P, etc., emerging from the surface SW of the companion and meeting at a point P in the atmospheres of the primary component. These rays lie within the quadrilateral such as $PSO'W$ and enter the boundary of the atmosphere of the component at points $T_0, \tau, T_1, \tau', T_2$, etc. The surfaces of the companion such as SW will be different for different points P in the atmosphere. The radiation field at P is estimated by calculating the source function whose contribution comes from self-radiation of the primary and due to the incident radiation from the surface SW of the secondary facing the primary. We need to estimate geometrical length of the ray segments such as $P\tau$, $P\tau'$, etc. inside the atmosphere so that the transfer of radiation along these segments is estimated and its contribution to the source function at the point P due to the incident radiation at τ, τ', etc. The length of the segments such as $P\tau$ in SPO' is given in appendix A of Peraiah & Srinivasa Rao (1998). So, for a given density distribution we need to calculate the optical depth along the segments $P\tau$, $P\tau'$, etc. The source function at points such as P due to the irradiation is calculated using the one-dimensional rod model given in Wing (1962) and Sobolev (1963). Let us consider segment AB which has two rays oppositely directed to each other (see Fig. 2). The optical depth τ is given by

$$\tau = \tau(X) = -\int_{\mathrm{L}}^{X} \sigma(X')\,\mathrm{d}X'; \quad \tau(0) = T, \tag{11}$$

where σ (X') is the extension coefficient and T is the total optical depth. The optical depth is measured in the direction opposite to that of the geometrical segment [for more details refer to Peraiah & Srinivasa Rao (1998)].

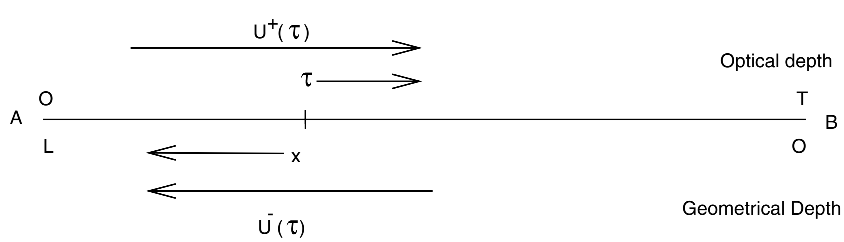

Figure 2. Schematic diagram for the rod model.

4 CALCULATION OF TOTAL SOURCE FUNCTION (S)

Finally, we calculate the total source function by adding self-radiation of the primary star $S_s(x, r)$ and irradiation from the secondary component $S_{\mathrm{I}}(\tau)$ which is calculated using one-dimensional rod model described in Peraiah & Srinivasa Rao (1998) and obtain

$$S = S_S + S_{\mathrm{I}}. \tag{12}$$

5 METHOD OF CALCULATING THE LINE FLUXES

We calculate the set of source functions at the points of intersection of the ray parallel to the line of sight and the shell boundaries as shown in Fig. 1. These source functions are used to calculate the emergent specific intensities at infinity (or at the observer's point) by using the following formula

$$I_{n+1}(r) = I_0(n)\mathrm{e}^{-\tau} + \int_o^\tau S(t)\mathrm{e}^{-[-(\tau-t)]}\,\mathrm{d}t, \tag{13}$$

where $I_n(r)$ corresponds to the specific intensity of the ray passing through between shell numbers n and $n+1$ and corresponding to perpendicular to the axis OO' at different radii. $I_0(n)$ corresponds to the incident intensity at the boundary of the shell and τ is the optical depth in the sector along the path of the ray. The source function $S(t)$ is calculated by linear interpolation between $S(t_n)$ and $S(t_{n+1})$. The specific intensity at the boundary of each shell is calculated by equation (13), and integral is evaluated by Simpson's rule. Finally, we calculate the flux by the integral

$$F(x) = 2\pi \int_A^B I(P, x) P\,\mathrm{d}P, \tag{14}$$

where P is the perpendicular distance from the centre of the star O to the ray along the line of sight.

5.1 Boundary condition

The incident radiation at Q, the bottom of the atmosphere (see Fig. 1), is given as

$$I_s(\tau = T,\ \mu_j) = 1. \tag{15}$$

The incident radiation from the secondary is given in terms of I_s in the ratio of I_s, where I is as given in Peraiah and Srinivasa Rao (1998, equation 28).

5.2 Velocity law

The velocities of expansion are measured in terms of mean thermal units, and uniform expansion velocity law is assumed. If V_A and V_B are the velocities at inner A and outer radius B of the star, respectively, then the velocity at any shell boundary $V_r = V_A + [(V_B - V_A)/(B - A)](r - A)$. This can also be written at inner radius A ($\tau = \tau_{\max} = T$) with the velocity V_A and at outer radius $B(\tau = 0)$ with the velocity V_B.

The velocities of expansion of the gas are expressed in terms of mean thermal units V_T (mtu) given by

$$V_T = \left[\frac{2kT_e}{m_i}\right]^{\frac{1}{2}}, \tag{16}$$

where k is the Boltzmann constant, T is the temperature and m_i is the mass of the ion. The envelope is divided into equal shells, and the velocity at inner radius A of the star is V_A (i.e. $n = 1$ or $\tau = T$) and V_B is the velocity at outer radius B (i.e. $n = 100$ or $\tau = 0$). They are given in terms of mtu or V_T as

$$V_A = \frac{v_A}{V_T} \tag{17}$$

$$V_B = \frac{v_B}{V_T}, \tag{18}$$

where v_A and v_B are the velocities at the inner radius A, outer radius B at radial point r, respectively, in units of mtu of gas. The line profiles are calculated at quadrature, and the line connecting the two stars is perpendicular to the line of sight (see Fig. 1). The line profile fluxes (F_q/F_{c}) are calculated against the normalized frequency q, where

$$q = x/x_{\max} \quad \text{where} \quad x = (\nu - \nu_0)/\Delta\nu, \quad x_{\max} = |x| + V_B, \quad \Delta\nu = \nu_0\frac{v_T}{C}, \tag{19}$$

$$F_q = F(x_q) \tag{20}$$

and

$$F_{\mathrm{C}} = F(x_{\max}). \tag{21}$$

6 RESULT AND DISCUSSION

The line transfer equations (1) and (2) are solved using discrete space theory of radiative transfer as a first part of the calculation and the derivation described in Appendix A.

In the second part of the calculation, the atmosphere of the primary whose centre is at O is divided into N (=100) equal shells as shown in Fig. 1. The separation of the components as r_1/R where r_1 is the outer radius of the primary and R $(= OO')$ is the separation of the centres of gravity of the components. We have considered two cases of separation, $r_1/R = 1/2$ and 1/5, and the atmospheric extension is set to be equal to stellar radius or $B/A = 2$ where B and A are the radii of the outer and inner radii of the atmosphere. For a given model, parameters shown in Table 1, the optical depths along segments like $P\tau$, $P\tau'$ are calculated using one-dimensional rod model described in Peraiah & Srinivasa Rao (1998).

A sample of results are presented in Figs 3 and 4 for different parameters. The figures are self-explanatory as far as the parameters are concerned. Fig. 3 contains the source functions S_s and S $(= S_s + S_I)$ for various parameters shown in the figure across the atmosphere. In Fig. 3, panel A represents the source functions against the radius r for a static medium $V_A = 0$, $V_B = 0$ and $\epsilon = 0$, $\beta = 0$ for incident radiation $I = 1$, 5 and 10. In general, the source function is highly dependant on sphericity, terminal velocity, thermalization parameter ϵ, continuum thermalization parameter β and the line optical depth. The source function S_s represented by the continuous line (which does not include reflected radiation) decreases as the radius of the star increases in the scattering medium with $T = 1 \times 10^4$. In the larger optical depths, the velocity gradient leads to an increase in the escape probability and then liberates the photons, otherwise photons will be trapped in the static atmosphere. With the uniform velocity law, we observe that the source functions decrease slowly towards the outer layers. When the reflected radiation is added S_I, the source function for $r_1/R = 1/2$ (dotted line in Fig. 3) and $r_1/R = 1/5$ (dash–dotted line in Fig. 3) is considerably enhanced as these source functions include the incident radiation from the companion along OO'.

Using the source functions shown in Fig. 3, we have calculated the line profiles and plotted against the normalized frequency points q (equation 19). In the case of static medium, we obtained symmetric double-peaked emission lines with broad central absorption which can be seen in panel A of Fig. 4. Panels B and C of Fig. 4 show the expansion velocity case. When expansion velocities are introduced $V_B = 2$ and 5 mean thermal units, we obtain P cygni type profiles with blueshifted absorption. However, the emission part of the profile confines more or less to the centre of the line formed in static medium. The reason for this is that the absorption core is formed in the portion of the atmosphere which is directly in between the star and the observer. As the medium is moving towards the observer, there will be a Doppler shift of the frequencies of the line photons towards the blue side of the centre of the line. The photons that are emitted in the side lobes of the atmosphere are merely scattered, and the Doppler effect due to the velocities in the farther part and nearer part (with respect to the obsever's point) will nearly counter each other maintaining approximate symmetric emission about the line centre. Therefore, asymmetry caused by the Doppler shifts is minimal in the emission part of the line. In the case of irradiation, the emission peaks are reduced due to reflection as we have seen in

Table 1. Input model parameters.

Parameter	Input parameter value	Description of the parameter
σ	6.6525×10^{-25} cm^2	Thomson scattering coefficient
ρ	10^{14} cm^{-3}	Electron density
A	10^{12} cm	Inner radius of the atmosphere of the primary component
B	2×10^{12} cm	Outer radius of the atmosphere of the primary component
B/A	2	Ratio of the outer to the inner radii of the atmosphere of the primary component and whose reflection effect is being studied
N	100	The atmosphere of the primary component is divided into a number of shells
ϵ	0	Probability per scatter that a photon is thermalized by collisional de-excitation (see equation 6)
β	0	Ratio of absorption coefficient in continuum to that in the line
T		Total optical depth
V_A		Initial velocity of expansion in units of mtu at inner radius of the primary component
V_B		Final velocity in units of mtu at outer radius of the primary component
$r_1 = B$	2×10^{12}	Outer radius of the atmosphere of the primary component
R		distance between centres of gravity of the two components of the binary system
r_1/R		Ratio of the radius of the component to that of the line joining
S_s		Source function due to self-radiation (see equation 3)
S_I		Source function due to irradiation (see description in Section 3)
I		Ratio of incident radiation to that of self-radiation of the star
S		Total source function (see equation 12)
q		x/x_{max} (see equation 19)
F_q/F_c		Ratio of the line flux at the normalized frequency q to that in the continuum or at x_{max} (see equations 20 and 21)
With Ref		With reflection
Without Ref		Without reflection

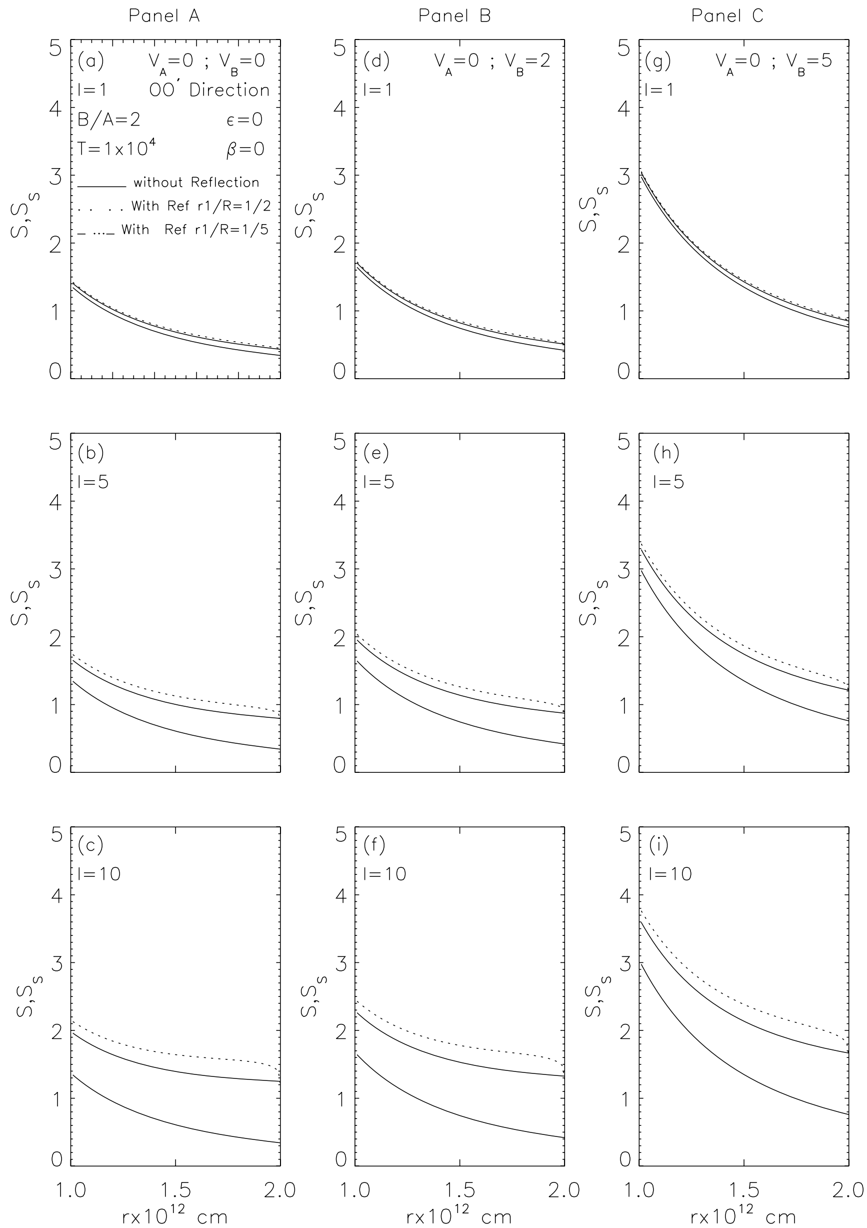

Figure 3. Source functions S and S_S (which are normalized to intensity units) with respect to radius in a scattering medium for shown parameters in panel A. Panel B is same as panel A but $V_A = 0$ and $V_B = 2$ mtu. Panel C is same as panel B but $V_A = 0$ and $V_B = 5$ mtu.

complete redistribution case (Peraiah & Srinivasa Rao 1998). Here, we are not able to distinguish the curves between the cases $r_1/R = 1/2$ and $r_1/R = 1/5$ due to small differences in the quantities.

7 CONCLUSIONS

We have studied the effect of PRD function in irradiated and moving atmospheres of close binary stars. We calculated the line source functions and their fluxes with and without reflection and obtained double-peaked emission lines. The variability displayed by the synthetic profiles occurs near the top of the line and reflects mainly on the motion of the atmosphere of the primary component. The obtained double-peaked emission lines from the model are similar of those observed in the spectrum of a long period Algols (the orbital period greater than 6 d) which are compact binaries (e.g. TT Hydrae, AU Monocerotis and RZ Ophiuchi). We also calculated the line profiles for the proximity of two components and noted that irradiation from the secondary contributes more radiation to the primary component when it is close to the primary component.

The present work is encouraging to investigate the irradiation effects in close binary systems to study in different mediums like partially scattering medium (i.e. $\epsilon = 1 \times 10^{-4}$, $\beta = 0$) and absorbing medium (i.e. $\epsilon = 1 \times 10^{-4}$, $\beta = 1 \times 10^{-4}$) for bringing out importance of reflection effect in close binaries.

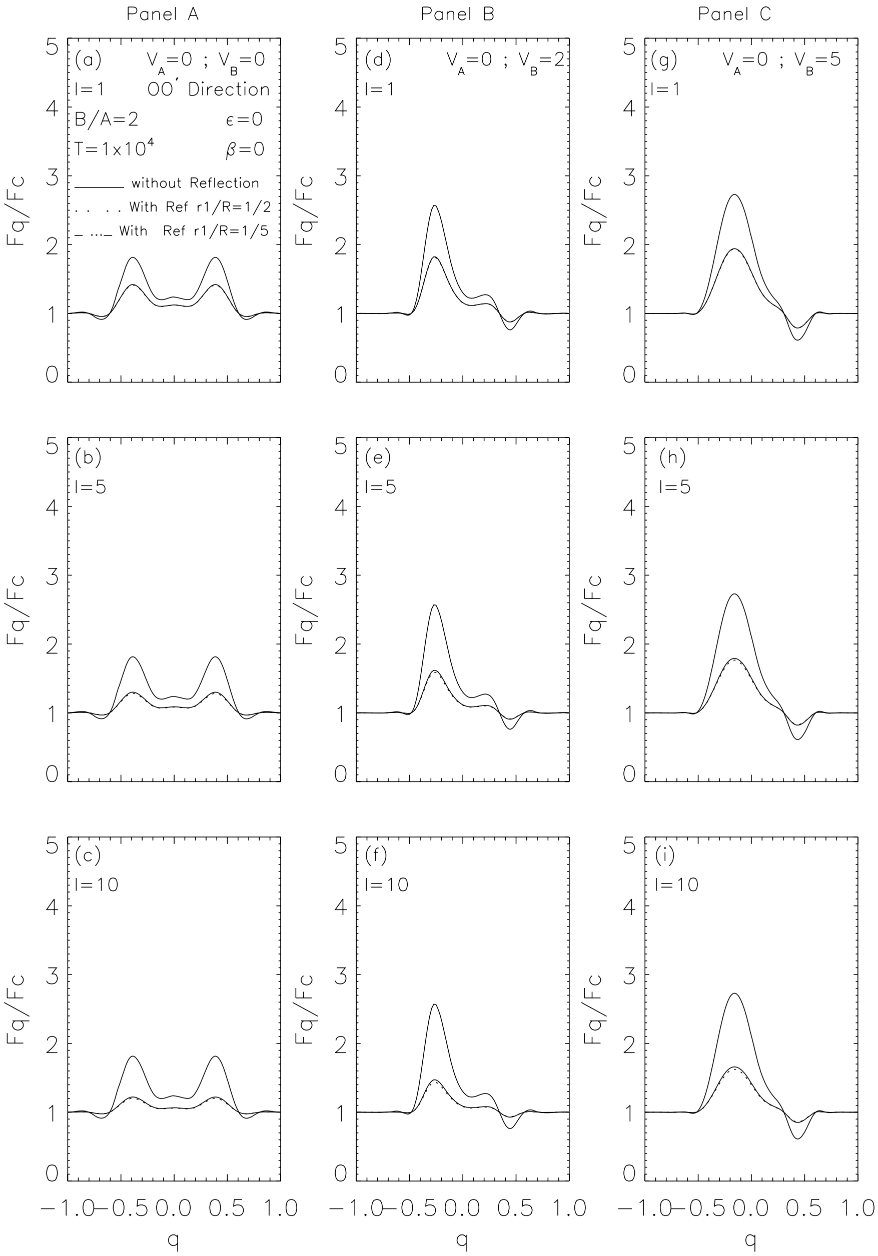

Figure 4. Line profiles of S and S_S with respect to the normalized frequency q in scattering medium for shown parameters in panel A. Panel B is same as panel A but $V_A = 0$ and $V_B = 2$ mtu. Panel C is same as panel B but $V_A = 0$ and $V_B = 5$ mtu.

ACKNOWLEDGMENTS

The author thanks the anonymous referee for his comments and helpful suggestions which helped in improving this paper. He also thanks Dr K. E. Rangarajan for his constructive comments on the radiative transfer section, which helped to improve the clarity of the manuscript, and Dr B. A. Varghese for careful reading and correcting the entire manuscript.

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APPENDIX A:

We need to find the solution of equation (1) and thus adopted the 'CELL' method described by Peraiah (1984) to solve the equation. This is done by suitable discretization in frequency, angle as radius. For frequency discretization, we choose the discrete points x_i and weights a_i such that

$$\int_{-\infty}^{+\infty} \phi(x) f(x)\,\mathrm{d}x \simeq \sum_{i=-I}^{I} a_i f(x), \quad \sum_{i=-I}^{I} a_i = 1. \tag{A1}$$

The angle discretization is done with abscissa $\{\mu_j\}$ and weights $\{C_j\}$ such that

$$\int_0^1 f(x)\,\mathrm{d}\mu \simeq \sum_{j=1}^{m} C_j f(\mu_j), \quad \sum_{j=1}^{m} C_j = 1. \tag{A2}$$

We choose the radial point r_n and r_{n+1} as discrete points for radial discretization. The integration of equation (1) is done on the '≪Cell≫' bounded by

$$[r_n, r_{n+1}][\mu_{j-\frac{1}{2}}, \mu_{j+\frac{1}{2}},][x_i, x_{i+1}]. \tag{A3}$$

The quantity $\mu_{j+\frac{1}{2}}$ is chosen by the relation

$$\mu_{j+\frac{1}{2}} = \sum_{k=1}^{j} C_k \quad j = 1, 2, 3, \ldots, m. \tag{A4}$$

We have chosen μs and Cs to be roots and weights of Gauss–Legendre quadrature.

Now the line transfer equation (1) after the radial integration

$$\begin{aligned} &\mathbf{M}\left[\mathbf{U}^+_{n+1} - \mathbf{U}^+_{\mathbf{n}}\right] + \rho_{\mathrm{c}}\left[\mathbf{\Lambda}^+\mathbf{U}^+_{n+\frac{1}{2}} - \mathbf{\Lambda}^-\mathbf{U}^-_{n+\frac{1}{2}}\right] + \tau_{n+\frac{1}{2}}\Phi^+_{n+\frac{1}{2}}\mathbf{U}^+_{n+\frac{1}{2}} \\ &\quad = \tau_{n+\frac{1}{2}}\mathbf{S}^+_{n+\frac{1}{2}} + \frac{1}{2}(1-\epsilon)\tau_{n+\frac{1}{2}}\left[\mathbf{R}^{++}\mathbf{W}^{++}\mathbf{U}^+ + \mathbf{R}^{+-}\mathbf{W}^{+-}\mathbf{U}^-\right]_{n+\frac{1}{2}}, \end{aligned} \tag{A5}$$

and another similar equation for (2) can be written as

$$\begin{aligned} &\mathbf{M}\left[\mathbf{U}^-_{n} - \mathbf{U}^-_{\mathbf{n+1}}\right] - \rho_{\mathrm{c}}\left[\mathbf{\Lambda}^+\mathbf{U}^-_{n+\frac{1}{2}} + \mathbf{\Lambda}^-\mathbf{U}^+_{n+\frac{1}{2}}\right] + \tau_{n+\frac{1}{2}}\Phi^-_{n+\frac{1}{2}}\mathbf{U}^-_{n+\frac{1}{2}} \\ &\quad = \tau_{n+\frac{1}{2}}\mathbf{S}^-_{n+\frac{1}{2}} + \frac{1}{2}(1-\epsilon)\tau_{n+\frac{1}{2}}\left[\mathbf{R}^{-+}\mathbf{W}^{-+}\mathbf{U}^+ + \mathbf{R}^{--}\mathbf{W}^{--}\mathbf{U}^-\right]_{n+\frac{1}{2}}, \end{aligned} \tag{A6}$$

various quantities in above equations are explained below.

$\rho_{\mathrm{c}} = (\Delta r/\bar{r})_{n+\frac{1}{2}}$ is the curvature factor, $r_{n+\frac{1}{2}}$ is the average radius of the shell bounded by the radii r_n and r_{n+1} and $\Delta r = r_{n+1} - r_n$.

$$S^+_{n+\frac{1}{2}} = \left[\rho\beta + \epsilon\phi^+_k\right]_{n+\frac{1}{2}} B'_{n+\frac{1}{2}}\delta_{kk'}, \tag{A7}$$

$$\mathbf{M} = [\mathbf{M}_m\delta_{mm'}], \quad \mathbf{M}_m = \left[\mu_j\delta_{jj}\right], \tag{A8}$$

$$\Lambda^{\pm} = \left[\Lambda^{\pm}_m\delta_{mm'}\right] \tag{A9}$$

$\Lambda^{\pm}$ are the curvature matrices given in Peraiah (2002).

$$\mathbf{U}^+_n = 4\pi r_n^2\left[I^+_{1,n}, I^+_{2,n}, \ldots, I^+_{\mathrm{I},n}\right]^{\mathrm{T}}, \tag{A10}$$

$$\Phi^+_{n+\frac{1}{2}} = \left[\Phi^+_{kk'}\right]_{n+\frac{1}{2}} = \left[\beta + \phi^+_k\right]_{n+\frac{1}{2}}\delta_{kk'}, \tag{A11}$$

$(i, j) = k = j + (i-1)J$, $1 \leqslant k \leqslant K$, $K = IJ$, $I=$ total number of frequency points and J is the total number of angle points.

$$(\phi_i W_k)_{n+\frac{1}{2}} = a_{i,n+\frac{1}{2}}c_j. \tag{A12}$$

Now, we will define following quantities $R^{++}, R^{-+}, R^{-+}, R^{-}$ as follows.

$$\begin{aligned} R^{-+}_{i,i'}(r) = \Big\{&R[x_i - \mu_1 V(r), x_{i'} + \mu_1 V(r), r], \ldots, R[x_i - \mu_m V(r), x'_{i'} \\ &+ \mu_m V(r), r]\Big\}^{\mathrm{T}}. \end{aligned} \tag{A13}$$

The procedure described in Peraiah (2002, chapter 6) is Applied, and one gets the cell operators of transmission and reflection operators of $t(n+1, n), t(n, n+1), r(n+1, n)$ and $r(n, n+1)$. These are given by

$$
\begin{aligned}
\mathbf{t}(n+1, n) &= \mathbf{G}^{+-}[\Delta^{+}\mathbf{A} + \mathbf{g}^{+-}\mathbf{g}^{-+}],\\
\mathbf{t}(n, n+1) &= \mathbf{G}^{-+}[\Delta^{-}\mathbf{D} + \mathbf{g}^{-+}\mathbf{g}^{+-}],\\
\mathbf{r}(n+1, n) &= \mathbf{G}^{-+}\mathbf{g}^{-+}[\mathbf{E} + \Delta^{+}\mathbf{A}],\\
\mathbf{r}(n, n+1) &+ \mathbf{G}^{+-}\mathbf{g}^{+-}[\mathbf{E} + \Delta^{-}\mathbf{D}],
\end{aligned}
\tag{A14}
$$

and the source terms are

$$
\begin{aligned}
\Sigma^{+}_{n+\frac{1}{2}} &= \mathbf{G}^{+-}\tau[\Delta^{+}S^{+} + \mathbf{g}^{+-}\Delta^{-}S^{-}],\\
\Sigma^{-}_{n+\frac{1}{2}} &= \mathbf{G}^{-+}\tau[\Delta^{-}S^{-} + \mathbf{g}^{-+}\Delta^{+}S^{+}],
\end{aligned}
\tag{A15}
$$

where $\mathbf{E}$ is the unit matrix and

$$
\begin{aligned}
\mathbf{G}^{+-} &= [\mathbf{E} - \mathbf{g}^{+-}\mathbf{g}^{-+}]^{-1},\\
\mathbf{g}^{+-} &= \frac{\tau}{2}\Delta^{+}\mathbf{Y}_{-}.
\end{aligned}
\tag{A16}
$$

$\mathbf{g}^{-+}$ and $\mathbf{G}^{-+}$ are obtained by interchanging the $+$ and $-$ signs in $\mathbf{G}^{+-}$ and $\mathbf{g}^{+-}$. Other matrices are

$$
\begin{aligned}
&\mathbf{A} = \mathbf{M} - \frac{\tau\mathbf{Z}_{+}}{2}, \quad \mathbf{D} = \mathbf{M} - \frac{\tau\mathbf{Z}_{-}}{2},\\
&\boldsymbol{\Delta}^{+} + \left[\mathbf{M} + \frac{\tau\mathbf{Z}_{+}}{2}\right]^{-1}, \quad \boldsymbol{\Delta}^{-} = \left[\mathbf{M} + \frac{\tau\mathbf{Z}_{-}}{2}\right]^{-1},\\
&\mathbf{Y}_{+} = \frac{\rho_{c}\Lambda^{-}}{\tau} + \frac{\mathbf{R}^{-+}\mathbf{W}^{-+}}{2},\\
&\mathbf{Y}_{-} = -\frac{\rho_{c}\Lambda^{-}}{\tau} + \frac{\mathbf{R}^{+-}\mathbf{W}^{+-}}{2},\\
&\mathbf{Z}_{+} = \phi^{+} - \frac{\mathbf{R}^{++}\mathbf{W}^{++}}{2} + \frac{\rho_{c}\Lambda^{+}}{\tau},\\
&\mathbf{Z}_{-} = \phi^{-} - \frac{\mathbf{R}^{--}\mathbf{W}^{--}}{2} - \frac{\rho_{c}\Lambda^{+}}{\tau}.
\end{aligned}
\tag{A17}
$$

Using the above operators, namely transmission, reflection and the scheme of internal and emergent radiation field, we obtain the solution of the line transfer equation in the observer's frame of the expanding atmospheres.

This paper has been typeset from a TEX/LATEX file prepared by the author.