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Non-parallel propagation of hydromagnetic surface waves in the presence of steady shear-flows

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Abstract. The combined effect of non-parallel propagation and steady shear-flows on the properties of hydromagnetic surface waves is examined for two different orderings of physical parameters that are expected at the edge of a hot and dense coronal loop and an isolated, photospheric magnetic flux-tube. It is found that a finite angle of inclination of the wavevector with respect to the magnetic field generally facilitates the propagation of surface waves by relaxing restrictions imposed on their phase velocities by the shear-flow. Non-parallel propagation, along with a shear-flow, can also give rise to *backward propagating surface modes* that may be subject to *negative energy instabilities*. Such a backward surface mode appears when the magnitude of the shear-flow exceeds a certain critical value. Such a critical flow is much larger than the magnitudes of the observed flows in solar coronal loops. Negative energy instabilities of hydromagnetic surface waves may not, therefore, play an important role in the energetics of the hot solar coronal loops. For a photospheric magnetic flux-tube, the critical flow for the appearance of a backward propagating slow surface mode is found to increase with the increase of the angle of propagation of the waves. The surface mode that is most prone to negative energy instabilities, therefore, seems to be the one that propagates parallel to the magnetic field lines.

Key words: Magnetohydrodynamics (MHD) – plasmas – waves – Sun: magnetic fields

1. Introduction

Hydromagnetic surface waves are believed to play important role in transporting mechanical energy from the subphotospheric layers of the Sun to the upper solar atmosphere and finally dissipating that energy into heat in the solar corona; see reviews by Hollweg (1990a,b), Roberts (1991) and Goossens (1991). Properties of these hydromagnetic surface waves are known to change considerably in the presence of steady shear-flows (see, Nakariakov & Roberts 1995; see also Satya Narayanan & Somasundaram 1985; Somasundaram & Satya Narayanan 1987; Satya Narayanan 1991; Nakariakov, Roberts & Mann et al. 1996). Particularly interesting in these studies is the appearance of *backward propagating waves* that are also the 'negative energy waves (NEW)' (Joarder et al. 1997). Such waves may be subject to *dissipative instabilities* in the presence of viscous, thermal or resistive diffusion or by resonant absorption process (cf. Ryutova 1988; Ruderman & Goossens 1995; Ruderman et al. 1996; Tirry et al. 1998; Andries et al. 2000). Most of these previous studies were confined to cases where all the magnetic fields, directions of the flows and the wavevector are parallel to each other and are also parallel to the interface of discontinuity in the plasma and the field parameters. In this paper, we remove one of these limitations by studying the important effect of non-parallel propagation of hydromagnetic surface waves in the presence of shear-flows. The effect of non-parallel propagation on hydromagnetic waves in the absence of shear-flows has already been analysed in detail in the literature (e.g. Somasundaram & Uberoi 1982; Uberoi 1982; Rae & Roberts 1983; Jain & Roberts 1991; Joarder & Roberts 1992; Satya Narayanan 1997). We extend these earlier analyses by including a steady shear-flow. We here note that Tirry et al. (1998) and Andries et al. (2000) have recently incorporated non-parallel propagation in their analysis of resonant instability of hydromagnetic surface waves in a thin, non-uniform transition layer of a zero - β plasma. In this paper, we confine ourselves to a single interface of discontinuity between two finite plasma - β media.

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2. The dispersion relation

Let x=0 be the interface of discontinuity separating two compressible, magnetic media (denoted by the suffix "e" in the region $x < 0$ and the suffix "0" in the region $x > 0$) that are infinitely extended in the y- and the z- directions with their Alfven speeds and the acoustic speeds being $v_{Ae},\ v_{A0}$ and $c_{se},\ c_{s0}$, respectively. Both the magnetic fields $(\mathbf{B_e}, \mathbf{B_0})$ and the steady flows $(\mathbf{U_e}, \mathbf{U_0})$ in the two media are directed along the z-axis, which is in the plane of the interface. The equilibrium densities ρ_e and ρ_0 of the two media are connected by the condition of total pressure balance, namely,

$$\frac{\rho_e}{\rho_0} = \frac{2c_{s0}^2 + \gamma v_{A0}^2}{2c_{se}^2 + \gamma v_{Ae}^2}, \quad (1)$$

where γ is the ratio of specific heats in both the media. We consider $\gamma = 5/3$ in our numerical examples in Sect. 3.

Following Roberts(1981a), we now linearize the ideal MHD equations about the above equilibrium. Elimination from the linearized equations of all variables except v_x, the amplitude of the x-component of velocity, yields the following second order ordinary differential equation:

$$\frac{d^2 v_x}{dx^2} - k_z^2 M_{(e,0)}^2 v_x = 0, \tag{2}$$

where,

$$M_{(e,0)}^2 = \frac{\left(v_{c(e,0)}^{+2} - \Omega_{(e,0)}^2\right)\left(v_{c(e,0)}^{-2} - \Omega_{(e,0)}^2\right)}{\left(v_{A(e,0)}^2 + c_{s(e,0)}^2\right)\left(c_{T(e,0)}^2 - \Omega_{(e,0)}^2\right)}, \tag{3}$$

and $\Omega_{(e,0)} = \omega/k_z - U_{(e,0)}$ are the Doppler-shifted, field-aligned phase-speeds in the two media. The speeds

$$c_{T(e,0)} = c_{s(e,0)} v_{A(e,0)} / \left(c_{s(e,0)}^2 + v_{A(e,0)}^2\right)^{1/2} \tag{4}$$

are the magnetoacoustic "cusp speeds" (e.g. Roberts 1981a) in the media "e" and "0", respectively. The quantities $v_{c(e,0)}^{\pm}$ are given by (Rae & Roberts 1983; Joarder & Roberts 1992)

$$v_{c(e,0)}^{\pm}(\theta) = \left\{ \frac{1}{2}\left(v_{A(e,0)}^2 + c_{s(e,0)}^2\right)\sec^2\theta \pm \frac{1}{2}\left[\left(v_{A(e,0)}^2 + c_{s(e,0)}^2\right)^2 \sec^4\theta - 4 v_{A(e,0)}^2 c_{s(e,0)}^2 \sec^2\theta\right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}, \tag{5}$$

that are simply the magnetoacoustic speeds along the magnetic fields $\mathbf{B}_{(\mathbf{e},\mathbf{0})}$, with "+" referring to the fast waves and "-" to the slow waves. In Eq. (5), θ is the angle of inclination of the wave vector $\mathbf{k}$ with the z-axis.

In the following, we restrict ourselves only to the *surface modes* (cf. Roberts 1981a) of the above equilibrium. This requires, that the amplitudes $v_x(x)$ evanescent away from the interface in either of the two media, thus implying that both M_0 and M_e are real and positive. Such assumption, of course, excludes *body waves*, for which $M_0^2 < 0$ (cf. Roberts 1981b), from our analysis.

Considering further that both the normal component of displacement and the perturbation in the total (gas + magnetic) pressure are continuous across the interface x = 0, we finally arrive at the dispersion relation for the hydromagnetic surface waves at the interface. This dispersion relation reads:

$$\rho_e\left(v_{Ae}^2 - \Omega_e^2\right)M_0 + \rho_0\left(v_{A0}^2 - \Omega_0^2\right)M_e = 0. \tag{6}$$

The dispersion relation reduces to the dispersion relation derived by Nakariakov & Roberts (1995) for $\theta = 0$. In the absence of shear flows ($U_{(e,0)} = 0$), Eq. (6) reduces to the dispersion relation derived by Joarder & Roberts (1992) in a limit, where the wavelengths of perturbations are sufficiently small ($k_y a \to \infty, M_0 > 0$) compared to the half width "a" of their model prominence slab.

3. Properties of the surface modes

The properties of the surface modes, that are revealed by the solutions of the dispersion relation (6), depend on the ordering of various physical speeds in the two media "e" and "0", respectively. For briefly explaining the nature of these surface modes, we consider two specific examples of such orderings of speeds that were earlier considered by Edwin & Roberts (1982). These two cases are described in the following.

3.1. $v_{Ae} > v_{A0} > c_{s0} > c_{T0} > c_{se} > c_{Te}$

This case is representative of situations found in dense solar coronal loop structures embedded in a rarer magnetic medium with a higher Alfven speed. We note that, for a purely parallel ($\theta = 0$) propagation, no surface wave solutions of Eq. (6) exists in this case with or without steady shear-flows (Edwin & Roberts 1982, 1983; Nakariakov & Roberts 1995). For non-parallel ($\theta \neq 0$) propagation, surface wave solutions may, however, exist even in the absence of steady flows (Uberoi 1982; Somasundaram & Uberoi 1982). The properties of such surface waves in the presence of steady shear-flows are discussed below.

Consider first the case where there is no flow in region "e" external to the loop ($U_e = 0$) and the material in region "0" inside the loop flows in the negative z - direction, i.e., $U_0 < 0$. In this paper, we mostly confine ourselves to the discussion of waves propagating in the positive z - direction ($k_z > 0$). With $U_0 < 0$, we then consider a coronal flow in a direction opposite to the direction of the field- aligned phase propagation of the waves. In this case, a surface wave solution of Eq. (6) must satisfy the condition

$$v_{A0} - |U_0| < c < v_{Ae}, \tag{7}$$

along with the conditions $M_{(e,0)} > 0$. It is evident that the *slow surface waves*, requiring $c < c_{T0} - |U_0|$, cannot exist in such a situation. *Fast surface waves*, requiring $v_{c0}^- < \Omega_0 < v_{c0}^+$, may, however, exist. Depending on the strength of the flow $|U_0| = -U_0$, there may be two such fast surface modes, namely, (i) the *(externally fast) fast surface mode* and (ii) the *(externally slow) fast surface mode*; see Joarder & Roberts (1992) for a discussion of such fast surface waves in a somewhat different context.

(i) *(Externally fast) Fast surface mode*: The field aligned phase speed $c \equiv \omega/k_z$ for these waves satisfies the condition

$$max(v_{ce}^-(\theta), v_{A0} - |U_0|) < c < min(v_{c0}^+(\theta) - |U_0|, v_{Ae}). \tag{8}$$

Such waves 'disappear' when the strength of the flow-speed is large enough so that

$$|U_0| \geq (v_{c0}^+(\theta) - v_{ce}^-(\theta)). \tag{9}$$

The mode that 'disappears' become radiative and their energy 'leaks' at a large distance $|x| \to \infty$ from the interface (Roberts & Webb, 1979).

Fig. 1a depicts the variation of phase speed c with the magnitude of the flow speed $|U_0|$ for the (externally fast) fast surface mode propagating at a particular angle $\theta = 10°$ with respect to the direction of the magnetic field $\mathbf{B}_{\mathbf{0},\mathbf{e}}$. For the particular

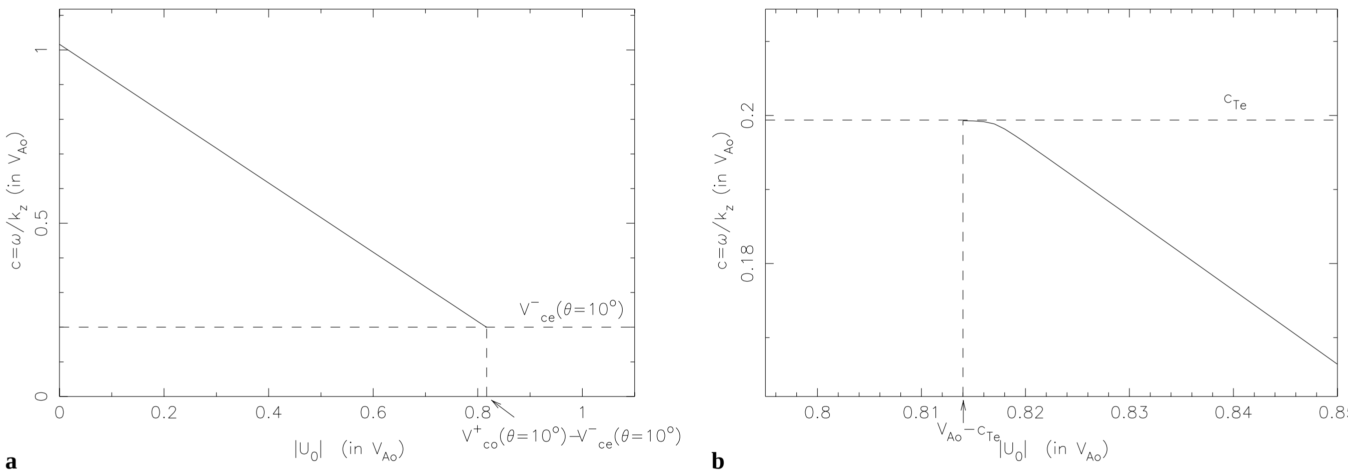

Fig. 1a and b. Field aligned phase-speed c vs. magnitude of the flow speed $|U_0| = -U_0$ for **a** the *(externally fast) fast surface mode* and **b** the *(externally slow) fast surface mode* in a case, where $v_{Ae} > v_{A0} > c_{s0} > c_{T0} > c_{se} > c_{Te}$, $U_0 < 0$, and for an arbitrarily chosen angle of inclination $\theta = 10°$. The numerical values chosen for various physical parameters are: $v_{A0} = 1.0, v_{Ae} = 2.5, c_{s0} = 0.3, c_{se} = 0.2, \rho_e/\rho_0 = 0.175$ and $U_e = 0.0$. Note that, in **a**, the (externally fast) fast surface mode 'disappears' when $|U_0| \approx 0.8165 v_{A0} = v^+_{c0}(\theta = 10°) - v^-_{ce}(\theta = 10°)$. Note also the upper cut-off $c = c_{Te}$ in the phase speed c of the (externally slow) fast surface mode **b**. This mode can propagate only when the magnitude of the flow speed exceeds a value $|U_0| = v_{A0} - c_{Te} \approx 0.8006 v_{A0}$.

choice of parameters considered in this diagram (see the figure caption below), the fast surface wave disappears for a flow speed $U_0 \leq -0.8165$ satisfying Eq. (9). The phase speed of the mode at this 'lower cut-off' is given by $c = v^-_{ce}(\theta = 10°)$.

(ii)*(Externally slow) Fast surface mode*: This mode appears when the strength of the flow satisfies the condition: $(v_{A0} - |U_0|) < c_{Te}$, with its phase speed c lying in a domain

$$(v_{A0} - |U_0|) < c < min(v^+_{c0}(\theta) - |U_0|, c_{Te}). \tag{10}$$

Fig. 1b demonstrates the variation of the phase - speed c for this fast surface mode with the magnitude of the flow speed $|U_0|$, again for a particular angle of propagation $\theta = 10°$. Eq. (10) shows that the field-aligned phase - speed c of this mode becomes negative as $|U_0|$ exceeds the Alfven speed v_{A0}; i.e., the mode becomes a *backward propagating fast surface mode* that changes its direction of propagation to be carried along with a strong oppositely directed flow. Such a super-Alfvenic downflow speed is excluded in our present analysis as they are rarely met in the solar coronal situation (Nakariakov & Roberts 1995; see also a summary of recent TRACE observations of coronal flows by Schrijver et al. 1999).

As θ approaches $\pi/2$, $v^-_{ce}(\theta)$ approaches c_{Te} and $v^+_{c0}(\theta)$ approaches infinity. Both the (externally fast and slow) fast surface modes still exist with their phase propagation windows given by:

$$max(c_{Te}, v_{A0} - |U_0|) < c < v_{Ae}, \tag{11}$$

for the *(externally fast) fast surface mode* and

$$(v_{A0} - |U_0|) < c < c_{Te}, \tag{12}$$

for the *(externally slow) fast surface mode*. Note that, unlike in the case of a finite propagation angle θ, the lower cut-off for the (externally fast) fast surface wave is now removed. The mode can now propagate for a much larger flow speed $|U_0|$ unless the flow becomes so large ($\approx 2v_{A0}$; see Nakariakov & Roberts 1995) so that the Kelvin-Helmholtz instability (e.g., Chandrasekhar 1961) sets in. We exclude a discussion of such instability in the present paper and confine ourselves to much smaller magnitudes of the flow - speed U_0.

In this limit of $\theta \to \pi/2$, the phase - speed c of the (externally fast) fast surface wave assumes the form

$$c_a = -\left[\frac{\rho_0}{\rho_0 + \rho_e}\right]|U_0| + \left\{\left[\frac{\rho_0 v_{A0}^2 + \rho_e v_{Ae}^2}{\rho_0 + \rho_e}\right] - \left[\frac{\rho_0 \rho_e}{(\rho_0 + \rho_e)^2}\right]|U_0|^2\right\}^{1/2}. \tag{13}$$

With $U_e = 0$, Eq. (13) coincides with the expression given by Chandrasekhar (1961) for the phase speed of an *incompressible surface wave* propagating along the direction of the magnetic field in the presence of shear-flows. Such asymptotic behaviour of certain compressible surface waves was pointed out earlier by Roberts (1981a), Somasundaram & Uberoi (1982), Joarder & Roberts (1992).

Consider now the case where the coronal flow is in the direction of the field- aligned phase propagation of the waves, i.e., $U_0 > 0$. Only one hydromagnetic surface mode, namely, the (externally fast) fast surface mode is of some interest. The phase speed c of the mode satisfies the condition

$$(v_{A0} + U_0) < c < min[(v^+_{c0}(\theta) + U_0), v_{Ae}]. \tag{14}$$

Fig. 2 depicts the variation of the longitudinal phase - speed c with the increase of the flow speed U_0 for these fast surface waves for a particular angle of propagation $\theta = 10°$. This figure shows, that the mode can propagate as long as the flow - speed satisfies the condition $U_0 < (v_{Ae} - v_{A0})$. As θ approaches $\pi/2$,

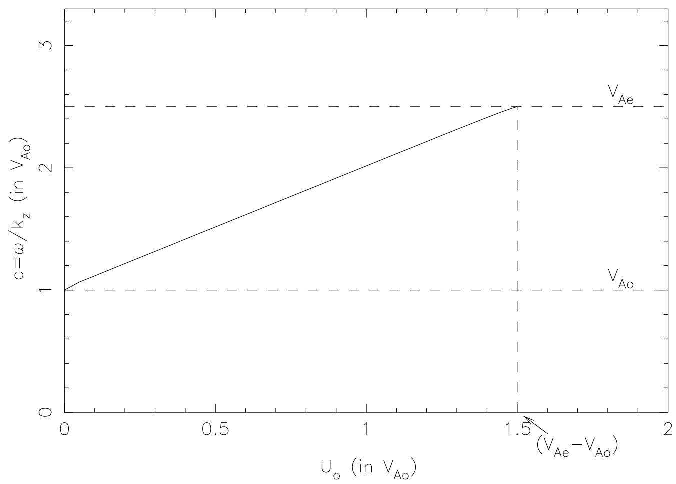

Fig. 2. Field-aligned phase-speed c vs flow-speed U_0 for the (externally fast) fast surface mode in a case, where, $v_{Ae} > v_{A0} > c_{s0} > c_{T0} > c_{se} > c_{Te}$ and $U_0 > 0$. The angle of propagation is arbitrarily chosen to be $\theta = 10°$. The numerical values of various other parameters are the same as in Fig. 1. Note, that the (externally fast) fast surface mode can only propagate when $U_0 < (v_{Ae} - v_{A0}) = 1.5v_{A0}$.

the limiting value for the phase speed c of this (externally fast) fast surface wave is again given by Eq. (13), but with $|U_0| = U_0$.

We also note that, there is *backward surface mode* propagating in the direction of the flow as the flow speed exceeds the Alfven speed v_{A0} of the internal plasma. We identify this mode as the backward propagating part of the (externally slow) fast surface mode discussed in the case of $U_0 < 0$.

3.2. $v_{Ae} = 0, v_{A0} > c_{se} > c_{s0} > c_{T0}$

This case is similar to the situation observed in an isolated, cool, photospheric flux-tube embedded in a field-free photospheric surrounding. As in Nakariakov & Roberts (1995), there is no flow in the region $x > 0$ so that $U_0 = 0$. The field-free medium at $x < 0$, however, has a downflow $U_e < 0$. If we assume the z-axis to point towards the upper atmosphere of the Sun, the negative flow in region "e" may then represent the granular/ supergranular downdraught motion surrounding the photospheric flux tube.

For purely parallel ($\theta = 0$) phase-propagation, we get two surface modes, namely, a *hydromagnetic fast surface mode and a hydromagnetic slow surface mode*. The propagation band for the fast surface mode is given by

$$c_{s0} < c < (c_{se} - |U_e|). \quad (15)$$

The fast surface waves 'disappear' for $|U_e| \geq (c_{se} - c_{s0})$ in this particular case of purely parallel propagation of the waves. For a non-parallel propagation $\theta \neq 0$, Eq. (15) for the propagation band of these waves modifies to

$$v_{c0}^{-} < c < min(c_{se}sec\theta - |U_e|, v_{A0}). \quad (16)$$

This property of the fast surface mode is shown in Fig. 3a, where, the magnitude of the steady downdraught is chosen such that $|U_e| = c_{se}sec(\theta = 10°) - v_{c0}^{-}(\theta = 10°)$. We see, in Fig. 3a, that the fast surface mode is absent for a propagation angle $\theta < 10°$, but can propagate freely for $\theta > 10°$. As θ approaches $\pi/2$, $v_{c0}^{-}(\theta)$ approaches c_{T0} and $c_{se}sec\theta$ approaches infinity, so that, the mode now propagates irrespective of the value of the downdraught speed in the non-magnetic region ($x < 0$).

Apart from the *fast surface mode*, we also get a *slow surface mode* for the above ordering of the physical speeds, even in the case of a purely parallel propagation ($\theta = 0$). The phase speed of this slow surface mode changes its sign from positive to negative values as the absolute value of the downdraught speed in the non-magnetic region increases through a critical speed U_{ec}. Beyond this critical downdraught speed U_{ec}, the slow surface mode becomes a *backward propagating mode* or a *negative energy mode* that may be subject to various dissipative instabilities (e.g. Joarder et al. 1997). The effect of a non-parallel propagation $\theta \neq 0$ is to suppress such negative energy instabilities of the slow surface mode by increasing the magnitude of the critical velocity threshold. For a small angle of propagation θ, this effect can be expressed by a simple formula:

$$U_{ec}^2(\theta) \approx \frac{2c_{se}^2 v_{A0}^4}{(2c_{s0}^2 + \gamma v_{A0}^2)^2} [\{1 + (2c_{s0}^2 + \gamma v_{A0}^2)^2 sec^2\theta / v_{A0}^4\}^{1/2} - 1]. \quad (17)$$

Fig. 3b demonstrates this particular effect of the angle of propagation θ on the appearance of the backward propagating slow surface wave for a particular value of the downdraught velocity $|U_e| = 0.55v_{A0}$, which is also the magnitude of the critical velocity for an angle of propagation $\theta = 10°$; see Eq. (17). This figure shows that the waves that propagate with $\theta > 10°$ are no longer the backward propagating waves. For nearly normal ($\theta \to \pi/2$) propagation, the asymptotic value of the phase speed c of such *forward propagating slow surface waves* is given by

$$c_a \approx -\left[\frac{\rho_e}{\rho_0 + \rho_e}\right]|U_e| + \left\{\left[\frac{\rho_0}{\rho_0 + \rho_e}\right]v_{A0}^2 - \left[\frac{\rho_0\rho_e}{(\rho_0 + \rho_e)^2}\right]|U_e|^2\right\}^{1/2}, \quad (18)$$

which coincides with the expression for the phase - speed of an incompressible surface wave (Chandrasekhar 1961) with $U_0 = 0$ and $v_{Ae} = 0$. This asymptote is shown in Fig. 3b.

4. Conclusions

We have examined the combined effect of non-parallel propagation and steady shear-flows on the properties of hydromagnetic surface waves. Two different cases are considered to study this effect. The first case is similar to the situations found in a hot coronal loop. The steady flow in the loop may be either anti - parallel or parallel to the direction of the longitudinal phase-propagation of the waves. We have examined both the possibilities in Sect. 3.1. The second case (in Sect. 3.2) is similar to the situation found in an isolated photospheric flux-tube surrounded by a granular/supergranular downdraught.

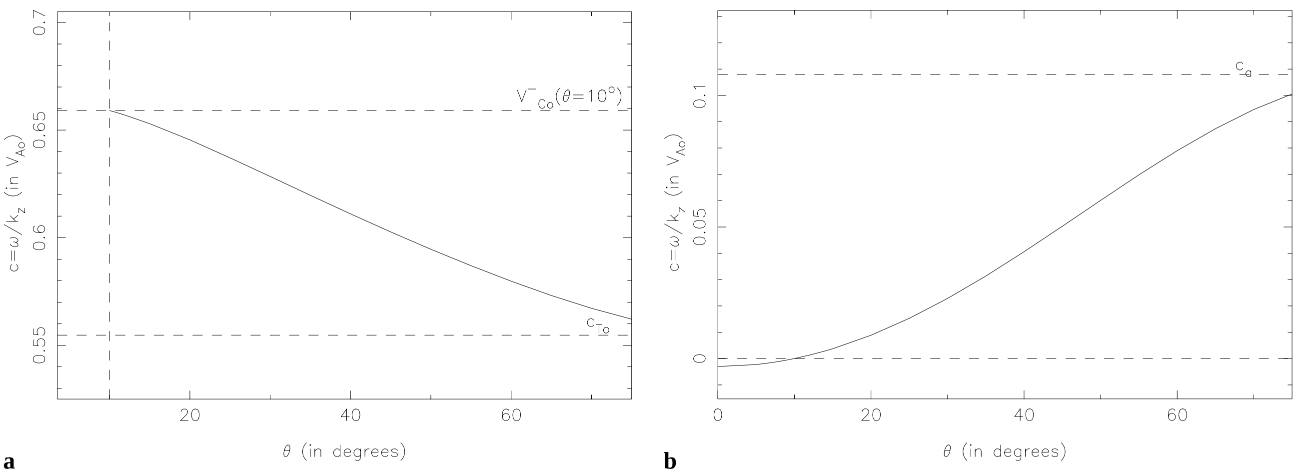

Fig. 3a and b. Field-aligned phase speed c vs. angle of propagation θ for **a** *the fast surface mode* and **b** *the slow surface mode* in a case where $v_{Ae} = 0, v_{A0} > c_{se} > s_{s0} > c_{T0}$ and $U_e < 0$. Numerical values of various parameters chosen to draw this diagram are $v_{A0} = 1.0, c_{s0} = 0.666666, c_{se} = 0.75, \rho_e/\rho_0 = 2.27$ and $U_0 = 0.0$. **a** is drawn for an arbitrarily chosen downdraught velocity $U_e = v^-_{c0}(\theta = 10^\circ) - v^+_{ce}(\theta = 10^\circ) \approx -0.1025 v_{A0}$. Note that, the fast surface wave propagates only when $\theta > 10^\circ$. For θ approaching 90°, the phase speed c of the fast surface mode asymptotically approaches $c_{T0} = 0.5547$. **b** is drawn for a particular downdraught velocity $U_e = U_{ec}(\theta = 10^\circ) = -0.5513$. The slow surface wave is a 'backward propagating wave' for $\theta < 10^\circ$. For $\theta > 10^\circ$, the mode becomes forward propagating with its phase speed c asymptotically approaching $c_a \approx 0.108$, in agreement with Eq. (18) of the main text.

In both the cases we studied, we have considered only surface waves, thus ignoring the body waves in magnetic structures. We also ignored the dispersion effects of finite thickness of such structures. These limitations of our study do not allow us to directly compare our results to the observations of solar magnetic flux-tubes. Some important conclusions regarding the physical properties of hydromagnetic surface waves can, nevertheless, be drawn from our study.

One such finding is that, a non-parallel propagation vector facilitates the propagation of surface waves either by introducing new surface modes in a situation where none were available to propagate in a direction purely parallel to the magnetic field, or, by widening the propagation window for an already existing surface mode. The (externally fast) fast surface mode in Sect. 3.1, for example, owes its existence solely to a non-zero angle of propagation θ of the waves. An increasing steady flow in a direction opposite to the direction of the field-aligned propagation of the waves tends to suppress this mode by narrowing down its propagation band; see Eq. (9). An increasing angle of propagation, on the other hand, widens the propagation window of this (externally fast) fast surface mode. The mode can only be fully suppressed by a super-Alfvenic flow in the loop in the direction of the longitudinal propagation of the waves; see Fig. 2. Such an extremely large flow of Alfvenic or super-Alfvenic magnitude has not, so far, been observed in solar coronal loops (Schrijver et al. 1999).

A non-parallel propagation also widens the propagation window for the fast surface mode at the edge of an isolated photospheric flux-tube (see Sect. 3.2), while the effect of a steady downdraught motion surrounding the flux-tube is to reduce the size of this propagation band.

The (externally slow) fast surface mode (Fig. 1b) in a coronal loop and the hydromagnetic slow surface mode (Fig. 3b) in an isolated photospheric flux-tube are of particular interest. Both these surface modes change into backward propagating (or, negative energy) surface modes as the magnitude of the steady flow, directed opposite to the direction of wave propagation, exceeds a certain critical value. For the (externally slow) fast surface mode (Sect. 3.1), this critical speed is always given by the Alfven speed v_{A0} inside the loop. Such an unusually large magnitude of the critical velocity of the fast surface mode, together with the absence of a backward slow surface mode (for which the critical velocity is expected to be of the order of the acoustic speed in the loop), may suggest that the negative energy instabilities of MHD surface waves may not play an important role in the heating and the wave propagation in solar coronal loops.

For the backward slow surface mode at the interface of a photospheric flux- tube and its non - magnetic environment, the critical speed of the steady downdraught motion is sensitive to the angle of propagation θ of the waves. Our result in Sect. 3.2 (see Eq. (17) and Fig. 3b) suggests that a non-parallel propagation, in this particular case, tends to suppress the backward surface mode by increasing its critical velocity threshold. The mode that is most prone to negative energy instabilities is then the backward slow surface mode propagating parallel to the magnetic field, that has already been investigated by Nakariakov & Roberts (1995) and Joarder et al. (1997). Detailed calculations, involving more realistic flux-tube geometries, is needed to examine the consequences of such negative energy instabilities of the longitudinally propagating backward slow surface mode in solar photospheric flux-tubes.

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