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One dimensional prominence model

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Abstract. Based on reasonable assumptions and mathematical approximations a one dimensional, analytical model for solar quiescent prominences is constructed, which is in both magneto-hydrostatic and thermal equilibrium. Thermal equilibrium here is a balance among thermal conduction, radiation and wave heating. The wave heating (H) is assumed to be equal to a constant (E_H) times the product of pressure (p) and density (ρ). We find the limit on the value of E_H for existence of prominence type solution. For given values of E_H, temperature at the center of prominence (T_0), gas pressure at the center of prominence (p_0) and the temperature at the edge of prominence(T_*), we found the following limits on the variables for the existence of the equilibrium: (1) the lower limit on the value of gas pressure at the edge of prominence (p_*), (2) the upper and lower limits on the length of the magnetic field line from the center to the edge of the prominence and (3) the upper limit $[W_0 \sec\phi_0]_{max}$ on the value of $W_0 \sec\phi_0$ where W_0 is the width of the prominence and ϕ_0 is the shear angle.

For specified values of T_0, T_*, p_0, E_H and for $W_0\sec\phi_0 < [W_0\sec\phi_0]_{max}$ there exist, in general, two types of solutions. In Type 1 solution, equilibrium is nearly isobaric and the magnetic field is strong and nearly horizontal. This type of solution is physically inadmissible when the value of $W_0 \sec\phi_0$ falls bellow a certain limit defined in the text. Conditions in this solution approach those in a real prominence as $W_0\sec\phi_0$ approaches $[W_0\sec\phi_0]_{max}$. In Type 2 solution, there is a large variation of gas pressure from the center to the edge, and the magnetic field is weak and nearly vertical. Conditions in this solution also approach those in a real prominence as $W_0\sec\phi_0$ approaches $[W_0\sec\phi_0]_{max}$. The physical characteristics of Type 1 and Type 2 solutions simulate those of 'normal' and 'inverse' prominences respectively as observed by Bommier et al. (1994).

Key words: Sun: prominences – Sun: corona

1. Introduction

Prominence is a structure in the corona which has a temperature about hundred times lower and density about hundred times higher than in the normal corona. Many varieties of structure fall in this category. Among them the "quiescent prominences" are long lived and slowly changing prominence which occur in or near dying active regions or in the quiet regions of the Sun. The main observed features of quiescent prominences have been reviewed by Tandberg-Hanssen(1974), Hirayama(1985) and Zirker(1989). Observed values of plasma and magnetic parameters in quiescent prominence are listed by Engvold et al. (1990), Jansen et al. (1990).

The central problem posed by quiescent prominence is to explain how the quasi steady state of prominence is possible mechanically and thermally. The earlier works on structure of quiescent prominences were aimed at the question of mechanical equilibrium. The earliest and the most famous among them is the model of Kippenhahn-Schlüter(1957). In this model the support is by magnetic tension and the model assumes an isothermal structure for the prominence, so does not link prominence plasma thermally to the ambient corona (Milne et al. 1979). The important question of thermal equilibrium has been neglected in this model. Some other models like Orrall and Zirker (1961), Chiuderi et al. (1979) have addressed the problem of thermal equilibrium, but neglected the support mechanism. Attempts to combine the above two problem has been made by Milne et al. (1979) and Low et al. (1981).

The object of the present work is to study the limits on the existence of the magneto-hydrostatics and energy balance in a simple equilibrium model of a prominence and also to study the strength of magnetic field and width of the prominence in terms of plasma parameters at the center and at the edge of prominence. To begin with, simple assumptions are made to get an analytical solution of the equations describing the system. The advantage of the analytical solution lies in the fact that boundary conditions can be imposed in a straight forward manner and physical properties can be illustrated directly. This will be useful in gaining the confidence needed for the future numerical treatment of the realistic problem.

In Sect. 2, basic assumptions are described, from these and the basic equations of magneto-hydrostatics, the equations describing the system are derived. In Sect. 3 we describe the method of integration of these equations and boundary conditions. In Sect. 4 the conditions for the existence of solutions are derived. In Sect. 5 the nature of the solutions is discussed. In Sect. 6 conclusions are summarized.

2. Basic assumptions and the equations describing the structure

Quiescent prominences last for 1–300 days. In the quiescence state flows are observed but with speeds (typically of order of 0.5 km $\sec^{-1}$ to 3 km $\sec^{-1}$) much less than free fall speed (of about 100 km $\sec^{-1}$). Thus to a first approximation the plasma is in magneto hydrostatic equilibrium with a rough balance between the magnetic forces, plasma pressure and gravitational forces (Priest1989).

The energy equation expresses a balance among thermal conduction, radiative loss and existing but unidentified heating processes.

Thermal conduction along the magnetic field is primarily by electrons and for fully ionized H-plasma, Priest(1982) gives the equation for finding thermal conductivity along the magnetic field ($K_\parallel$) as $K_\parallel = 1.8\times10^{-10}T^{2.5}/\ln\Lambda$, in prominence conditions we have taken $\ln\Lambda = 10$. The ratio of thermal conductivity across the magnetic field to that of thermal conductivity along the magnetic field as given by Priest(1982) is $\frac{K_\perp}{K_\parallel} = \frac{2\times10^{-31}n^2}{T^3B^2}$. In the main body of prominence $n\approx10^{16}$ particles m^{-3}, T≈6000 K and $B\approx8\times10^{-4}$ T using these values, the value of the fraction ($\frac{K_\perp}{K_\parallel}$) is of the order of 10^{-4}. Hence thermal conductivity perpendicular to the magnetic field is neglected. We define $K_0 = 1.8\times10^{-11}$.

Expression for radiative loss given by Cox and Tucker (1969), Hildner(1974) are valid for temperatures greater than 20,000 K. In the central region of prominence where the temperature is in the range of 4400–8000 K, whereas at the edge of the prominence temperature is in the range of 8000–12,000 K (Jansen et al. 1990). For this range of temperature Peres et al. (1982), expresses the radiative loss in the form $L_\nu = \chi\rho^2T^\alpha$ where χ and α are step wise continuous functions of T for different ranges of T the value of χ and α are given bellow.

T(K)	χ	α
$4400\leq T\leq8000$	4.49×10^{-30}	11.7
$8000\leq T\leq20000$	1.76×10^{-8}	6.15

To get an analytical solution, we choose α as 11.0 instead of 11.7 in the temperature range of 4400–8000 K and as 6.0 instead of 6.15 in the temperature range of 8000–20,000 K. Using the condition of continuity of radiative loss at T = 8000 K, χ in the range of 8000–20,000 K is calculated as $\chi = 4.49\times10^{-30}\times8000^5 = 1.47\times10^{-10}$ the new values of χ and α are given bellow.

T(K)	χ	α
$4400\leq T\leq8000$	4.49×10^{-30}	11.0
$8000\leq T\leq20000$	1.47×10^{-10}	6.0

The exact form of wave heating is not known. Generally it is assumed as a constant times density. This assumption is ad hoc and justification is in mathematical simplicity. On the same ground, we assume that heating H is proportional to the product of density and pressure, i.e. $H = E_H p\rho$, where E_H is a constant.

A rectangular Cartesian coordinate system is used with x-axis along the length of the prominence, y-axis perpendicular to the length in the horizontal plane and z-axis is in vertical direction. We assume that all the plasma parameters and the magnetic field components vary in the direction of y only. Based on the above assumptions, the basic equations of magneto-hydrostatics reduce to the following set of equations.

$$\boldsymbol{B} = [C_1, C_2, B_Z], \tag{1}$$

$$\frac{dB_Z}{dy} = \frac{\mu_0\rho g}{C_2}, \tag{2}$$

$$p + {B_z}^2/2\mu_0 = constant = p_0, \tag{3}$$

$$p = \frac{\tilde{R}\rho T}{\tilde{\mu}}, \tag{4}$$

$$\frac{d}{dy}\left[\left(\frac{K_0{C_2}^2T^{2.5}}{{C_1}^2+{C_2}^2+{B_z}^2}\right)\frac{dT}{dy}\right] = \chi\rho^2T^\alpha - E_H p\rho, \tag{5}$$

where $\tilde{\mu}$ is the mean molecular weight, μ_0 is the magnetic permeability of free space and all other symbols have their usual meanings.

Thus we have four equations for four unknowns B_z, p, ρ and T. In order to solve the above set of equations we have to specify constants C_1, C_2 and four boundary conditions. Symmetry demands that both the temperature gradient and the vertical component of the magnetic field vanish at the center of the prominence, so that

$$B_z = \frac{dT}{dy} = 0,\ \ T = T_0,\ \ p = p_0 \ \ \text{at}\ \ y = 0. \tag{6}$$

For the remaining conditions if W_0 is the width of the prominence then,

$$T = T_* \quad at \quad y = W_0/2. \tag{7}$$

The above set of equations reduce to a system of two equations for two unknowns B_z, T in the form

$$\frac{dB_z}{dy} = \frac{\mu_0 g}{C_2}\left(\frac{\tilde{\mu}}{\tilde{R}T}\right)\left[p_0 - B_z^2/2\mu_0\right]\ , \tag{8}$$

$$\frac{d}{dy}\left[\left(\frac{K_0{C_2}^2T^{2.5}}{{C_1}^2+{C_2}^2+{B_z}^2}\right)\frac{dT}{dy}\right] = \left(\frac{\tilde{\mu}}{\tilde{R}}\right)^2\frac{(p_0-{B_z}^2/2\mu_0)^2}{T}\left[\chi T^{\alpha-1} - E_H\tilde{R}/\tilde{\mu}\right]. \tag{9}$$

To obtain the relation between T and B_z, we use the formula

$$\frac{d}{dy} = \frac{dB_z}{dy}\frac{d}{dB_z} = \frac{\mu_0 g}{C_2}\left(\frac{\tilde{\mu}}{\tilde{R}T}\right)[p_0 - {B_z}^2/2\mu_0]\frac{d}{dB_z}. \tag{10}$$

Using the above relation and Eq. (9), we get the following differential equation relating f and B_z where $f = T^{2.5}$.

$$\frac{d}{dB_z}\left[\left(\frac{p_0-{B_z}^2/2\mu_0}{{C_1}^2+{C_2}^2+{B_z}^2}\right)\frac{df}{dB_z}\right] = \frac{2.5(p_0-{B_z}^2/2\mu_0)}{(\mu_0 g)^2K_0}\left[\chi f^{\frac{\alpha-1}{2.5}} - E_H\tilde{R}/\tilde{\mu}\right]. \tag{11}$$

By integrating the above equation, we can derive the relation between f and B_z.

3. Method of integration

Eq. (11) can be written in the form

$$\frac{d}{dB_z}\left[\left(\frac{p_0 - {B_z}^2/2\mu_0}{{C_1}^2+{C_2}^2+{B_z}^2}\right)\frac{df}{dB_z}\right]^2 = \frac{5(p_0-{B_z}^2/2\mu_0)^2}{(\mu_0 g)^2 K_0({C_1}^2+{C_2}^2+{B_z}^2)} \times\frac{d}{dB_z}\left[\frac{\chi 2.5}{\alpha+1.5}f^{\frac{\alpha+1}{2.5}} - E_H\tilde{R}f/\tilde{\mu}+K_1\right],$$

where K_1 is a constant of integration. The quantity

$$Q=\frac{(p_0-{B_z}^2/2\mu_0)^2}{{C_1}^2+{C_2}^2+{B_z}^2},$$

is a continuous and smoothly varying function of B_z, so by using the mean value theorem, the above equation can be integrated to get the following differential equation.

$$\frac{df}{dB_z}=\sqrt{\frac{5[Q]}{K_0(\mu_0 g)^2}}\left(\frac{{C_1}^2+{C_2}^2+{B_z}^2}{p_0-{B_z}^2/2\mu_0}\right) \times\sqrt{\frac{\chi 2.5}{\alpha+1.5}f\left(\frac{\alpha+1.5}{2.5}\right)-E_H\tilde{R}f/\tilde{\mu}+K_1}, \qquad (12)$$

where $[Q]$ represents the mean value of Q in the range of value of B_z. Using Eqs. (10) and (12) we get

$$\frac{df}{dy}=\frac{\tilde{\mu}}{C_2\tilde{R}}\sqrt{\frac{5[Q]}{K_0}}\left(\frac{{C_1}^2+{C_2}^2+{B_z}^2}{f^{0.4}}\right) \times\sqrt{\frac{\chi 2.5}{\alpha+1.5}f(\frac{\alpha+1.5}{2.5})-E_H\tilde{R}f/\tilde{\mu}+K_1}. \qquad (13)$$

Eqs. (12) and (13) are elliptical. For integration, the values of α and K_1 are needed. By making use of the boundary conditions, the value of α and K_1 can be calculated.

3.1. Conditions at the center of the prominence

Let T_0 be the temperature at the center of the prominence. From observations it is found that T_0 is in the range of 4400–8000 K (Engvold et al. 1990, Jansen et al. 1990). For this range of temperature $\alpha=\alpha_1=11$ and $\chi=\chi_1=4.49\times10^{-30}$. From Eqs. (6) and (13) we get

$$K_1=-\frac{\chi_1{f_0}^5}{5}+\frac{E_H\tilde{R}f_0}{\tilde{\mu}}.$$

At the center of the prominence $\frac{df}{dy}=0$. From observation it is clear temperature should be minimum at the center. Hence one must have $(\frac{d^2f}{dy^2})_{y=0}>0$. Applying this condition in the above equation, we get $\chi_1{f_0}^4>E_H\tilde{R}/\tilde{\mu}$ i.e. $E_H<\frac{\chi_1\tilde{\mu}{f_0}^4}{\tilde{R}}$. This tells us that at the center of the prominence, the rate of heating should be less than the radiative loss. To get an analytical solution, we have taken $E_H=\frac{\tilde{\mu}\chi_1{f_0}^4}{5\tilde{R}}$ i.e. heating at the center to be one-fifth of the rate of the radiative loss at the center. Substituting this the values of E_H and K_1 Eqs. (12) and (13) take the forms

$$\frac{df}{dB_z}=\sqrt{\frac{\chi_1[Q]}{K_0(\mu_0 g)^2}}\left[\frac{{C_1}^2+{C_2}^2+{B_z}^2}{p_0-{B_z}^2/2\mu_0}\right]\sqrt{f^5-f{f_0}^4}, \qquad (14)$$

$$\frac{df}{dy}=\frac{\tilde{\mu}}{C_2\tilde{R}}\sqrt{\frac{\chi_1[Q]}{K_0}}\left(\frac{{C_1}^2+{C_2}^2+{B_z}^2}{f^{0.4}}\right)\sqrt{f^5-f{f_0}^4}. \qquad (15)$$

Eqs. (14) and (15) are valid for $4400\,\mathrm{K}\le T_0\le T\le T_*\le 8000\,\mathrm{K}$. Equations for $T>8000\,\mathrm{K}$ will be discussed later in the Sect. (3.3). Thus we can integrate the Eqs. (14) and (15) in the above mentioned temperature range for T.

3.2. Integration for $4400K\le T_0\le T\le T_*\le 8000\,\mathrm{K}$

Eq. (14) can be written as

$$\int_{f_0}^{f}\frac{df}{\sqrt{f^5-f{f_0}^4}} = \sqrt{\frac{\chi_1[Q]}{K_0(\mu_0 g)^2}}\int_0^{B_z}\frac{[{C_1}^2+{C_2}^2+{B_z}^2]dB_z}{p_0-{B_z}^2/2\mu_0}$$

let I = $\int_{f_0}^{f}\frac{df}{\sqrt{f^5-f{f_0}^4}}$ and $u=\frac{f}{f_0}$. It can verified that

$$I=\frac{1}{\sqrt{2}{f_0}^{1.5}}[F(\phi_1|\alpha_1)-\frac{F(\phi_2|\alpha_1)}{\sqrt{3+2^{1.5}}}],$$

where $F(\phi_1|\alpha_1)$ and $F(\phi_2|\alpha_1)$ are elliptical integrals of first kind, ϕ_1, ϕ_2 and α_1 are defined bellow

$$\sin^2(\phi_1)=\frac{(u+1)/\sqrt{2}+\sqrt{2}/(u+1)-\frac{3}{\sqrt{2}}}{(u+1)/\sqrt{2}+\sqrt{2}/(u+1)-2}$$

$$\sin^2(\phi_2)=\frac{(u+1)/\sqrt{2}+\sqrt{2}/(u+1)-\frac{3}{\sqrt{2}}}{(u+1)/\sqrt{2}+\sqrt{2}/(u+1)-\sqrt{2}}$$

$$\sin^2(\alpha_1)=\frac{2+2^{1.5}}{3+2^{1.5}}.$$

It can also be verified that

$$\int_0^{B_z}\frac{[{C_1}^2+{C_2}^2+{B_z}^2]dB_z}{p_0-{B_z}^2/2\mu_0} = -2\mu_0 B_z+\frac{[{C_1}^2+{C_2}^2+2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times\ln\left[\frac{\sqrt{p_0}+B_z/\sqrt{2\mu_0}}{\sqrt{p_0}-B_z/\sqrt{2\mu_0}}\right].$$

Thus we get

$$\left[F(\phi_1|\alpha_1)-\frac{F(\phi_2|\alpha_1)}{\sqrt{3+2^{1.5}}}\right] = \sqrt{\frac{2\chi_1{f_0}^3[Q]}{K_0[\mu_0 g]^2}}\times\left(-2\mu_0 B_z+\frac{[{C_1}^2+{C_2}^2+2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times\ln\left[\frac{\sqrt{p_0}+B_z/\sqrt{2\mu_0}}{\sqrt{p_0}-B_z/\sqrt{2\mu_0}}\right]\right). \qquad (16)$$

This equation defines a relation among T_0, T, B_z and other constants, which is valid for both T_0, T in the range of 4400–8000 K. At $T = T_* = 8000$ K we define $u = u_8 = (8000/T_0)^{2.5}$ and $B_z = B_{z8}$. Applying this condition in Eq. (16) we get the following relation.

$$\left[F(\phi_{1*}|\alpha_1) - \frac{F(\phi*_2|\alpha_1)}{\sqrt{3+2^{1.5}}}\right]$$
$$= \sqrt{\frac{2\chi_1 {f_0}^3[Q]}{K_0[\mu_0 g]^2}}$$
$$\times\left(-2\mu_0 B_{z8} + \frac{[{C_1}^2 + {C_2}^2 + 2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}}\right.$$
$$\left.\times \ln\left[\frac{\sqrt{p_0} + B_{z8}/\sqrt{2\mu_0}}{\sqrt{p_0} - B_{z8}/\sqrt{2\mu_0}}\right]\right), \quad (17)$$

where ϕ_{1*} and ϕ_{2*} are the values of ϕ_1 and ϕ_2 when the value of u is replaced by u_8. From Eq. (16) it is found that it is not possible to express B_z in terms of u or vice versa. Hence we again use the mean value theorem to integrate Eq. (15). From Eq. (15) we have

$$\int_{f_0}^{f} \frac{f^{0.4} df}{\sqrt{f^5 - f{f_0}^4}} = \frac{\tilde{\mu}}{C_2\tilde{R}}\sqrt{\chi_1[Q]/K_0}[B^2]y\ ,$$

where $[B^2]$ is the mean value of B^2 in the range of value of B_z. It can be verified that

$$I = \int_{f_0}^{f} \frac{f^{0.4} df}{\sqrt{f^5 - f{f_0}^4}}$$
$$= \frac{1}{{f_0}^{1.1}}\int_1^u \frac{du}{u^{0.1}\sqrt{u^4-1}} = \frac{F(\phi_3|\alpha_2)}{\sqrt{2}{f_0}^{1.1}[u^{0.1}]}\ ,$$

where $F(\phi_3|\alpha_2)$ is an elliptical integral of first kind, $\cos(\phi_3) = 1/u$: and $\alpha_2 = 45^0$, $[u^{0.1}]$ is the mean value of u in the range $[1, u_8]$. This relation among T_0, T, B_z and y is valid for both T_0 and T in the range of 4400–8000 K. At $T = 8000$ K, we have $B_z = B_{z8}$ and $y = y_8$, we get following relation.

$$F(\phi_{3*}|\alpha_2) = \frac{\tilde{\mu}{f_0}^{1.1}}{C_2\tilde{R}}\sqrt{2\chi_1[Q]/K_0}[u^{0.1}][B^2]y_8\ , \quad (18)$$

where $\cos(\phi_{3*}) = 1/u_8$ and $\alpha_2 = 45^0$.

3.3. *Integration for* $T_0 < 8000$ K *and for* $8000 < T \le T_* \le 20,000$ K

From observations it has been found that the temperature at the edge of the prominence is in the range of 8000–12,000 K Engvold et al. (1990). The calculation done in Sect. 3.2 is valid here also up to $T = 8000$ K. To continue the calculation further for higher temperature, we assume that there is no sudden change in the values of density, pressure, magnetic field components, wave heating and radiative loss at $T = 8000$ K. Thus we have to use the same value of E_H which we have used earlier. In the above mentioned temperature range for T, $\alpha = \alpha_2 = 6$ $\chi = \chi_2 = 1.47\times 10^{-10}$.

From Eqs. (12) and (13) after substituting the same earlier value of E_H and α we get the following equations.

$$\frac{df}{dB_z} = \sqrt{\frac{5[Q]}{K_0(\mu_0 g)^2}}\left(\frac{{C_1}^2 + {C_2}^2 + {B_z}^2}{p_0 - {B_z}^2/2\mu_0}\right)$$
$$\times\sqrt{\frac{\chi_2}{3}f^3 - \frac{\chi_1 f{f_0}^4}{5} + K_1}, \quad (19)$$

$$\frac{df}{dy} = \frac{\tilde{\mu}}{C_2\tilde{R}}\sqrt{\frac{5[Q]}{K_0}}\left(\frac{{C_1}^2 + {C_2}^2 + {B_z}^2}{f^{0.4}}\right)$$
$$\times\sqrt{\frac{\chi_2}{3}f^3 - \frac{\chi_1 f{f_0}^4}{5} + K_1}\ , \quad (20)$$

where K_1 is the constant of integration, which can be determined by assuming the continuity of temperature gradient at $T = 8000\,K$. Thus we that we get $K_1 = -2\chi_2 8000^{7.5}/15$. Eq. (19) can be written as

$$\int_{f_8}^{f} \frac{df}{\sqrt{f^3 - \frac{3{f_0}^4 f}{5(8000^5)} - \frac{2(8000^{7.5})}{5}}}$$
$$= \sqrt{\frac{5[Q]\chi_2}{3K_0(\mu_0 g)^2}}\int_{B_{z8}}^{B_z}\frac{[{C_1}^2 + {C_2}^2 + {B_z}^2]dB_z}{(p_0 - {B_z}^2/2\mu_0)}\ .$$

Let $$I = \int_{f_8}^{f}\frac{df}{\sqrt{f^3 - \frac{3f{f_0}^4}{5(8000^5)} - \frac{2(8000^{7.5})}{5}}}$$
$$= \frac{1}{{f_0}^{0.5}}\int_{u_8}^{u}\frac{du}{\sqrt{u^3 - \frac{3u}{5({u_8}^2)} - \frac{2({u_8}^3)}{5}}}\ .$$

Let β be the real root of the equation

$$P(u) = u^3 - \frac{3u}{5({u_8}^2)} - \frac{2{u_8}^3}{5} = 0\,,$$

it can be verified that β is the only real root of the equation $P(u) = 0$ and $\beta < u_8$. Following Abramowitz and Stegun (1970),

$$\int_{\beta}^{x}\frac{du}{\sqrt{u^3 - \frac{3u}{5({u_8}^2)} - \frac{2({u_8}^3)}{5}}} = \frac{F(\phi|\alpha)}{\Lambda_3}\ ,$$

where $F(\phi|\alpha)$ is an elliptical integral of first kind ϕ, α and Λ_3 are defined as follows.

$$\Lambda_3 = \left[\left(\frac{dP}{du}\right)_{u=\beta}\right]^{0.25} : \ \cos(\phi) = \frac{{\Lambda_3}^2 - (x-\beta)}{{\Lambda_3}^2 + (x-\beta)} :$$
$$G = \left(\frac{d^2P}{du^2}\right)_{u=\beta} : \ \sin^2(\alpha) = 0.5 - \frac{G}{8{\Lambda_3}^2}\ .$$

Therefore by using the above formula we get

$$I = \frac{1}{\Lambda_3 f_0^{0.5}}[F(\phi_4|\alpha_3) - F(\phi_5|\alpha_3)]\ ,$$

where ϕ_4 and ϕ_5 are defined as follows:

$$\cos(\phi_4) = \frac{{\Lambda_3}^2 - (u-\beta)}{{\Lambda_3}^2 + (u-\beta)} : \ \cos(\phi_5) = \frac{{\Lambda_3}^2 - (u_8-\beta)}{{\Lambda_3}^2 + (u_8-\beta)} .$$

Thus we get the following equation

$$\frac{1}{\Lambda_3 f_0^{0.5}}[F(\phi_4|\alpha_3) - F(\phi_5|\alpha_3)]$$
$$= C_5 \left(-2\mu_0 B_z + \frac{[{C_1}^2 + {C_2}^2 + 2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times \ln\left[\frac{\sqrt{p_0} + B_z/\sqrt{2\mu_0}}{\sqrt{p_0} - B_z/\sqrt{2\mu_0}}\right]\right)$$
$$-C_5 \left(-2\mu_0 B_{z8} + \frac{[{C_1}^2 + {C_2}^2 + 2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times \ln\left[\frac{\sqrt{p_0} + B_z 8/\sqrt{2\mu_0}}{\sqrt{p_0} - B_z 8/\sqrt{2\mu_0}}\right]\right)$$

where $C_5 = \sqrt{\frac{5[Q]\chi_2}{3K_0(\mu_0 g)^2}}$. Substituting for the last term from the Eq. (17), the above equation after simplification reduces to

$$F(\phi_{1*}|\alpha_1) - \frac{F(\phi{*}_2|\alpha_1)}{\sqrt{3+2^{1.5}}} + \frac{\sqrt{1.2}}{\Lambda_3 u_8}[F(\phi_4|\alpha_3) - F(\phi_5|\alpha_3)]$$
$$= \sqrt{\frac{2\chi_1 {f_0}^3[Q]}{K_0[\mu_0 g]^2}} \left(-2\mu_0 B_z + \frac{[{C_1}^2 + {C_2}^2 + 2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times \ln\left[\frac{\sqrt{p_0} + B_z/\sqrt{2\mu_0}}{\sqrt{p_0} - B_z/\sqrt{2\mu_0}}\right]\right) .$$

This equation defines a relation among T_0, T, B_z and other constants, which is valid for $T_0 < 8000K$ and $T_0 < T_* < 20,000$ K. At the edge of the prominence we have $T = T_*$, $u = u_* = (\frac{T_*}{T_0})^{2.5}$, $B_z = B_{z*}$. Then the above equations reduce to

$$F(\phi_{1*}|\alpha_1) - \frac{F(\phi{*}_2|\alpha_1)}{\sqrt{3+2^{1.5}}} + \frac{\sqrt{1.2}}{\Lambda_3 u_8}[F(\phi_{4*}|\alpha_3) - F(\phi_5|\alpha_3)]$$
$$= \sqrt{\frac{2\chi_1 {f_0}^3[Q]}{K_0[\mu_0 g]^2}} \left(-2\mu_0 B_{z*} + \frac{[{C_1}^2 + {C_2}^2 + 2p_0\mu_0]\sqrt{\mu_0}}{\sqrt{2p_0}} \times \ln\left[\frac{\sqrt{p_0} + B_{z*}/\sqrt{2\mu_0}}{\sqrt{p_0} - B_{z*}/\sqrt{2\mu_0}}\right]\right) , \quad (21)$$

where ϕ_{4*} is the value of ϕ_4 when u is replaced by u_*. Similarly from Eq. (20) we get

$$\frac{F(\phi_{3*}|\alpha_2)}{[u^{0.1}]} + \frac{\sqrt{1.2}[u^{0.4}]}{\Lambda_3 u_8}[F(\phi_{4*}|\alpha_3) - F(\phi_5|\alpha_3)]$$
$$= \frac{\tilde{\mu} {f_0}^{1.1}}{C_2 \tilde{R}} \sqrt{\frac{\chi_1[Q]}{2K_0}}[B^2]W_0 , \quad (22)$$

where W_0 is the width of the prominence and $[u^{0.4}]$ represents the mean value of $u^{0.4}$ in the range $u_8 \le u \le u_*$.

From the 1^{st} mean value theorem we have: If f and ϕ are integrable on $[a, b]$ and $\phi(x) \ge 0$ then

$$\int_a^b f\phi dx = \Lambda \int_a^b \phi dx ,$$

where $m = inf f(x) \le \Lambda \le sup f(x) = M$. Also we have

$$m \int_a^b \phi dx \le \int_a^b \phi f dx \le M \int_a^b \phi dx .$$

We have applied the above formula, while we are integrating Eq. (11), with $f = Q$ where Q is defined as

$$Q = \frac{(p_0 - {B_z}^2/2\mu_0)^2}{{C_1}^2 + {C_2}^2 + {B_z}^2} .$$

It can be verified that $Q(B_z)$ is a monotonically decreasing function, with $M = {p_0}^2/[{C_1}^2 + {C_2}^2]$ and $m = {p_*}^2/{B_*}^2$.

Here we defined mean value of Q as $[Q] = (M + m)/2 = ({p_0}^2/[{C_1}^2 + {C_2}^2] + {p_*}^2/{B_*}^2)/2$ similarly mean value of B^2 is defined as $[B^2] = ({C_1}^2 + {C_2}^2 + B^2)/2$.

3.4. *Specifying the boundary conditions*

The components of magnetic field C_1, C_2 and B_z can also be represented in terms of the strength of the magnetic field (B), dip angle (θ) and the shear angle (ϕ) which is defined as the angle between y-axis and the direction of the magnetic field. The relation among these quantities at the edge of the prominence is

$${B_*}^2 = {C_1}^2 + {C_2}^2 + {B_{z*}}^2 ,$$
$$\tan\theta_* = B_{z*}/\sqrt{{C_1}^2 + {C_2}^2}, \ \tan\phi_0 = C_1/C_2 ,$$

the inverse relation is

$$C_1 = B_* \cos\theta_* \sin\phi_0, \ \ C_2 = B_* \cos\theta_* \cos\phi_0,$$
$$B_{z*} = B_* \sin\theta_* ,$$

where the suffix $*$ indicates the value at the edge of the prominence.

The problem in hand is to find out the relations among the following quantities p_0, ρ_0, T_0 p_*, ρ_*, T_*, B_*, θ_*, ϕ_0 and W_0. By making use of equation of state $\rho = \frac{\tilde{\mu}p}{\tilde{R}T}$ and the relation $p_* = p_0 - \frac{(B_* \sin\theta_*)^2}{2\mu_0}$, the set of unknown parameters to be determined reduce to p_0, T_0, T_*, B_*, θ_*, ϕ_0 and W_0. Defining the parameter

$$\xi = \frac{(B_* \sin\theta)^2}{2\mu_0 p_*} ,$$

so that $p_0/p_* = 1 + \xi$ or

$$B_*^2 = \frac{2\mu_0 p_0 \xi \csc^2\theta_*}{[1+\xi]} ,$$

the unknowns to be determined become p_0, T_0, T_*, ξ, θ_*, W_0 and ϕ_0.

Substituting the expressions for $[Q]$, $[B^2]$ and B_* the Eqs. (21) and (22) become

$$\sqrt{(1+\xi)^2 \sec^2\theta_* + 1} \left(-1 + \frac{(\xi \csc^2\theta_* + 1)}{\sqrt{\xi(\xi+1)}} \times \log\left[\sqrt{1+\xi} + \sqrt{\xi}\right]\right) = \frac{{K_1}^*(1+\xi)\csc\theta_*}{p_0} , \quad (23)$$

where

$$K_1{}^*=\frac{g}{2{T_0}^{3.75}}\sqrt{\frac{K_0}{\chi_1}}\times\left(\left[F(\phi_{1*}|\alpha_1)-\frac{F(\phi*_2|\alpha_1)}{\sqrt{3+2^{1.5}}}\right]\right.$$
$$\left.+\frac{\sqrt{1.2}}{\Lambda_3 u_8}\left[F(\phi_{4*}|\alpha_3)-F(\phi_5|\alpha_3)\right]\right) . \quad (24)$$

$$W_0\sec\phi_0=\frac{K_2{}^*(1+\xi)\cos\theta_*}{p_0(1+\cos^2\theta_*)\sqrt{(1+\xi)^2\sec^2\theta_*+1}} \quad (25)$$

where

$$K_2{}^*=\frac{4\tilde{R}}{\tilde{\mu}{T_0}^{2.75}}\sqrt{\frac{K_0}{\chi_1}}\times\left(\frac{F(\phi_{3*}|\alpha_2)}{[u^{0.1}]}+\frac{\sqrt{1.2}[u^(0.4)]}{\Lambda_3 u_8}\right.$$
$$\times\left[F(\phi_{4*}|\alpha_3)-F(\phi_5|\alpha_3)\right]) . \quad (26)$$

The parameters K_1^* and K_2^* are the functions of T_0 and T_*. Thus we can find the values of ξ and θ_* from the Eq. (23) and (25) if we specify the values of p_0, T_0, T_*, ϕ_0 and W_0. Since $W_0\sec\phi_0$ appears as a single term, it is enough to specify the values of p_0, T_0, T_* and $W_0\sec\phi_0$ to solve the above problem. For arbitrarily chosen values of all these parameters the existence of the solution cannot be assured.

4. Condition for the existence of solution

In order to solve the above Eqs. (23) and (25) for ξ and θ_* we have to specify the values of p_0, T_0, T_* and $W_0\sec\phi_0$. The observed values of p_0, T_0 and T_* for prominences can be chosen from the table given by Engvold et al. (1990) or Jansen et al. (1990).

Now we reverse the problem as follows: For a given values of p_0 T_0 and T_* what will be the extremum values of $W_0\sec\phi_0$, for which Eqs. (23) and (25) are satisfied. Thus we have to find the extremum of $W_0\sec\phi_0$ subject to the condition G = 0, where

$$G=-\frac{K_1{}^*(1+\xi)\csc\theta_*}{p_0}+\sqrt{(1+\xi)^2\sec^2\theta_*+1}$$
$$\times\left(-1+\frac{(\xi\csc^2\theta_*+1)}{\sqrt{\xi(\xi+1)}}\log\left[\sqrt{1+\xi}+\sqrt{\xi}\right]\right) . \quad (27)$$

For this the condition to be satisfied is,

$$\frac{\partial W_0\sec\phi_0}{\partial\xi}\frac{\partial G}{\partial\theta_*}-\frac{\partial W_0\sec\phi_0}{\partial\theta_*}\frac{\partial G}{\partial\xi}=0 .$$

Substituting for $W_0\sec\phi_0$ and G from Eqs. (25) and (27) respectively, the following relation between ξ and θ_* can be obtained.

$$\frac{(2\csc^2\theta_*-3)}{2\xi}\left[\xi\csc^2\theta_*+1-\frac{\sqrt{\xi(\xi+1)}}{\log\left[\sqrt{1+\xi}+\sqrt{\xi}\right]}\right]$$
$$-\cot^2\theta_*(1+2\cot^2\theta_*)+\left(\frac{2(1+\xi)^2\sec^2\theta_*+\sin^2\theta_*}{4\xi^2}\right)$$
$$\times\left[\frac{(\xi\csc^2\theta_*+1)\sqrt{\xi(\xi+1)}}{\log\sqrt{\xi+1}+\sqrt{\xi}}+(\xi\csc^2\theta_*-2\xi-1)\right]=0 . \quad (28)$$

Table 1. Values of $ew=\frac{K_1^*[W_0\sec\phi_*]_{max}}{K_2^*},\xi,\theta_*$ $Be=\frac{B_*10^3}{\sqrt{K_1^*}}$ $\beta=\frac{2\mu_0 p_0}{{B_*}^2\sin\theta_*}$ for various values of K_1^*/p_0.

$\frac{K_1^*}{p_0}$	ew	ξ	θ_*	Be	β
10	4.9	8.59	9.85	2.77	0.19
9	4.39	7.69	10.58	2.71	0.21
8	3.89	6.78	11.44	2.64	0.23
7	3.38	5.87	12.47	2.57	0.25
6	2.87	4.94	13.71	2.49	0.28
5	2.37	4.01	15.25	2.41	0.33
4	1.86	3.08	17.19	2.33	0.39
3	1.35	2.14	19.61	2.25	0.49
2	0.85	1.23	22.32	2.19	0.69
1.32	0.529	0.651	23.228	2.1957	1
1.321	0.528	0.65	23.227	2.1958	1
0.9853	0.38	0.40	22.30	2.25	1.33
0.6721	0.25	0.20	19.36	2.39	1.98
0.2904	0.10	0.04	10.81	3.11	4.70
0.1555	0.05	0.01	6.14	4.09	9.03

For a given T_0, T_* and p_0 we can find the values of K_1^* and $K_2{}^*$. The value of ξ and θ_* corresponding to the extremum value of $W_0\sec\phi_0$ is obtained by solving simultaneously the equations G = 0 and Eq. (28). The extremum value of $W_0\sec\phi_0$ is obtained by substituting the values of K_2^*, p_0, ξ and θ_* in the Eq. (25). This value of $W_0\sec\phi_0$ turns out to be the maximum value of $W_0\sec\phi_0$ which we denote it by $[W_0\sec\phi_0]_{max}$. For various values of K_1^*/p_0 the values of $K_1^*/K_2^*[W_0\sec\phi_0]_{max}$ and corresponding values of ξ, θ_* and other parameters are tabulated in Table 1.

5. The nature of solution and comparison with observations

There exist two types of equilibrium solutions for the value of $W_0\sec\phi_0$ in the range

$$\frac{K_2^*}{\sqrt{8}p_0}\le W_0\sec\phi_0\le[W_0\sec\phi_0]_{max} .$$

In the Type 1 solution the values of ξ and θ_* are lower than those in Type 2 solution, for the same value of $\frac{K_1^* W_0\sec\phi_0}{K_2^*}$ (Figs. 1,3). Solution of Type 1 does not exist for

$$W_0\sec\phi_0<\frac{K_2^*}{\sqrt{8}p_0} .$$

It can also be seen that in Type 1 solution the strength of the magnetic field is higher compared to that in Type 2 solution (Fig. 4). The value of gas pressure at the edge is higher in Type 1 solution than in Type 2 solution. Type 1 solutions are similar to the conditions in lower regions of the solar atmosphere and Type 2 solutions are similar to the conditions of higher regions of solar atmosphere. Both the solutions converge to a single solution at $W_0\sec\phi_0=[W_0\sec\phi_0]_{max}$.

Table 2. Values of $\xi_{\max}$ and θ_* for various values of K_1^*/p_0

$\frac{K_1^*}{p_0}$	$(\xi)_{max}$	θ_*
5	110.25	50.5
4	40.83	51.6
3	14.87	53.08
2	5.09	55.1
1.322	2.21	57.03

We find from Table 1 that for a given value of T_0 and T_* or equivalently for a given value of K_1^* and K_2^* the increase of central pressure results in the decrease in the value of $[W_0 \sec\phi_0]_{max}$ and ξ. In the region of lower p_0, the magnetic pressure dominates over plasma pressure. With the increase of p_0, the dip angle at the edge of the prominence corresponding to the maximum value of $W_0 \sec\phi_0$ increases and the strength of the magnetic field at the edge of the prominence corresponding to maximum value of $W_0 \sec\phi_0$ decreases. The maximum value of this dip angle at the edge corresponding to the maximum value of $W_0 \sec\phi_0$ is $\theta_* \approx 23$ and is reached for the value of $K_1^*/p_0 = 1.32$ where the value of $\frac{2\mu_0 p_0}{{B_*}^2 \sin\theta_*} = 1$. The minimum value of B_* is also reached close to this point. With the further increase of the value of p_0 the dip angle decreases, the strength of magnetic field increases and the plasma pressure dominates over the magnetic pressure. The value of ξ also decreases, and $p_0 \to p_*$. The range of values of $W_0 \sec\phi_0$ over which Type 1 solution exists is

$$\left[\frac{K_2^*}{\sqrt{8p_0}}, [W_0 \sec\phi_0]_{max}\right] .$$

This range gradually decreases with the increase of p_0. The value of θ_* also gradually decreases. Thus for higher value of p_0 the Type 2 solution is more likely to exist than Type 1 solution.

5.1. Study of variation of ξ and p_* with respect to the variation of $W_0 \sec\phi_0$ for a given value of K_1^*/p_0

The relation between ξ and $\frac{K_1^* W_0 \sec\phi_0}{K_2^*}$ is shown in Fig. 1. The relation between p_* and $\frac{K_1^* W_0 \sec\phi_0}{K_2^*}$ is shown in Fig. 2. For given set of values of T_0, T_* and p_0 the maximum value of ξ is reached where $\frac{\partial G}{\partial\theta_*} = 0$ (G is defined in eq (27)). This corresponds to the equation

$$\frac{K_1^*(1+\xi)}{p_0 \sin\theta_*}\left[\frac{(1+\xi)^2\sec^4\theta_* + 1}{[(1+\xi)^2\sec^2\theta_* + 1]^{1.5}}\right] = 2\xi\csc^2\theta_* \frac{\log\left[\sqrt{1+\xi}+\sqrt{\xi}\right]}{\sqrt{\xi(\xi+1)}} . \quad (29)$$

The simultaneous solution of the equations G = 0 and (29) gives the maximum value of ξ and the corresponding value of θ_*. For the various values of K_1^*/p_0 we have calculated these values and are shown in Table 2. It is evident that the maximum value of ξ for a given K_1^*/p_0 corresponds to the minimum value of p_* and the maximum value of B_{z*} for existence of equilibrium.

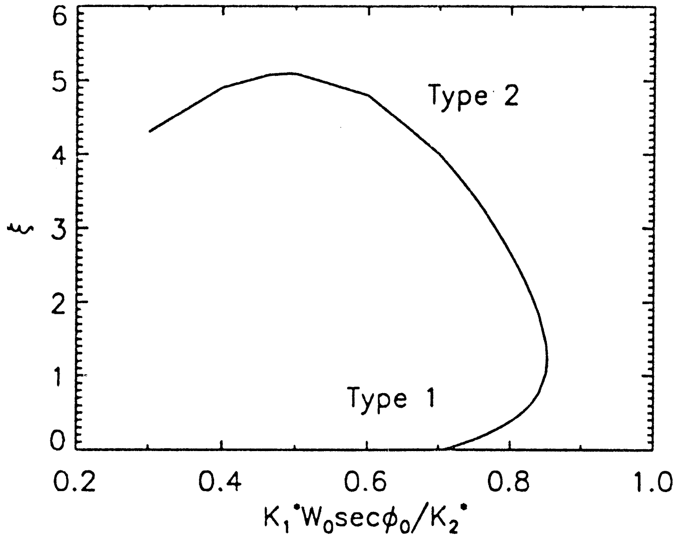

Fig. 1. Variation of $\xi(= \frac{p_0}{p_*} - 1)$ with $\frac{K_1^*(W_0 \sec\phi_0)}{k_2^*}$ for $\frac{K_1^*}{p_0} = 2$.The maximum value of $\xi \approx 5.0$.

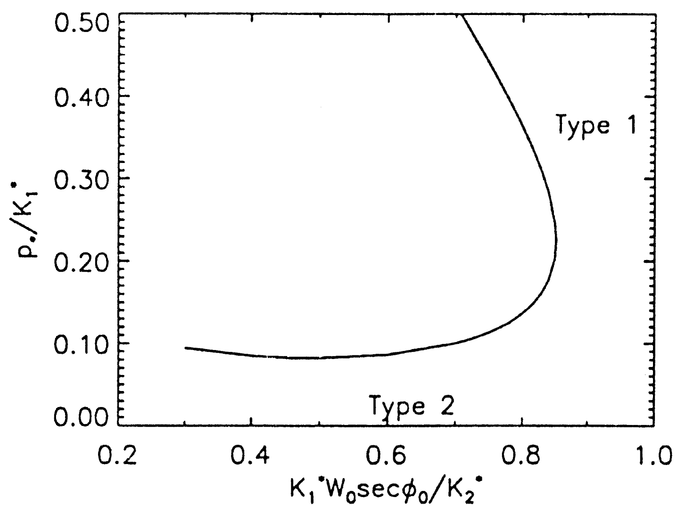

Fig. 2. Variation of $\frac{p_0}{K_1^*}$ with $\frac{K_1^*(W_0 \sec\phi_0)}{k_2^*}$ for $\frac{K_1^*}{p_0} = 2$. The maximum value of $\frac{p_*}{K_1^*} = 0.5$, minimum value of $\frac{p_*}{K_1^*} \approx 0.082$,

5.2. Study of variation of the magnetic field line length(s) with respect to the variation of $W_0 \sec\phi_0$ for a given value of K_1^*/p_0

The value of the magnetic field line length ‘s’ can be calculated by using the Eq. (30) for the various values of $W_0 \sec\phi_0$ the corresponding value of ‘s’ is shown in Fig. 5.

$$s = \frac{K_1^*(1+\xi)}{p_0[1+\cos\theta_*]\sqrt{(1+\xi)^2\sec^2\theta_* + 1}} . \quad (30)$$

We have to find the extremum of ‘s’ for a given K_1^*/p_0 subject to the condition $G = 0$. Following the procedure similar to that described in Sect. 4 for finding the maximum value of $W_0 \sec\phi_0$ we have calculated the maximum value of ‘s’ for various values of K_1^*/p_0. These are tabulated in the Table 3.

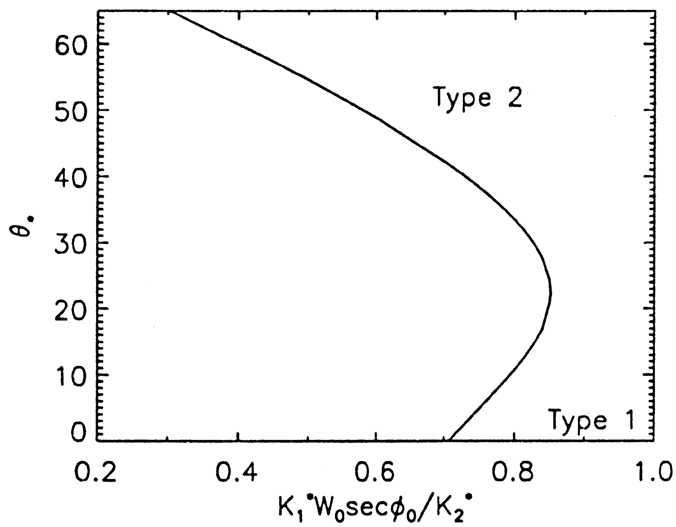

Fig. 3. Variation of θ_* with $\frac{K_1^*(W_0 \sec\phi_*)}{k_2^*}$ for $\frac{K_1^*}{p_0} = 2$.

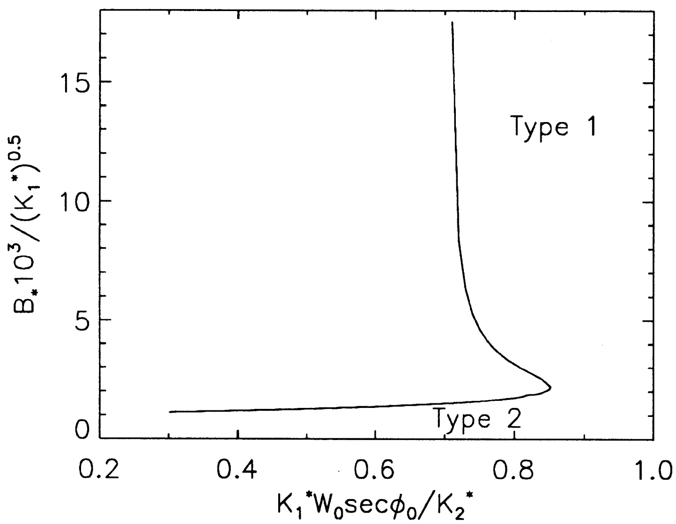

Fig. 4. Variation of strength of the magnetic field B_* with $\frac{K_1^*(W_0 \sec\phi_0)}{k_2^*}$ for $\frac{K_1^*}{p_0} = 2$.

Table 3. Values of $(s)_{max}$ for various values of K_1^*/p_0

$\frac{K_1^*}{p_0}$	ξ	θ_*	$\frac{K_1^* S_{max}}{K_2^*}$	$\frac{K_1^* W_0 \sec\phi_*}{K_2^*}$
10	12.03	10.89	4.94	4.89
9	10.78	11.74	4.37	4.39
8	9.52	12.76	3.93	3.88
7	8.25	13.98	3.43	3.38
6	6.97	15.49	2.93	2.87
5	5.68	17.39	2.42	2.36
3	3.09	23.17	1.40	1.34
2	1.83	27.58	0.90	0.84
1.21	0.92	31.57	0.51	0.47
0.95	0.66	32.59	0.39	0.35

It is found that 's' will have a minimum value (s_{min}) which is reached when the value of both ξ and θ_* are 0. For a given set of T_0, T_* and p_0 the minimum value of 's' is equal to $\frac{K_2^*}{p_0\sqrt{8}}$. This situation corresponds to an isobaric plasma of pressure p_0, maintained at a central temperature T_0 and at an edge temperature of T_* in the presence of a strong horizontal magnetic field. Since we have assumed that thermal conduction is only along the magnetic field line, the length $s_{min}/2$ is the minimum length of separation between the temperatures T_0 and T_* required to maintain the thermal equilibrium of an isobaric plasma of pressure p_0. In this case due to the absence of vertical component of magnetic field, the mechanical equilibrium is not possible.

Due to the presence of vertical component of magnetic field, the length of the magnetic field line increases, both thermal and mechanical equilibrium are maintained. With the increase of $W_0 \sec\phi_0$ the length of the magnetic field line increases. It reaches its maximum value close to the maximum value of $W_0 \sec\phi_0$. This is the maximum distance over which the temperature difference of T_* to T_0 can be maintained for existence of both thermal and mechanical equilibrium of plasma maintained at a central pressure p_0.

5.3. *Nature of the solution for various values of T_0, T_*, θ_* and p_0*

In Table 4 we have given values of the strength of the magnetic field at the edge of the prominence (B_*) and gas pressure at the edge of the prominence (p_*), for various values of θ_* and for various combinations of T_0, T_* and p_0. From Table 4 we draw the following conclusions.

With the increase of θ_*, keeping (T_0, T_*, and p_0) fixed, the strength of the magnetic field decreases and gas pressure at the edge decreases.

With the increase of T_*, keeping (T_0, p_0 and θ_*)fixed, it is found that the strength of the magnetic field increases and the gas pressure at the edge decreases.

With the increase of T_0, keeping (T_*, p_0 and θ_*) fixed, it is found that the strength of the magnetic field decreases, p_* increases.

With the increase of p_0, keeping (T_0, T_* and θ_*) fixed, the strength of the magnetic field increases p_* also increases.

It is also found that the B_* is sensitively depend on T_0 and θ_* than p_0 and T_*.

5.4. *Comparison with observations*

Bommier et al. (1994), from their observations concluded the following properties of the inverse and normal type of prominences.

(1) For the Inverse prominences

- the strength of magnetic field ranges from 2.3 G–15.1 G with an average of 7.5 G
- the shear angle ϕ_0 ranges from 6 to 84 degree with an average of 54^0
- the dip angle θ_* ranges from 0–45 degree with an average of 29^0

Table 4. Values 0f ξ_*, B_* and p_* for various values of θ_*

(T_0 = 4400 K T_* = 8000 K p_0 = 1.012×10^{-3} Pascal)

θ_*	ξ	B_* in Gauss	$p_* \times 10^4$ in Pascal
5	0.88	3.96	5.38
10	3.80	2.58	2.11
15	12.69	.74	1.88
20	38.72	.25	1.46

(T_0 = 4400 K T_* = 12,000 K p_0 = 1.012×10^{-3} Pascal)

θ_*	ξ	B_* in Gauss	$p_* \times 10^4$ in Pascal
5	1.01	4.1	5.04
10	4.65	2.65	1.79
15	16.85	1.89	0.57
20	56.06	1.46	0.17

(T_0 = 6000 K T_* = 8,000 K p_0 = 1.012×10^{-3} Pascal)

θ_*	ξ	B_* in Gauss	$p_* \times 10^4$ in Pascal
5	0.1372	2.010	8.90
10	0.32	1.44	7.64
15	0.58	1.18	6.39
20	0.94	1.03	5.23

(T_0 = 4400 K T_* = 12,000 K p_0 = 2×10^{-3} Pascal)

θ_*	ξ	B_* in Gauss	$p_* \times 10^4$ in Pascal
5	0.35	4.13	1.48
10	1.03	2.91	9.85

– the electron density, ranges from (2.5 to 63)×10^9 cm^{-3} with an average of 2.1×10^{10} cm^{-3}

(2) For Normal Prominences

– the average magnetic field is found to be 13.2 G
– the average ϕ_0 is found to be 37^0
– the average value of θ_* is zero
– the average electron density is found to be 8.7×10^9 cm^{-3}

Comparing our solution with these observational results we conclude the following

Type 1 solution simulates normal prominence as B_* is high and θ_* is small

Type 2 solution simulates the inverse prominence result as B_* is low and θ_* is high

Type 2 solution is exist for the large value of N_0 corresponding to high particle density and pressure which is in agreement with our conclusion drawn previously.

6. Conclusions and discussion

For given values of temperature at the center of the prominence (T_0), temperature at the edge of prominence (T_*) from these values the values of the parameter K_1^* and K_2^* can be found out

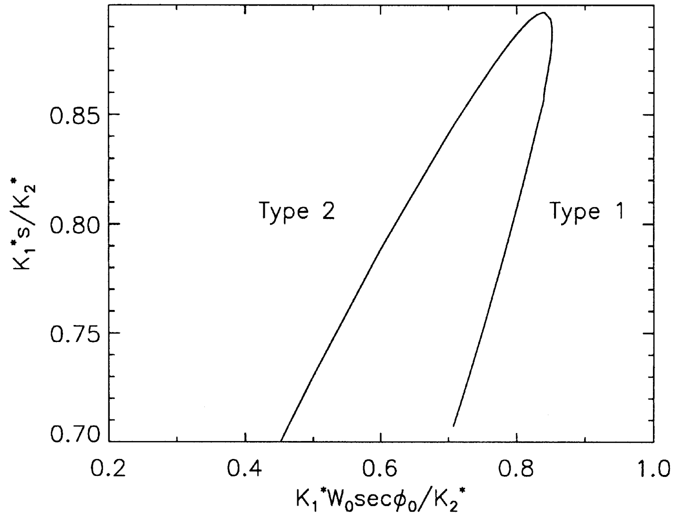

Fig. 5. Variation of 's' with $\frac{K_1^*(W_0 \sec\phi_0)}{k_2^*}$ for $\frac{K_1^*}{p_0} = 2$.

from the Eqs. (24) and (26).Using these values and the value of gas pressure at the center of prominence p_0, we have found the maximum value of $W_0 \sec\phi_0$ (where W_0 is the width of the prominence and ϕ_0 is the shear angle) up to which the solutions for the Eqs. (23) and (25) will exist. For the value of $W_0 \sec\phi_0 \leq \frac{K_2^*}{p_0\sqrt{8}}$, there exists only one type of solution. For the value of $W_0 \sec\phi_0$ in the range $\frac{K_2^*}{p_0\sqrt{8}} \leq W_0 \sec\phi_0 < [W_0 \sec\phi_0]_{max}$, there exist two type of solutions. Both the solutions coincide at $W_0 \sec\phi_0 = [W_0 \sec\phi_0]_{max}$.

One having lower value of ξ (which is nothing but $p_0/p_* - 1$ where p_* is the plasma pressure at the edge of the prominence) and θ_* (dip angle at the edge of the prominence) corresponds to the lower region of solar atmosphere is called Type 1 solution. The other solution corresponds to the higher region of solar atmosphere is called Type 2 solution. In the Type 1 solution an equilibrium of isobaric plasma in the presence of strong and horizontal magnetic field can change over to a prominence type of solution with the increase of $W_0 \sec\phi_0$. Similarly in the Type 2 solution, an equilibrium with a high difference of pressure in the presence of a weak magnetic field having dominant vertical component can change over to a prominence type of solution with the increase of $W_0 \sec\phi_0$.

It is found that the range of values of $W_0 \sec\phi_0$ over which Type 1 solution exists decreases with the increase of p_0. Thus for the higher value of p_0 the Type 2 solution is more likely to exist than Type 1 solution. Similar conclusions were drawn from Bommier et al. (1994). This appears to be contradictory with our earlier conclusion that Type 1 solution is similar to the condition in the lower region of solar atmosphere, and Type 2 solution correspond to the higher regions in solar atmosphere. But this is not the case for the following reason. From the integration of eq (2), we have

$$[\rho] = \frac{2{B_*}^2(1+\cos^2\theta_*)}{\mu_0 g[K_2^*(1+\xi)]}\sqrt{[(1+\xi)^2\sec^2\theta+1]}\sin\theta_*$$

where $[\rho]$ represent mean density and is proportional to $\sin\theta_*$ from the above equation. In Type 2 solution and in the inverse type of prominence the value of θ_* is large when compared to Type 1 solution or in normal prominence, that is why the value of $[\rho]$ in Type 2 solution or in inverse type of prominence is greater than that of Type 1 solution or of normal prominences.

So far in our study, from the given set of values for T_0 and T_* we have calculated the values of K_1^* and K_2^*, the value of K_1^*/p_0 in turn determines the nature of the solution,they in turn determine the other parameters. While calculating K_1^* and K_2^* to get an analytical solution we have assumed that heating at the center of prominence is equal to one-fifth of the radiation loss at the center. From the Eq. (24) we can derive K_1^* as,

$$K_1^* = g\sqrt{\frac{K_0}{10}}\int_{f_0}^{f_*}\frac{df}{E}$$

where

$$E = \sqrt{\int_{f_0}^{f_*}\frac{[\chi\rho^2T^\alpha - E_H p\rho]df}{\rho^2 T}}\ , \quad \text{and } f = T^{2.5}$$

If we vary the value of heating rate at the center or the value of E_H, within the permissible range as mentioned previously, the value of K_1^* will vary, but the nature of the solution will be the same. It can be seen that with the increase of E_H the value of K_1^* will increase as seen from the above equation. It must be remembered that these results are only qualitative since it is based on simplifying assumptions about heating function and radiative loss. The real situations will be quantitatively different.

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