Empirical Determination of Threshold Partial Wave Amplitudes in $p\,p \to p\,p\,\omega$

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Abstract

Using the model independent irreducible tensor approach to ω production in pp collisions, we show theoretically that, it is advantageous to measure experimentally the polarization of ω, in addition to the proposed experimental study employing a polarized beam and a polarized target.

Threshold production of light as well as heavy mesons in NN collisions has attracted considerable attention [1, 2] in recent years, as the reactions are sensitive to the short range NN interaction and involve only a few partial waves. Experimental studies have reached a high degree of sophistication with measurements of spin observables in charged [3] as well as neutral [4] pion production in $\vec{p}\vec{p}$ collisions. Amongst the various proposed theoretical models including those which invoke subnucleonic degrees of freedom, the Julich meson exchange model [5] may be said to have yielded theoretical predictions which are nearer to data. Although this model was more successful in the case of charged pions [3], it failed to provide an overall satisfactory reproduction of the data on neutral pions [4]. Both Moskal et al[1]., and Hanhart [1], have remarked that " apart from rare cases, it is difficult to extract a particular piece of information from the data". In this context, a model independent approach [6] which was developed using irreducible tensor techniques [7] has been employed [8] to analyze the data [4] on $\vec{p}\vec{p} \to pp\pi^0$ and Deepak, Haidenbauer and Hanhart [8] have recently found that the Julich model deviates very strongly from empirically extracted estimates for the $^3P_1 \to {}^3P_0 p$ and to a lesser extent for the $^3F_3 \to {}^3P_2 p$. They also find that the Δ degree of freedom is important for the quantitative understanding of the reaction. The analysis has reiterated once again the importance of Δ contribution which has been noted in several earlier studies [9]. The rich spin structure [10] of $NN \to N\Delta$ and $N\Delta \to N\Delta$ has also been analyzed. Of the sixteen amplitudes associated with $NN \to N\Delta$ as many as ten are second rank spin tensors and Ray [11] has drawn attention to their importance based on a partial wave expansion model where he found that "the total and differential cross-section reduced by about one half, the structure in the analyzing powers increased dramatically, the predictions of D_{NN} became much too negative, while that for D_{LL} became much too positive and the spin correlation predictions were much too small when all ten of the rank 2 tensor amplitudes were set to zero, while the remaining six amplitudes were unchanged." The study of meson production has also focussed attention on the "missing resonance problem" [12] which refers to the predicted [13] highly excited N^* states which have not been seen in πN scattering. Moreover heavy meson production not only probes distances [2] which are shorter than that in the case of pion production, but also the strange quark content of the nucleon. In particular the cross-section ratio for $NN \to NN\omega/\phi$ has been measured [14] in view of the dramatic violations [15] of the OZI [16] rule observed in $\bar{p}p$ collisions. Heavy meson production has also attracted attention in the context of dilepton spectra and medium modifications [17]. In particular the total cross-sections for $pp \to pp\omega$ have been measured [18]at five c.m. energies in the range 3.8 MeV to 30 MeV above threshold and the total and differential cross-sections [19] at 92 and 173 MeV above threshold. There is also a proposal [20] to study experimentally the heavy meson production in $\vec{N}\vec{N}$ collisions. A model independent irreducible tensor formalism [21] has recently been developed to analyze such measurements where it was also pointed out that the polarization of ω can be studied by looking at the decay $\omega \to \pi^0\gamma$.

The purpose of the present paper is to point out that measurements of the differential cross-section together with the spin polarization of ω and the analyzing powers are sufficient to determine empirically the leading partial wave amplitudes at threshold without any discrete ambiguities. The partial wave amplitudes not only depend on the c.m. energy E at which the reaction takes place but also on the invariant mass W give in natural units by

$$W = (E^2 + M_\omega - 2EE_\omega)^{1/2}, \tag{1}$$

of the two protons in the final state, where E_ω denotes the energy of the meson and M_ω its rest mass. If $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ denote respectively the initial and final relative momenta between the two protons in their respective c.m frames, we have

$$E^2 = 4(p_i^2 + M^2),\; W^2 = 4(p_f^2 + M^2), \tag{2}$$

where M denotes the rest mass of the proton. Choosing W (or equivalently E_ω)and the polar angles (θ_f, φ_f) of $\boldsymbol{p}_f$ together with the polar angles (θ, φ) of the meson momentum $\boldsymbol{q}$ in the c.m frame as the five independent kinematical variables, we may write the unpolarized differential cross section, as

$$\begin{aligned} \frac{d^5\sigma_0}{dW d\Omega d\Omega_f} &= (2\pi)^{-5} \frac{W\, E_\omega\,(E - E_\omega)}{16\, p_i}\, q\, p_f \mathtt{T}r(TT^\dagger) \\ &= \frac{1}{4}\mathtt{T}r(\mathcal{M}\mathcal{M}^\dagger), \end{aligned} \tag{3}$$

in a kinematically complete experiment, where T denotes the on-energy-shell transition-matrix for the reaction, $T^\dagger$ its hermitian conjugate and $\mathtt{T}r$ denotes trace. Following [7] we may express $\mathcal{M}$, in a model independent way, as

$$\mathcal{M} = \sum_{s_f,s_i=0}^{1} \sum_{\lambda=|s_f-s_i|}^{(s_f+s_i)} \sum_{S=|1-s_f|}^{(1+s_f)} \sum_{\Lambda=|S-s_i|}^{(S+s_i)} ((S^1(1,0) \otimes S^\lambda(s_f, s_i))^\Lambda \cdot \mathcal{M}^\Lambda(Ss_f s_i; \lambda)), \tag{4}$$

where the irreducible tensor amplitudes $\mathcal{M}^\Lambda_\nu(Ss_f s_i; \lambda)$ of rank Λ are given by

$$\begin{aligned} \mathcal{M}^\Lambda_\nu(Ss_f s_i; \lambda) &= W(1 s_f \Lambda s_i; S\lambda)[\lambda] \sum_{j,ll_f L l_i} f^j_{Ss_f s_i, ll_f L l_i}\, W(s_i l_i S L; j\Lambda) \\ &\times ((Y_l(\hat{\boldsymbol{q}}) \otimes Y_{l_f}(\hat{\boldsymbol{p}}_f))^L \otimes Y_{l_i}(\hat{\boldsymbol{p}}_i))^\Lambda_\nu, \end{aligned} \tag{5}$$

where (l_i, s_i) and (l_f, s_f) characterize respectively initial and final states of the NN system in terms of their relative orbital angular momentum and total spin quantum numbers, j denotes the total angular momentum which is conserved and l, the orbital angular momentum of the emitted spin 1 meson. The channel spin quantum number S in the final state is the resultant of combining the spin 1 of the ω with s_f and likewise l and l_f combine to give L.The partial wave amplitudes

$$\begin{aligned} f^j_{Ss_f s_i, ll_f L l_i} &= (4\pi)^{-2}(-1)^{L+l_i+s_i-j}[j]^2[S][1]^{-1}[s_f]^{-1} \\ &\times \langle (((ll_f)L(1s_f)S)j||T||(l_i s_i)j\rangle, \end{aligned} \tag{6}$$

depend only on the c.m. energy E, and invariant mass W of the two nucleon system in the final state. The above equations are valid for all c.m energies E. In particular, at threshold, we may set $l_f = 0$. In view of the observed [19] anisotropic angular distribution of ω at 173 MeV excess energy above threshold, we may take into consideration both $l = 0$ and 1. We then have to consider only two irreducible tensor amplitudes

$$\mathcal{M}^1_\nu(101; 1) = (12\sqrt{3}\pi)^{-1} Y_{1\nu}(\hat{\boldsymbol{p}}_i) f_1 \tag{7}$$

$$\mathcal{M}^1_\nu(100; 0) = (12\pi)^{-1} Y_{1\nu}(\hat{\boldsymbol{q}}) f_2 + (6\sqrt{5}\pi)^{-1}(Y_1(\hat{\boldsymbol{q}}) \otimes Y_2(\hat{\boldsymbol{p}}_i))^1_\nu f_3, \tag{8}$$

where the short hand notation f_1, f_2, f_3 is used for convenience to denote threshold partial wave amplitudes shown in Table I. After integration with respect to $d\Omega_{p_f}$, the unpolarized differential cross-section is obtained as

$$\frac{d^3\sigma_0}{dW d\Omega} = \frac{1}{192\pi^2}[a_0 + \frac{9}{10}\, a_2\, cos^2\theta], \tag{9}$$

where an experimental measurement of (9) readily enable us to determine the coefficients

Table 1: Threshold partial wave amplitudes for $pp \to pp\omega$.

Partial Wave Amplitudes	Initial State	Final State
$f_1 = f^1_{101;0001}$	$^3P_1$	$(^1Ss)\,^3\mathcal{S}_1$
$f_2 = f^0_{100;1010}$	$^1S_0$	$(^1Sp)\,^3\mathcal{P}_0$
$f_3 = f^2_{100;1012}$	$^3D_2$	$(^1Sp)\,^3\mathcal{P}_2$

$$a_0 = [|f_1|^2 + 3|f_2 + \frac{1}{\sqrt{10}} f_3|^2] \tag{10}$$

$$a_2 = [|f_3|^2 - 2\sqrt{10}\,\Re(f_2 f_3^*)] \tag{11}$$

The state of polarization of ω, with c.m energy E_ω, may be defined in terms of its spin density matrix ρ whose elements are given by

$$\rho_{mm'} = \frac{1}{4}\sum_{s_f m_f}\int d\Omega_f \langle s_f m_f; 1m|\mathcal{M}\mathcal{M}^\dagger|1m'; s_f m_f\rangle \tag{12}$$

$$= \frac{\mathtt{Tr}\rho}{3}\sum_{k=0}^{2}(-1)^q C(1k1; m'-qm)[k]t^k_q, \tag{13}$$

in terms of Fano statistical tensors t^k_q of rank k such that $\mathtt{Tr}\rho$ is given by (9) and $t^0_0 = 1$.

It is advantageous now to express the vector and tensor polarizations t^1_q and t^2_q in the transverse frame which is a right handed frame whose Z-axis is chosen along $\boldsymbol{p}_i \times \boldsymbol{q}$ with $\boldsymbol{p}_i$ along X-axis. It may be noted that the polar angles of $\boldsymbol{q}$ in this frame are $(\frac{\pi}{2}, \theta)$ if we continue to use θ to denote the angle between $\boldsymbol{q}$ and $\boldsymbol{p}_i$, i.e. $\boldsymbol{q}\cdot\boldsymbol{p}_i = q\,p_i\,cos\theta$. We then have

$$\mathtt{Tr}\rho\; t^1_0 = \frac{3}{64\pi^2}\sqrt{\frac{3}{5}}\,b\,sin\theta\,cos\theta \tag{14}$$

$$\mathtt{Tr}\rho\; t^2_0 = \frac{1}{384\pi^2}\sqrt{\frac{1}{2}}\,[c_0 + \frac{18}{10}\,c_2\,cos^2\theta] \tag{15}$$

$$\mathtt{Tr}\rho\; t^2_{\pm 2} = \frac{1}{256\pi^2}\sqrt{\frac{1}{3}}\,[d_0 - 12\,d_2\,cos^2\theta \mp 6\,i\,d_3\,sin2\theta] \tag{16}$$

in the transverse frame. The coefficients are given by

$$b = -\Im(f_2 f_3^*) \tag{17}$$

$$c_0 = [-|f_1|^2 + 6|f_2 + \frac{1}{\sqrt{10}} f_3|^2] \tag{18}$$

$$c_2 = a_2 \tag{19}$$

$$d_0 = [|f_1|^2 + 6|f_2 + \frac{1}{\sqrt{10}} f_3|^2] \tag{20}$$

$$d_2 = [|f_2|^2 + \frac{1}{4}|f_3|^2 - \frac{1}{\sqrt{10}}\Re(f_2 f_3^*)] \tag{21}$$

$$d_3 = [|f_2|^2 - \frac{1}{5}|f_3|^2 - \frac{1}{\sqrt{10}}\Re(f_2 f_3^*)] \tag{22}$$

Thus the experimental measurement of differential cross-section (9) and t^k_q enable us to determine empirically

$$
\begin{aligned}
|f_1|^2 &= \frac{1}{2}(d_0 - c_0) && (23)\\
|f_2|^2 &= \frac{1}{36}(d_0 + c_0 + 8d_2 + 16d_3) && (24)\\
|f_3|^2 &= \frac{20}{9}(d_2 - d_3) && (25)\\
\Re(f_2 f_3^*) &= \frac{\sqrt{10}}{36}(d_0 + c_0 - 8d_2 - 4d_3) && (26)\\
\Im(f_2 f_3^*) &= -b && (27)
\end{aligned}
$$

Thus it is seen from our model independent theoretical analysis that priority should be given to measure the polarization of ω in $pp \to pp\vec{\omega}$. This enables us to determine empirically, not only the strengths of the partial wave amplitudes f_1, f_2, f_3 but also the relative phases between f_2 and f_3. Thus $|f_2 + \frac{1}{\sqrt{10}} f_3|$ is known except for an overall phase. The proposed study of $\vec{p}\vec{p} \to pp\omega$ can then be used to determine the relative phase between $(f_2 + \frac{1}{\sqrt{10}} f_3)$ and f_1 as follows.

If $\boldsymbol{P}$ and $\boldsymbol{Q}$ denote respectively the beam and target polarizations, the differential cross-section for $\vec{p}\vec{p} \to pp\omega$ is given by

$$\frac{d^3\sigma}{dW d\Omega} = \int d\Omega_{p_f} \mathtt{Tr}(\mathcal{M}\rho^i \mathcal{M}^\dagger) \tag{28}$$

where

$$\rho^i = \tfrac{1}{4}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{P})(1 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{Q}), \tag{29}$$

This leads to [21]

$$\frac{d^3\sigma}{dW d\Omega} = \frac{d^3\sigma_0}{dW d\Omega}[1 + \boldsymbol{P} \cdot \boldsymbol{A}^B + \boldsymbol{Q} \cdot \boldsymbol{A}^T + \sum_{k=0}^{2}((P^1 \otimes Q^1)^k \cdot A^k)] \tag{30}$$

The vector analyzing powers $\boldsymbol{A}^B, \boldsymbol{A}^T$ and $\boldsymbol{A}$ are normal to the reaction plane containing $\boldsymbol{p}_i$ and $\boldsymbol{q}$. Thus

$$
\begin{aligned}
A_Z^B - A_Z^T &= \frac{1}{16\pi^2}\sqrt{\frac{1}{6}}\,\Im[f_1(f_2 + \frac{1}{\sqrt{10}} f_3)^*] sin\theta, && (31)\\
A_Z &= \frac{i}{32\pi^2}\sqrt{\frac{1}{3}}\,\Re[f_1(f_2 + \frac{1}{\sqrt{10}} f_3)^*] sin\theta && (32)
\end{aligned}
$$

in the transverse frame.

Since $|f_1|$ is known from (23) the relative phase between f_1 and $(f_2 + \frac{1}{\sqrt{10}} f_3)$ is determinable without any trigonometric ambiguity from (31) and (32). Thus f_1, f_2 and f_3 are determinable empirically except for an overall phase.

In summary, therefore, we advocate measurement of polarization of ω in $pp \to pp\vec{\omega}$ in addition to the proposed experiments [20] on $\vec{p}\vec{p} \to pp\omega$, as this will enable the complete empirical determination of the leading threshold amplitudes f_1, f_2 and f_3 without any discrete ambiguities.

Acknowledgments

One of us (G.R.) is grateful to Professors B.V. Sreekantan, R. Cowsik, J.H. Sastry and R. Srinivasan for facilities provided for research at the Indian Institute of Astrophysics and another (J.B.) acknowledges much encouragement for research given by the Principal Dr. T.G.S. Moorthy and the Management of K.S. Institute of Technology.

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