Bull. Astr. Soc. India (1982) 10, 73–110

Planetary nebulae

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Received 1982 May 1

Abstract. Recent research on planetary nebulae is reviewed, with special emphasis on ultraviolet and infrared observations. Various distance scales and statistics based on them are discussed and the empirical mass-radius relation due to Maciel & Pottasch (1980) is used to compile a new list of local planetaries. Recent determinations of the effective temperatures of central stars and their new locations on the H-R diagram are described. The implications of the chemical abundance studies of these nebulae are discussed in the light of current work on stellar evolution and nucleosynthesis.

Key words : planetary nebulae statistics—abundances—stellar evolution

1. Introduction

Ever since their discovery planetary nebulae have delighted all observers of the night sky because of their beautiful appearance. They have also been a source of considerable scientific knowledge because of their interesting physics. Messier catalogued four of these objects—the Dumbbell Nebula in Vulpecula (M 27), the Ring Nebula in Lyra (M 57), M 76 in Perseus and the Owl Nebula in Ursa Major (M 97). Historically, the first observation of a planetary nebula was made in 1790, when William Herschel looked at NGC 1514 and described it as "a most singular phenomenon". The scientific study of the nebulae began 1864 August 29, when William Huggins took a spectrum of the nebula NGC 6543 and discovered its unusual spectrum consisting of no continuum and a trio of emission lines at 4861 Å, 4959 Å and 5007 Å. He identified the weakest of them at 4861 Å as H_β. Huggins was also responsible for the first photographic spectra of planetary nebulae which showed the [O II] doublet at 3727 Å.

Recent years have witnessed enormous progress in the spectroscopy of planetary nebulae. Planetary nebulae have been observed in all wavelengths from far-ultraviolet to the radio. Observations in the new spectral regions have widened our perspective and produced valuable data for a more complete description of these objects. The early interest in the interpretation of the spectra from the viewpoint of an atomic physicist has given way to the current trend in which the implications of planetary nebula observations are studied more in the context of stellar and galactic evolution. This has, of course, been made possible by the refinement and a wide coverage of the observational data and a growing faith in our general understanding of the physics of these objects. Thus it has now become meaningful to talk about small differences, *e.g.* in abundances, and draw conclusions from them or compare them with predictions of the astrophysical theory.

The early work on the optical spectra of planetary nebulae provided the stimulus for much of the research on atomic processes in dilute gases which were responsible for producing the spectra. The photoionization theory of Zanstra outlining the basic physics of nebular radiation, the identification by Bowen of the green nebular lines as transitions between the metastable levels of the ground electron configuration of O^{++} and the series of papers by Menzel and associates on the physical processes in the gaseous nebulae form the foundation of the theory of gaseous nebulae on which all later advances have been based. It is not surprising that even today the attempts at elucidation of the physics of these objects stimulate a lot of work in the theory of atomic spectra and atomic collision processes.

Planetary nebulae were observed to be invariably associated with a blue star at the centre called the nucleus of the nebula. Following Zanstra's work it was also known that the nucleus is the original source of energy powering the nebula. With the advances in the theory of stellar evolution and detailed analysis of observational data it became quite clear that the nuclei of planetary nebulae represent a rather late stage in the evolution of a star not much more massive than the sun. The data also suggested that further evolution of these stars leads naturally to the white dwarf state. The nature of the precursor to the planetary nebula nucleus was not, however, immediately clear although the early work in the field suggested that red giants and long period variables are the promising candidates. During the last decade, considerable progress has taken place in our understanding of these late phases of stellar evolution and the evolutionary connection between planetary nebulae and their immediate precursors red giants, seems to have been clearly established. Although far from complete, a general picture of the evolution of low- and moderate mass stars has now emerged and planetary nebulae are seen to play a key role in this scenario.

The spectra of the nebular shells taken with slit spectrographs had suggested that the nebulae were in a state of expansion with moderate velocities of 20–40 km s^{-1}. Long-time-interval imaging has confirmed this idea by measuring directly increases in nebular sizes for many of the well studied objects. This expansion leads to the eventual dispersal of the stellar material that forms these shells, into the interstellar medium. Since this occurs late in the life of a star it is conceivable that part of the material thus returned has undergone the effects of stellar nucleosynthesis. This brings into focus the role of planetary nebulae in the general scheme of chemical evolution of galaxies. The exact manner in which chemical enrichment has occurred in galaxies is a subject of enormous current interest. In the last few years, the refined chemical analysis of the nebular spectra has indicated that planetary nebulae contribute significantly to this process of enrichment. Moreover, being the end products of stars over a sufficiently large mass interval, they record in their chemical aspects, although partially, the nucleosynthetic history of the Galaxy. Their observed number is sufficiently large so that meaningful statistical analyses of the data may be performed. They are bright and relatively easy to discover and observe. They form a largely homogeneous well defined group of objects.

The significant progress in our knowledge of planetary nebulae has been celebrated since 1967 by holding quinquennial symposia on the subject and the proceedings of the symposia record in detail the history of the development of our knowledge in the field. These volumes contain excellent summaries on all the different aspects of planetary nebula studies. Apart from these, several other conferences, a few on late-type stars and related objects as well as the ones discussing the results obtained with the international ultraviolet explorer satellite, deal with the recent work on planetary nebulae. To limit the review to manageable proportions, I shall primarily concern myself with the work done since the last IAU symposium on the subject held 1977. In section 2, I shall discuss the observational data and statistics of the nebulae. Section 3 will deal with the physical conditions, section 4 with chemical composition of these objects; and in section 5, I shall attempt to describe how these objects fit into the general scheme of stellar evolution and related evolutionary processes in galaxies.

2. Basic data and some statistics

(a) *Surveys*

Publication of the catalogue of galactic planetary nebulae by Perek & Kohoutek (1967) was the pioneering effort in the survey of these objects in the Galaxy. The survey has been continued and Kohoutek (1978) has updated it with a list of 226 new nebulae. The total number of actual nebulae catalogued is somewhat uncertain in view of the misclassifications pointed out by Kohoutek for at least 34 objects in the earlier catalogue. The ESO/Uppasala survey of the southern sky (Lauberts *et al.* 1981; Lauberts 1982) has discovered many new planetary nebulae. Interference filter photography of selected regions from the Palomar sky survey charts has also led to fresh identifications of new planetary nebulae (Weinberger & Sabbadin 1981). To date the most complete catalogue of planetary nebulae is the Strasbourg catalogue of Acker *et al.* (1981) containing a list of 1455 objects.

(b) *Distances*

Perhaps the most fundamental parameter needed to describe many of the physical properties of planetary nebulae is their distance. Distance is also important for statistical discussions of the data, *e.g.* the galactic distribution of planetary nebulae, their birthrate, etc. Unfortunately, only a few nebulae are directly accessible to astronomical distance measurements. The only planetary having a measured trigonometric parallax is the one nearest to us, namely NGC 7293. A second direct method, that could be applied in principle, is to combine the available angular expansion data on a few of the nebulae with their observed radial velocity of expansion leading to an estimate of the distance. However, this method is based on the crucial assumptions of uniform symmetric expansion and optical thinness of the shells, both of which may be erroneous for the individual cases considered. A few planetaries are known to be in binary systems and their distances can be determined by the method of spectroscopic parallax applied to the companion star. The classic example is NGC 246 whose nucleus has a G8-KO main sequence companion and for which

Minkowski (1959) derived a spectroscopic parallex of 430 pc. A few others have since been discovered to be in binary systems. HD 87892, the A type central star of NGC 3132, has a hot white dwarf companion discovered by Kohoutek & Laustsen (1977). Mendez & Niemela (1981) have confirmed that the A type central star of NGC 2346 is a single-lined spectroscopic binary. Seaton (1980) has presented convincing arguments showing the central star of NGC 1514 also to be a binary. Liller (1978) has given a list of seventeen objects which from their colours appear to be ordinary main sequence stars but have planetaries surrounding them. All of these stars may have hotter and fainter companions which excite these nebulae. Clearly, more work is needed to establish the binary nature of all of these systems. K 648 in M 15 is the only known planetary nebula in a globular cluster and NGC 2818 is a planetary very likely associated with the open cluster of the same name (Dufour & Hack 1978).

An indirect method which has received a great deal of attention in recent times is based on the use of interstellar extinction (Acker 1978, Pottasch 1980). If the interstellar extinction of a nebula were known and a measurement of the extinction as a function of distance for stars in its neighbourhood in the sky available, the two may be combined to obtain a distance to the nebula. This, of course, requires a rather complete coverage of the photometric data for the stars and even then the patchiness of the interstellar extinction may remain a potential problem. In all, the number of galactic planetaries with individual measurement of distances is small and the data are hardly adequate for further statistical analyses.

Distances to almost all planetary nebulae in the Galaxy are derived on the basis of an indirect method originally due to Shklovsky (1956). The method is based on the assumption that all planetary nebulae are similar objects in different stages of expansion which implies, in turn, that when fully ionized, all of them have the same mass. The distance D of the nebula is related to the ionized mass M_1, the angular size ϕ and the extinction-corrected measured flux in H_β at earth πF_{H_β} through

$$D = K^{0\cdot2}\, M_1^{0\cdot4}\, \phi^{-0\cdot6}\, (\pi F_{H_\beta})^{-0\cdot2}. \qquad ...(1)$$

Here K is a constant given by

$$K = \frac{3}{16\pi^2}\, \frac{1 + xy}{(1 + 4y)^2 m_H^2}\, \frac{\alpha_\beta^{eff}\, h\nu_\beta}{\epsilon}$$

where x is a measure of the fractional ionization of He^+ to He^{++} and equals $(N_{He^+} + 2N_{He^{++}}/(N_{He^+} + N_{He^{++}})$, y is the helium abundance by number, ϵ the volume filling factor and α_β^{eff} the effective recombination coefficient for the H_β line (see Osterbrock 1974 for the derivation).

For a fully ionized nebula that is optically thin in the Lyman continuum, $M_1 = M$, the total mass in the shell. In the Shklovsky method, M is treated as a constant and absorbed in K, so that

$$D = K^{0\cdot2}\, \phi^{-0\cdot6}\, (\pi F_{H_\beta})^{-0\cdot2}. \qquad ...(2)$$

Once this relation is calibrated using planetaries with known distances, it can be used for any nebula with a measured angular size and Hβ flux to infer its distance. Of course, for a nebula which is not optically thin, the distance will be an overestimate, although the weak dependence on M makes the actual distance rather insensitive to the choice of the nebular mass. There have been several calibrations of the distance scale by various authors. The most recent calibration based on statistical parallaxes is due to Cudworth (1974). Each calibration gives a value for the mass of the nebular shell and the various calibrations to date indicate that the mass lies in the range of 0.14 to 0.47 $M_{\odot}$. The difference in the distance scales due to the difference in the value of the calibraion mass is, therefore, within a factor of $(0.47/0.14)^{0.4}$ or 1.62.

Cahn & Kaler (1971, hereafter CK) used the Shklovsky method combining optical and radio data on more than 600 planetaries and derived distances. They used a calibration very close to the one obtained by Seaton (1968). They used $y = 0.14$, $\epsilon = 0.65$, $\alpha_{\beta}^{\rm eff} = 5.38 \times 10^{-14}$ cm^3 s^{-1} and the calibration mass was equal to 0.18 $M_{\odot}$. Milne & Aller (1975) and Milne (1979) carried out high-sensitivity radio observations of 332 planetary nebulae at 5 GHz. By combining the radio data with the Hβ fluxes they were able to derive the extinction to each nebula. Further, they assumed a He$^+$ ionization fraction of 0.5 and a helium abundance of 0.11, $\epsilon = 0.6$ and $T_e = 10^4$ K. By setting $M = 0.16\ M_{\odot}$, they determined a radio distance scale of planetary nebulae. Together with CK, their work represents the most extensive use of the Shklovsky method.

Acker (1978) adopted a different approach to the problem of the distance scale. By careful selection of a large amount of data and obtaining distances by kinematic methods and interstellar extinction measurements Acker was able to establish a set of accurate distances for 63 planetary nebulae. For each nebula a comparison of the distance thus obtained with distances from calibrated distance scales, keeping in view the physical property of the object, helped her establish mean coefficients for each scale. These were then used to derive a new synthetic distance scale. Distances to 330 planetary nebulae were obtained in this way. Judged by the statistical nature of the analysis, it appears that the distances in Acker's scale may at best be reliable to a factor of two. Recently, Pottasch (1981a) has questioned the validity of the Shklovsky method. By combining the best available distances of individual nebulae with precise determinations of their electron density, Pottasch (1980) has calculated the ionized mass of 28 nebulae. The ionized mass is seen to vary over a factor of 200 in this sample. Moreover, a plot of the ionized mass against the radius of the nebulae, under consideration, shows a striking linear relationship. Interpreted in a straightforward manner, this implies that most of these nebulae are optically thick to the ionizing radiation. As they expand and densities decrease the recombination rate goes down and the same photon luminosity ionizes a larger amount of matter. This continues until the entire shell is fully ionized when the increase in mass with the size of the ionized region should stop abruptly. The largest nebular masses in the sample considered by Pottasch are close to the values of the total shell masses obtained by calibrating distance scales using the Shklovsky method. Thus Pottasch concluded that it might no longer be possible to apply the Shklovsky method with any degree of confidence to large samples of planetary nebulae. Many of the nebulae considered optically thin in earlier discussions seem to actually have much smaller ionized masses indicating a situation quite the opposite. A change in the mass by a factor of 200 produces an error of a factor of 8.3 in the distance determination based on the Shklovsky method which is much larger than the differences between the various calibrated distance scales. That the Shklovsky method could be in error was also noted by Maciel & Pottasch (1980, hereafter MP) in a plot of the distances obtained by Milne & Aller (1975) using this method against the angular radii of the nebulae. Figure 1 is taken from MP and shows that there is a systematic

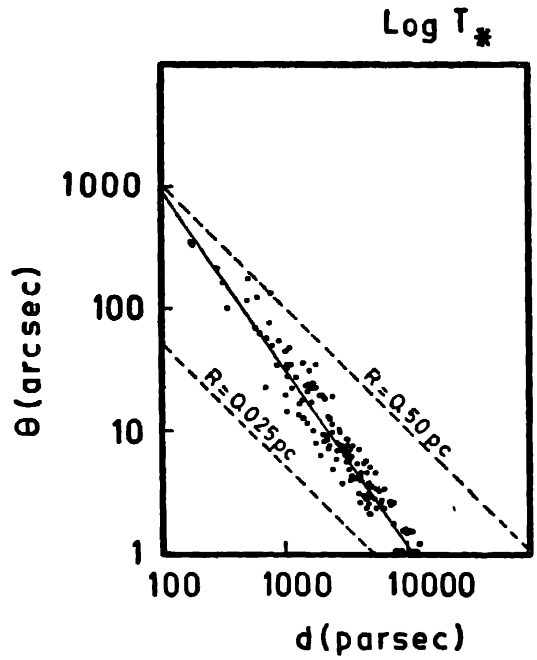

Figure 1. Angular radius θ (arcsec) vs distance *d* (parsec) of planetary nebulae from Milne & Aller (1975). See text. (Courtesy : Springer International, Heidelberg.)

trend in the angular sizes with the distance in the sense that all nearby nebulae are large while those farther away are small. The lines corresponding to constant linear radii are also shown. Since the surface brightness of a nebula decreases as the fifth power of the radius (Seaton 1968) larger nebulae or low surface brightness objects are selectively missed at large distances which explains why there are so few nebulae in the plot with large angular sizes and large distances. However, if the surveys are reasonably complete within one kiloparsec of the sun, the striking correlation seen in figure 1 cannot be explained away by observational selection effects. The Shklovsky distances thus suggest a rather special situation for the greater solar neighbourhood with a paucity of high surface brightness objects. Maciel & Pottasch obtained an empirical mass-radius relation by performing a least-squares fit to the data on masses and linear radii of the few tens of nebulae for which independent distance and electron density measurements are available. This relation is given by

$$M_1\ (M_{\odot}) = aR_1\ (\text{parsec}) + b, \qquad \ldots(3)$$

where $a = 1.225$ and $b = -0.0123$ are constants. If the Shklovsky distance to a nebula were denoted by D_0, then its true distance D would be given by

$$D = D_0 \left(\frac{M_1}{M}\right)^{0.4}, \qquad ...(4)$$

where M is the calibration mass in the particular distance scale used. Equations (3) and (4) can now be solved for M_1 and D. MP and Maciel (1981a) applied this method to the list of Milne & Aller (1975) and Milne (1979) and obtained new distances to 330 planetary nebulae.

(c) *Space density and height distribution*

The local space density of planetary nebulae as well as their height distribution can be obtained by considering a distance-limited sample of these objects. The usual practice has been to include all the objects with projected distances in the galactic plane of less than or equal to roughly a kiloparsec from the sun. Osterbrock (1973) used the data from CK while Cahn & Wyatt (1976) used a revised version of the same to derive these quantities. Since the distance measurement is based on the Shklovsky method, their quoted values refer only to optically thin systems. The criterion of optical thinness was satisfied by considering a size--limited subset of the data which then contained planetaries with radii in the range 0.08–0.40 pc. The new distances obtained from MP contain substantial changes, and using them a different list of local objects is derived. The property of optical thinness or thickness is not specifically used in this case, although the sample is limited in size in the range 0.01–0.40 pc as this is the range for which the mass-radius relationship has been clearly established. Table 1 lists the local planetaries from Cahn & Wyatt and from the new data obtained on an application of the MP procedure to CK. The samples are limited to nebulae whose projected distances on the plane are within 1100 pc of the sun. The distance z from the plane is also quoted for each nebula in both the lists. The local height distribution based on columns 3 and 5 of table 1 is displayed in figure 2. The straight line is a least-squares solution to the data points. In table 2 the local space density and the scale height of the z-distribution are given from various authors. The values quoted against Maciel (1981b) are also based on the new distances following MP although they were obtained by a straightforward application of a scale-factor determined through a comparison of the MP distances with those from CK. Both the space and surface densities are lower for the new list compared to the values obtained by Cahn & Wyatt while the scale height is somewhat increased. The various scale heights indicate a fairly strong concentration of the local nebulae to the galactic plane. There are some discrepancies amongst the values of the scale heights quoted in the literature (see Terzian 1980). These may be due to the mixed population characteristics of planetaries.

Calibration of the distance scale by Cudworth (1974) was notably different from the earlier calibrations due to O'Dell (1962) and Seaton (1966, 1968). The Cudworth calibration implied an increase in the distance by a factor 1.5 with respect to CK and hence a decrease in the space density by 3.4. Compelling arguments for an increase in the distance scale by a factor 1.3 have been presented by Weidemann (1977a). The observational Cudworth scale comes very close to fulfilling this requirement. Alloin *et al.* (1976) have discussed in detail the implications of the Cudworth

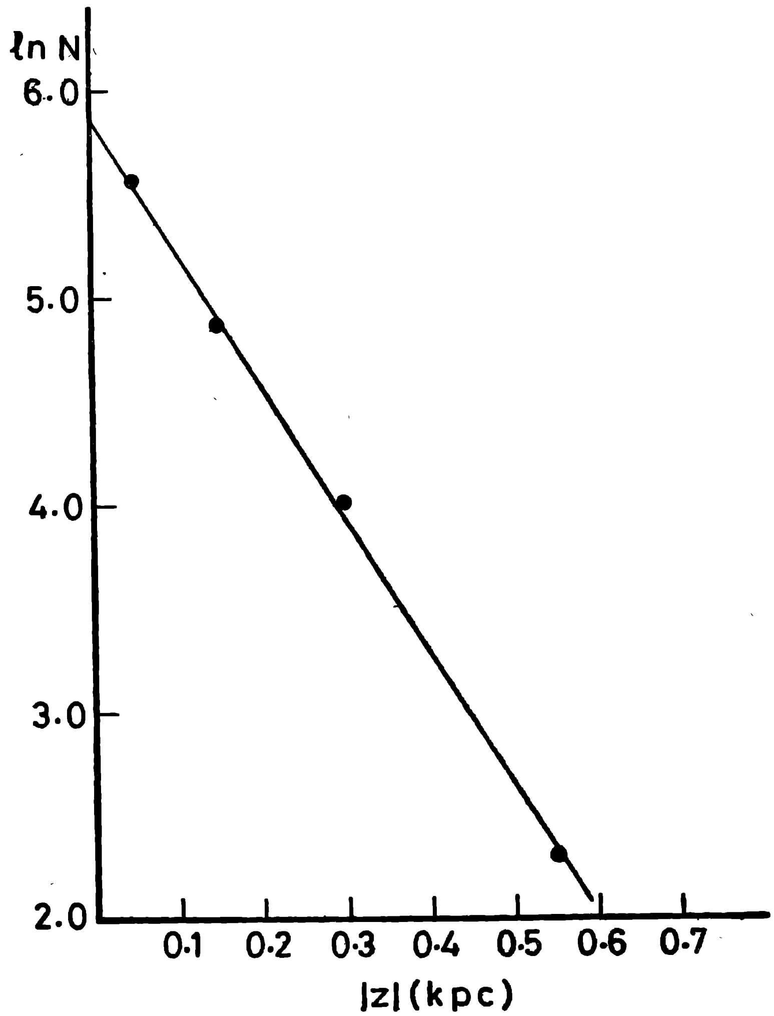

Figure 2. The z-distribution of planetary nebulae based on the data in table 1. N is the number of nebulae per unit interval of z (in absolute value).

scale. Specifically, this scale would imply a space density of 15 kpc^{-3} of planetary nebulae and a surface density equal to 5.8 kpc^{-2} for the solar neighbourhood, values that are much lower than those quoted in table 2.

Following Grieg (1971, 1972), Cudworth also considered two kinematically distinct classes of planetary nebulae—classes B and C. The z-distribution and the velocity dispersion perpendicular to the plane are quite different for the two classes. Class B nebulae perhaps correspond to a younger population with a mean distance above the plane $\langle |z| \rangle = 160$ pc and $\sigma_z = 15$ km s^{-1}. The class C nebulae show a lesser concentration to the plane with $\langle |z| \rangle = 320$ pc and $\sigma_z = 26$ km s^{-1}. This suggests that class B nebulae come from stars which are, on the average, more massive than the progenitors of class C nebulae. Several other lines of argument have emerged in the last few years to indicate that planetary nebule have substantial differences amongst themselves in chemical and kinematic properties. Since planetary nebulae come from stars with a range in their initial masses, the differences in their properties are traced to their different origins and different evolutionary courses pursued by their parent stars.

Table 1. Local list of planetaries

Cahn & Wyatt (1976)			This paper		
Name	q(kpc)	z(kpc)	Name	q(kpc)	z(kpc)
NGC 246	0.157	—0.576	NGC 246	0.113	—0.415
NGC 650/1	0.771	—0.143	NGC 650/1	0.701	—0.130
NGC 1360	0.210	—0.288	NGC 1360	0.214	—0.293
NGC 1514	1.014	—0.277	NGC 1514	0.482	—0.132
V–V1 2	0.514	0.019	V–V1 2	0.666	—0.024
V–V1 4	0.828	—0.033	V–V1 4	0.800	—0.031
V–V1 5	0.451	—0.0105	V–V1 5	0.486	—0.010
NGC 3242	1.013	0.634	NGC 3242	0.474	0.265
NGC 3587	0.293	0.452	NGC 3587	0.256	0.394
NGC 4361	0.827	0.789	NGC 4361	0.680	0.648
NGC 5189	0.612	—0.037	NGC 5189	0.768	—0.047
NGC 6302	0.890	0.016	NGC 6302	0.415	0.008
NGC 6720	0.874	0.218	NGC 6720	0.582	0.145
M 1 67	0.597	0.035	M1 67	0.988	0.057
NGC 6781	0.872	—0.045	NGC 6781	0.903	—0.047
NGC 6853	0.256	—0.017	NGC 6853	0.299	—0.019
A 69	0.491	0.010	A 69	0.282	0.006
A 72	0.746	—0.253	A 72	1.029	—0.349
NGC 7008	0.788	0.076	NGC 7008	0.846	0.081
NGC 7009	1.009	—0.695	NGC 7009	0.510	—0.352
NGC 7293	0.083	—0.128	NGC 7293	0.098	—0.151
SH1 118	1.056	0.046	NGC 40	0.690	0.120
A 5	1.052	—0.144	IC 418	0.684	—0.308
A 6	0.852	0.074	IC 2149	0.983	0.182
K2 1	0.980	—0.098	NGC 1535	0.456	—0.390
A 13	0.861	—0.129	NGC 2346	0.798	0.050
K2 2	0.355	0.030	NGC 2371	0.470	0.170
KA 24	0.360	0.095	NGC 2392	0.668	0.209
HA4 1	0.604	18.703	NGC 2440	1.087	0.046
K1 3	0.921	0.203	NGC 3132	0.684	0.150
NGC 6164	0.351	—0.0018	IC 3568	0.395	0.272
IC 2195	0.564	—0.047	NGC 3918	0.920	0.076
YM 16	0.473	0.017	HD 138403	0.682	—0.159
SH2 71	0.729	—0.017	MZ 3	1.024	—0.018
A 56	0.796	—0.049	PE1 5	0.866	0.018
A 61	0.771	0.203	NGC 6153	1.042	0.100
A 62	0.725	0.055	NGC 6210	0.237	0.184
K 1 6	0.696	0.272	NGC 6369	0.618	0.063
A 71	0.748	0.059	TC 1	1.023	—0.159
A 80	0.910	— 0.080	NGC 6445	1.028	0.070
JN 1	0.428	—0.244	NGC 6543	0.866	0.499
			HA1 13	0.783	—0.003
			NGC 6572	0.509	0.107
			NGC 6567	1.000	—0.011
			M3 28	0.914	—0.007
			BD+30°3639	0.797	0.070
			NGC 6826	1.074	0.238
			NGC 6891	0.684	—0.147
			M 1 79	0.963	—0.041
			M 2 53	0.872	—0.024
			NGC 7354	0.859	0.035
			NGC 7662	0.667	—0.212
			HA 4 1	0.434	13.426

Table 2. Local densities and scale heights

	σ (kpc^{-2})	n_0 (kpc^{-3})	z_0 (pc)
Cahn & Kaler (1971)	13	50	90
Cahn & Wyatt (1976)	19	80	115
Smith (1976)		150	110
Acker (1978)	13	48	
Maciel (1981)	12	41	144
This paper	16	46	156

(d) *Galactic distribution and total number*

Optical surveys of planetary nebulae suffer, at larger and larger distances, from incompleteness due to the effects of interstellar extinction. Thus our direct knowledge of the spatial distribution of these objects in regions of the plane far away from the solar neighbourhood is rather scanty. Cahn & Kaler (1971) considered the sample of optically thin planetaries (in their calibration all planetaries in the size interval 0.08–0.40 pc could safely be assumed to be optically thin) and studied their galactic distribution by fitting the data to an assumed density distribution due originally to Perek (1962). Their fit implied a very high density of planetaries at the galactic centre and a radial scale-factor of 1.1–1.4 kpc depending on R_0, the distance of the sun from the galactic centre. The total number of planetaries inferred from this distribution was correspondingly large being in the neighbourhood of 3–4 $\times$ 10^5. Alloin *et al.* (1976) gave arguments why this number may be too high. They assumed a flat-disk model of the Galaxy and an exponential surface density law of the form $\mu(R) = \mu_0 \exp(-\alpha R)$. By fitting the same data but using both the CK and Cudworth scales, they obtained radial scale-factors substantially higher than the CK determination. The total number of planetaries in the Galaxy came down by more than an order of magnitude. In table 3 we summarize the results on the total number of planetaries from various sources.

An alternative procedure of determining the total number of nebulae is to use the specific number κ defined as the number of nebulae per unit mass. If σ denotes the local surface density of nebulae and Σ the local mass density $\kappa = \sigma/\Sigma$. By assuming that the specific number is representative of the whole Galaxy, an estimate of the total number can be obtained by multiplying κ by the mass of the Galaxy. The values obtained in this way are displayed in column 4 of table 3.

Isaacman (1980) has conducted an extensive radio survey of planetary nebulae near the galactic centre. He finds that within 2° of the galactic centre the planetary nebula surface density is 23.9 deg^{-2}. This corresponds, for R_0 = 9 kpc to a $\sigma(R=0)$ = 969 kpc^{-2}. If an exponential law for the surface density distribution were now assumed, and the σ taken from column 2 of table 2 the radial scale-factor becomes 2.19 kpc. Assuming the galactic disk to extend to a radius of 15 kpc, the total number of planetaries works out to 29,000. The last row of table 3 quotes this result. Isaacman himself considered the specific number of nebulae in the galactic centre and scaling it up by the mass of the galaxy obtained for the total number 21,000.

Table 3. Total number of planetary nebulae and their birthrate

	κ ($M_{\odot}^{-1}$)	α (kpc^{-1})	Total number		b_{PH} (10^{-3}kpc^{-3} yr^{-1})
			Estimate 1	Estimate 2	
Cahn & Kaler (1971)		0.714		430,000	3.2
Cahn & Wyatt (1976)			38,000 ±	12,000	5.1 ± 1.0
Alloin, Cruz-González & Peimbert (1976)	1.73(—7)	0.30	22,500(CK)	12,700(CK) 5,700 (C)	3.1 (CK)
	7.7 (—8)	0.13	10,000 (C)	9,000 (CK) 4,000 (C)	0.63 (C)
Acker (1978)	1.7 (—7)		25,000		3.0
Cahn & Wyatt (1978)		0.250		45,000	
Isaacman (1981)	1.5(—7)		21,000		
Maciel (1981)	1.6 (—7)		21,000		2.0
This paper	1.6 (—7)	0.457	24,000	29,000	

(e) *Expansion velocity and lifetime*

The total timescale of evolution of a planetary nebula can be determined from the observed expansion velocities and the largest linear sizes determined from the distance data. By assuming a constant expansion velocity of 20 km s^{-1} and the largest linear size to be 0.7 pc based on his own calibration, O'Dell (1968) obtained a timescale of 35,000 yr for this evolution. It is more appropriate to only consider optically thin nebulae since the use of the Shklovsky method implies that the sizes of only these are correct. Seaton (1966) and CK determined the range of linear sizes for which the nebula is likely to be observed in the optically thin phase. In the CK scale, this range is between 0.04–0.4 pc. A nebula which is optically thick in reality will have its distance and radius overestimated by the Shklovsky method. Thus, although for uniform expansion the size distribution of nebulae should be reasonably flat, the observational procedure creates a surplus at a nebular size, set by the ionizing photon luminosity, by adding wrongly optically thick nebulae to size bins larger. Size bins smaller than this limit then show a deficit. To be reasonably sure that only optically thin nebulae are considered in the sample, CK chose the lower limit to be 0.08 pc. Ideally, the lifetime of an optically thin shell should have been given by

$$\tau = \int_{0.08}^{0.40} \frac{dr}{V(r)}, \qquad \ldots(5)$$

where $V(r)$ is the expansion velocity of the nebula as a function of the linear size. $V(r)$ has not been determined with much confidence yet and uniform expansion seems to be a plausible assumption from other observational considerations (Seaton 1966, Cahn & Wyatt 1976). Then, equation (5) reduces to $\tau = 0.32$ pc$/V$. Since the observational data on expansion velocities seem to favour a value of 20 km s^{-1} with an error of at most 5 km s^{-1} we adopt this value for V. The lifetime τ, then, turns out to be 16,000 yr. This timescale is extremely short compared to normal time scales encountered in stellar evolution theory. The suggestion in the data that the central stars of planetary nebulae also evolve on such a short timescale has always been a severe constraint on theories of evolution of these stars. We discuss this in some detail in section 5.

The birthrate of planetary nebulae in the solar neighbourhood can be obtained by dividing the local space/surface density of these objects by the lifetime. Since the lifetime is sufficiently short, it is generally assumed that for a steady population of planetary nebulae their birthrate should equal their deathrate. The last column of table 3 summarizes the observational estimates of the birthrate from various sources.

Assuming the total number of nebulae in the Galaxy to be 21,000 their galactic birthrate is approximately 1.3 yr^{-1}. This may be compared to the galactic birthrate of pulsars which, according to Taylor & Manchester (1977), equals 0.17 yr^{-1} while a more recent determination by Narayan & Vivekanand (1981) shows the rate to be lower by a factor of 3.5 and thus equal to 0.05 yr^{-1}.

(*f*) *Planetary nebulae in Local Group galaxies*

Planetary nebulae have been discovered in a number of Local Group galaxies. Jacoby (1980) has used the earlier surveys by Ford and associates (see Ford & Jacoby 1978 and references therein) combined with newer data on planetaries in LMC and SMC to determine the luminosity function of planetaries in the Magellanic Clouds. The similarity between these luminosity functions and the one constructed for the solar neighbourhood is quite remarkable. Extrapolating the MC luminosity function to fainter magnitudes the incomplete surveys in other galaxies were corrected for. The luminosity specific number of planetaries was found to be nearly constant in the eight Local Group galaxies surveyed. The total number of nebulae in each of these galaxies was derived. The same approach for our Galaxy led to the number 10,000 which is in excellent agreement with the estimates in table 3. This approach has suggested once more that the CK scale may be in error and the Cudworth scale is to be preferred.

3. Physical conditions

The observed line and continuum spectra of planetary nebulae have been, on the whole, successfully interpreted in terms of photoionization of the nebular shell by UV radiation from a central star followed by recombinations and collisional excitation by electrons of low-lying metastable levels of abundant ions. However, in several cases charge transfer reactions have been shown to be extremely important in determining the relevant levels of ionization of certain species. Similarly, dust is now known to play a significant role in absorbing the ultraviolet and optical radiation and reemitting in the infrared. The rapid acquisition of observational data, particularly through the spectral windows in UV and the infrared, has led to significant improvements in modelling the nebulae and obtaining better estimates of physical parameters describing them. The principal thrust of the new observations has been to derive a reliable set of abundances in the nebulae almost entirely from observations. This was not possible earlier because many of the important ions do not have lines in the visible region.

(a) *Diagnostics for electron temperatures and densities*

Over the years, the best estimates of the electron temperature T_e and the electron density N_e have come from the optical forbidden lines of O^{++}, N^{+}, S^{++}, O^{+} and S^{+}.

For the determination of T_e, the most convenient configuration is that of two or four equivalent p electrons resulting in three closely spaced fine-structure levels followed by two upper ones of sufficiently different excitation energies. Collisiona excitations from the ground level populate the two upper levels at rates proportional to the ionic and electronic densities and $T_e^{-1/2} \exp(-E/kT_e)$ where E is the excitation energy of the level. The constant of proportionality involves a dimensionless atomic parameter Ω, called the collision strength, and other numerical factors (see Osterbrock 1974). Since most excitations are followed by downward radiative transitions, the relative intensities of lines originating from the two upper levels, are principally a function of T_e. Thus, for the [O III] transitions ^{3}P–^{1}D and ^{1}D–^{1}S

$$\frac{I_{\lambda 4959} + I_{\lambda 5007}}{I_{\lambda 4363}} = 7.2 \exp(32{,}970/T_e) \left\{\frac{1 + 0.00054x}{1 + 0.063x}\right\}, \qquad \ldots(6)$$

where $x = 10^{-2} N_e/T_e^{1/2}$ and the quantity in braces is a correction factor for collisional deexcitation (Seaton 1975). For almost all nebulae $x < 1$ and the correction is insignificant. For the determination of N_e, the convenient term structure is provided by a system with 3 equivalent p electrons where the ground level is a $^4S_{3/2}$ term followed by two closely spaced upper levels of ^{2}D designation followed by another two closely-spaced ^{2}P terms. The excitation rates to the ^{2}D terms are nearly equal but the ratio of the line intensities $3 \rightarrow 1$ and $2 \rightarrow 1$ depends sensitively on N_e as collisional deexcitation preferentially depopulates $2 \rightarrow 1$ with respect to $3 \rightarrow 1$. In the low-density limit ($N_e \rightarrow 0$), each excitation is followed by a downward radiative transition and the intensity ratio of the two lines is simply the ratio of the statistical weights of the two upper levels in question, while in the high-density limit ($N_e \rightarrow \infty$), a Boltzmann population is set up amongst all the levels and the intensity ratio ($I_{3\rightarrow 1}/I_{2\rightarrow 1}$) becomes equal to the ratio of statistical weights multiplied by the corresponding A values of the two transitions. Between the two limits, $I_{3\rightarrow 1}/I_{2\rightarrow 1}$ is a function of N_e that can be calculated exactly from the atomic theory. This is displayed in figure 3 (Saraph & Seaton 1970) and shows the relevant intensity ratios for O^+, S^+, Cl^{++}, Ar^{+++} and K^{++++} all of which have 3 equivalent p electrons in their outermost shell. The most extensive recent determination of T_e and N_e from optical measurements alone is due to Barker (1978a). Temperature maps of planetary nebulae have been recently produced by using the [O III] lines (Reay & Worswick 1982).

The UV data have made other ions available for T_e and N_e measurements. In figure 4 the energy-level diagrams of C^{++}, O^{++} and Ne^{+++} are shown. The C III] lines at λλ1907, 1909 have been observed in a number of planetary nebulae by IUE and densities based on these measurements published (Feibelman 1981; Feibelman *et al.* 1980, 1981). Since C^+ has an ionization potential of 24.4 eV (higher than that of both O^0 and S^0), these lines probe directly regions of a nebula previously accessible only through the weaker [Ar IV] on [Cl III] lines for N_e measurements. Moreover, the semiforbidden character of the transitions makes the C III] lines very strong and their usefulness was recognised much earlier (Osterbrock 1970). The densities obtained so far from C III] lines show enhancements with respect to the optically obtained values. The [Ne IV] lines at λλ2422, 2424 are another set which have been used for density measurements for some of the nebulae (Seaton 1980). N_e can also

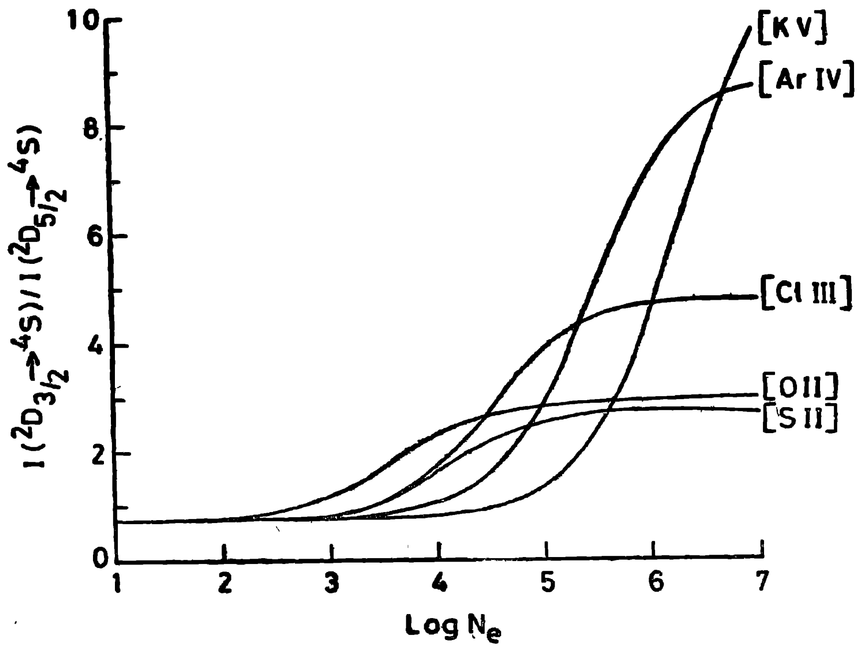

Figure 3. Ratio of line intensities vs log N_e from Saraph & Seaton (1970). (Courtesy : Blackwell Publishing Co., London.)

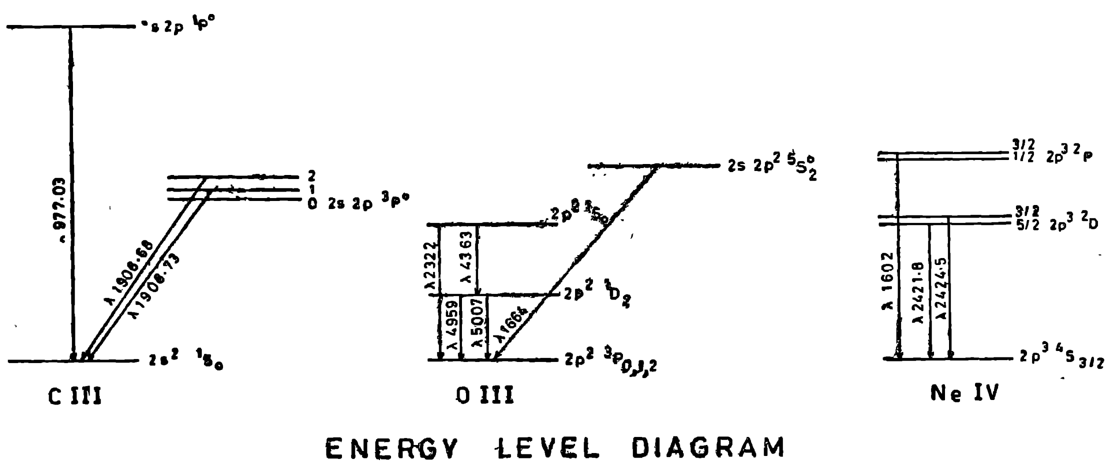

Figure 4. Energy level diagrams for C^{++}, O^{++} and Ne^{+++} showing the important UV transitions.

be measured by combining UV data on O III] λ1664 with the optical measurements of [O III] λλ 4959, 5007. At lower densities, $I(\lambda1664)/I(\lambda4959+\lambda5007)$ is a function of T_e and if the latter were already known from $I(\lambda4363)/I(\lambda5007)$, $I(\lambda1664)$ is exactly predictable from theory and its observed value provides a test. At higher densities $I(\lambda4959 + \lambda5007)$ decreases due to collisional deexcitation and $I(\lambda1664)/I(\lambda4959 + \lambda5007)$ increases with increasing N_e.

It is difficult to measure T_e from UV data alone although Harrington *et al.* (1979) have shown that the ratio of O IV] λ1402 to the He II Balmer α at λ1640 may be effectively used as a temperature indicator. However, the relative ionization of O^{++} to He^{+} has to be assumed before a reliable T_e is determined and this makes the method model-dependent.

(b) *Ionization structure*

Photoionization models of planetary nebulae have provided accurate descriptions of their thermal and ionization structures and have adequately interpreted the observed data by predicting accurate emission-line fluxes. The methods of such modelling have been extensively reviewed in the literature (Williams 1968; Flower 1968; Osterbrock 1971, 1974; Harrington 1978). One of the shortcomings of all these models has been their inability to reproduce correctly the strengths of the low-excitation lines, mainly the ones to O^+ and N^+. Lines of these ions always appear much weaker in the models compared to their observed values. It was recognized that abundance or density effects could not explain the discrepancy because any change in these parameters of a model would jeopardize the otherwise good agreement obtained for most of the other lines seen in the spectra. Williams (1973) considered optically thick condensations in planetary nebulae responsible for the emission of [O I] λ6300 and found that the process of charge exchange between O^+ and H° ($O^+ + H^\circ \rightarrow O^\circ + H^+$) modified the ionization structure of O^+ sufficiently to produce the required enhancement of [O I] intensities. Since then several other charge-transfer reactions have been considered and shown to be of great importance in determining the ionization structure of nebulae.

Pequignot *et al.* (1978) derived a consistent model for NGC 7027 by considering a host of charge-transfer reactions mostly involving protons. Some of these reactions are :

$$\begin{aligned} &N^+ + H^\circ \rightarrow N^\circ + H^+ \\ &O^{++} + H^\circ \rightarrow O^+ + H^+ \\ &H^{++} + H^\circ \rightarrow N^+ + H^+ \\ &Ne^{++} + H^\circ \rightarrow Ne^+ + H^+. \end{aligned} \qquad \ldots(7)$$

The major effect of these reactions is to reduce the ionization of the trace elements. The reaction rates are sufficiently large such that a low neutral density can considerably suppress X^{+n} and enhance the abundance of $X^{+(n-1)}$. The inclusion of these reactions significantly improved the theoretical model of NGC 7027 by matching the derived line intensities with their observed values. They also suggested that the charge-exchange rate between N^+ and H° was greatly overestimated by Steigman *et al.* (1971). This has since been confirmed by Butler & Dalgarno (1979). Butler *et al.* (1979) have also demonstrated from theoretical calculations that $O^{++} + H^\circ \rightarrow O^+ + H^+$ is very effective in controlling the ionization structure of O^+ while the corresponding rate for Ne^{++} — H° reaction was overestimated by Pequignot *et al.* Harrington *et al.* (1982) have shown the importance of $O^{+++} + H^\circ \rightarrow O^{++} + H^\circ$ in determining the fractional ionization of O^{++} in the inner hot regions of a planetary nebula. Butler *et al.* (1980) and Butler & Dalgarno (1980) have calculated rates for a variety of charge transfer reactions involving both H and He. The ionization and recombination rates for several of these reactions are found to be larger than the corresponding radiative rates by several orders of magnitude. Charge transfer, in these cases, controls the ionization structure. It is also true that since H is orders of magnitude more abundant than the trace ions, its ionization structure is unaffected by these processes although no detailed investigation in this respect has been made so far.

(e) *Infrared observations*

Unlike the case of UV observations, observations in the IR have, since the beginning, brought in big surprises. Besides elucidating many of the physical properties of planetary nebulae, they have also indicated the evolutionary connection between these objects and late-type giants and supergiants whose envelopes show common spectral features.

NGC 7027, the planetary nebula which provided the first data on infrared emission, is still perhaps the most studied infrared object in the sky. This nebula has been observed photometrically to 300 μm and its spectra up to 90 μm have been taken. Both photometric and spectroscopic data on several other planetary nebulae are also available. An infrared spectrum usually shows a continuum which, in the middle to far-infrared, is much stronger than was predicted by atomic processes alone and this surplus is attributed to thermal emission by grains contained in the nebula (Krishna Swamy & O'Dell 1968). In addition the spectrum shows recombination lines of H and He, fine-structure lines of various ions, lines in the quadrupole spectrum of H_2 plus some other features a few of which are yet to be reliably identified. Almost all of the spectral features were first seen in NGC 7027 and later in many other nebulae. The continuum in the region 2–4 μm is dominated by atomic processes but beyond 8 μm the dust continuum becomes prominent and continues to be so in far-IR regions.

The 8–13 μm region has been observed photometrically by Cohen & Barlow (1974, 1980) and spectroscopically by Aitken *et al.* (1979, 1981). The spectrum of NGC 7027 shows at least seven narrow emission features in this region at 3.28 μm, 3.4 μm, 3.5 um, 6.2 μm, 7.7 μm, 8.6 μm and 11.25 μm. These are not due to unresolved blends of atomic lines and their origin is not fully understood (see Aitken 1981). The features at 8.6 μm and 11.25 μm have since been seen in many other nebulae. Far-IR fine-structure lines due to [S III] (18.7 μm), [O IV] (25.9 μm), [Ne V] (24.3 μm), [O I] (63 μm) have been observed in NGC 7027 and a few other nebulae (Greenberg *et al.* 1977; Forrest *et al.* 1980; Melnick *et al.* 1981) and limits on [O III] 52 μm and 88 μm have been placed by Watson *et al.* (1981). Far-IR photometry of NGC 7027 extends to 300 μm (Telesco and Harper 1977) and many other nebulae have been observed to 108 μm by Moseley (1980). The continuum shows a peak at around 35 μm and declines steeply longward such that not much far-IR flux is seen beyond 70 μm. While the lines have provided data on ionization structure and shown the importance of density fluctuations in modelling and prediction, the broad features and the continuum, in general, have yielded information on the properties of dust and its origin, on certain evolutionary aspects of the nebulae and on the total dust-to-gas ratio in these nebulae.

Aitkin *et al.* (1979) and Aitken & Roche (1981) find that planetaries display, in varying degrees of prominence, four major characteristics in their continuum spectra in the 8–13 μm region, and that they can be broadly classified into different categories. The characteristics are (i) the 9.7 μm feature attributed to the presence of silicate grains in the nebula, (ii) a broad feature at 11 μm which is thought to be produced by SiC, (iii) a smooth continuum, and (iv) the two sharp features at 11.25 μm and 8.6 μm mentioned earlier. The spectra of some compact, and presumably young, planetary nebulae have the silicate feature very prominent and they are classed as oxygen-rich. SwSt 1, HB 12, IC 4997 and He 2–131 belong to this class. Several others like NGC 6572, IC 2501, NGC 6790 and IC 418 show the broad SiC feature which is also seen in carbon stars and, therefore, are carbon-rich. In addition, some of these show the 11.25 μm and 8.6 μm features. The classical planetary nebulae, of which the archetype is NGC 7027, display the narrow unidentified features prominently. These authors noted that the observational data are biased towards warm emitting grains and thus the absence of a feature does not preclude its existence in the nebula. For example, colder silicate dust may be contained in the outer regions of these objects without showing the 9.7 μm emission feature. On the whole, it appears that most planetary nebulae have a carbon-rich dust environment while the oxygen-rich material as exemplified by the occurrence of the 9.7 μm feature is confined to only the few compact ones and to very low excitation objects (VLE).

The continuum data can be used to determine the total IR flux from a nebula. Although a wide coverage in frequency is to be preferred, in general fluxes are observed only at two or three points. An emissivity function and the assumption that grains radiate like black bodies are needed to obtain the total IR flux from these measurements. Further, if the distance were known, the IR luminosity can also be calculated. The IR flux also yields a measurement of the temperature of the grains T_g. Cohen & Barlow (1974, 1980) used the data from their 10 μm and 20 μm photometry of planetary nebulae to derive T_g and L_{IR} using the distance scale of Acker (1978). They compared L_{IR} with the $L_{Ly\alpha}$ luminosities of the nebulae, having obtained the latter from an extrapolation of 5 GHz radio fluxes. Except for a few objects including NGC 7027, $L_{IR}/L_{Ly\alpha} \lesssim 1.5$. A least-squares fit to the data on fluxes ((F_{IR} vs F_{5Hz}) also yielded a slope of unity. This, the authors stressed, confirms the hypothesis that the thermal emission from dust is due, almost entirely, to the absorption of Lyα photons. The few cases, where $L_{IR} > L_{Ly\alpha}$, perhaps indicate that in addition to Lyα, ionizing stellar continuum photons are also absorbed by the dust.

For a list of 13 nebulae, Moseley (1980) has carried out photometry between 37 μm and 108 μm. These data have been combined with measurements in the middle IR to yield a more complete IR spectrum of these objects. According to Moseley, a major fraction of the far-IR flux comes from beyond 30 μm, peaking generally around 35 μm and dropping at longer wavelengths. This contradicts the results of Cohen & Barlow whose inferred dust temperatures implied black body peaks at less than 25 μm and thus much less flux was expected in the region beyond 30 μm. Moseley confirmed the earlier conclusion by Telesco & Harper (1977) that an emissivity function, ϵ_ν, of the form ν^n, fitted the data with $n \approx 2$. Since the emitting mass of dust is a sensitive function of dust temperature ($M_d \sim T_d{}^{-(4+n)}$ and since the longer wavelength peaks indicated $T_d \sim$ 70–90 K, much large mass of dust was derived in many of these nebulae. An enormous range in $L_{IR}/L_{Ly\alpha}$ was found varying from 1.0 for NGC 3242 to 18.6 for BD $+30°3639$. The difference of this ratio with Cohen & Barlow's estimates is at least a factor 2 and much larger in many cases. Moseley concluded that, due to this large range in $L_{IR}/L_{Ly\alpha}$, it was difficult to decide on the principal heating sources of the dust specially in those cases where $L_{IR} >> L_{Ly\alpha}$. For NGC 7027, the IR flux could perhaps be totally explained by heating due to stellar Lyc and trapped Lyα photons while these sources alone would be totally inadequate to explain the IR flux of BD +30°3639 specially because the central star in this nebulae seems too cool to have significantly higher UV flux. Moseley postulated the absorption of nonionizing flux by the dust in this case.

Following Cohen & Barlow (1980) a simple scenario for the IR behaviour of planetary nebulae may be given here. Since planetary nebulae are in a state of expansion, the densities and optical depths in dust and gas change with time. Moreover, since both dust and gas compete for the Lyc photons, it is obvious that their relative absorption efficiencies also vary with time and the excess of IR flux due to the absorption of Lyc photons should reflect this. For an expanding nebula of uniform density, $n_e R^3$ is a constant and if one also assumes a constant dust-to-gas ratio during the expansion, it follows that the UV optical depth of the dust $\tau_{UV} = n_d \sigma_{abs} R \sim n_e R \sim R^{-2}$. For a density bounded nebula, the optical depth in neutral hydrogen $\tau_{H^\circ} \sim n_{H^\circ} R \sim R^{-3}$, for it can be shown from considerations of ionization equilibrium that $n_{H^\circ} R^{-2} \sim n_e n_p \sim n_e^2$. Thus, as the expansion causes τ_{H° to decrease faster than τ_{UV}, the absorption by gas of the Lyc photons should decrease and $L_{IR}/L_{Ly\alpha}$ should increase with time. On the other hand, if the nebula were ionization-bounded, $\tau_{UV} \sim R^{-0.5}$ but τ_{H° remains steady since there is always neutral hydrogen to absorb the Lyc photons. In this case, $L_{IR}/L_{Ly\alpha}$ should decrease as grains absorb less and less of ionizing photons. A plot of $L_{IR}/L_{Ly\alpha}$ against n_e (rms) is shown for eleven nebulae from Moseley's list (figure 5). Although, there seems to be a convergence in the $L_{IR}/L_{Ly\alpha}$ values for densities around 4×10^3 cm^{-3}, there is an enormous spread in the luminosity ratio for high n_e. It is doubtful whether the simple scenario suggested by Cohen & Barlow represents the true behaviour. Moreover, if the characteristics of the dust and the ionizing star change during the evolution of the nebular shell, it may be meaningless to seek a correlation at all.

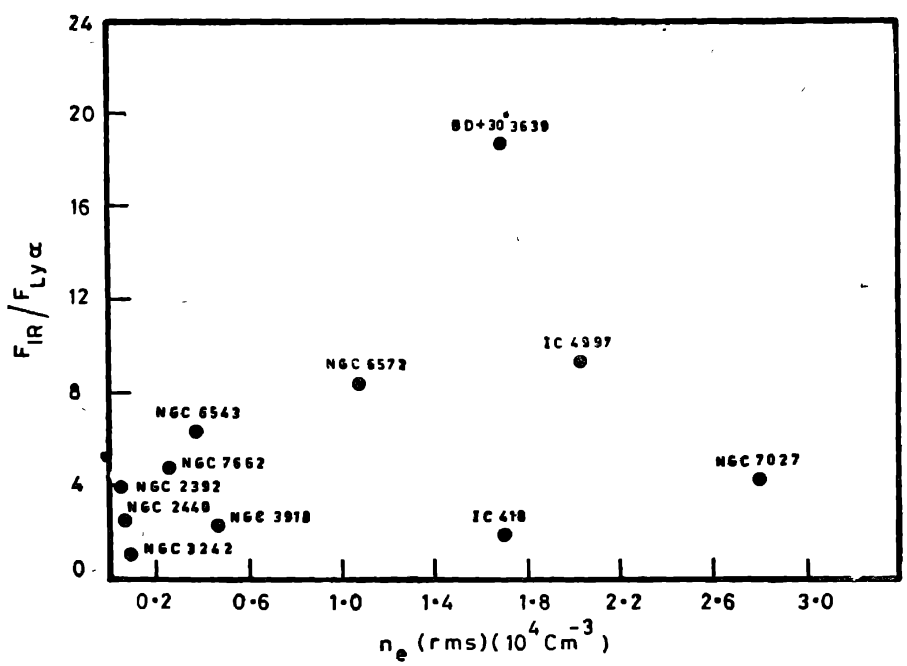

Figure 5. Ratio of the total infrared to Lyα flux vs n_e (rms) based on the data from Moseley (1980). The densities are from Torres-Peimbert & Peimbert (1977) and Natta & Panagia (1981).

Moseley's data have been subjected to further theoretical analysis by Natta & Panagia (1981). By modelling the dust emission in uniform density spherically symmetric well mixed nebulae and allowing for absorption by dust of a wide variety of UV radiation (Lyα, Lyc, nonionizing stellar, etc.), they found that the data implied a considerable variation in the properties of dust from nebula to nebula. In fact, the average grain size as well as the dust-to-gas ratio by mass correlated with the inverse of nebular radius. Since the decrease in dust-to-gas ratio was less steep than $\bar{a}^3$, where $\bar{a}$ is the average size of a grain, the second correlation implied that the number of dust particles actually increased with increasing nebular radius. These correlations have been interpreted as indicative of either a significant evolution in the properties of grains along with the nebular evolution or of a stratified distribution of grains within the envelope at the time of ejection with bigger grains in the inner regions and the smaller ones in the outer regions. As ionization proceeds outward with time more of the smaller grains appear within the nebula and the average size decreases while their numbers increase. In any case, these calculations show that the dust-to-gas ratio in planetaries is less than the same in the average interstellar medium and, therefore, planetary nebulae provide dust-deplated material to the Galaxy.

4. Chemical composition

One of the major aims of spectroscopic studies of planetary nebulae is to derive chemical composition. In recent years the UV and IR data have greatly aided the optical studies in this venture. There are essentially two ways of deriving abundances from the observed spectra :

(i) The intensities of various ionic emission lines are compared with the hydrogen recombination lines (generally Hβ), and if the appropriate T_e and N_e are known from the various forbidden line ratios, the ionic abundances, N^{+x}/N_{H^+}, can be calculated in a straightforward way. To derive the total abundance of an element by this procedure, all the relevant ionization stages should be either observed or inferred from ionization-correction schemes.

(ii) A more sophisticated approach is to make detailed models of nebulae based on the assumptions of photoionization and thermal equilibria with the stellar radiation field, gas density and chemical composition as inputs. The resulting ionization and temperature structures are then used to compute emission-line fluxes and these are compared with the observed line intensities. By varying the inputs the best fit is obtained and the abundances come out naturally from such a match. This method requires not only the knowledge of the nebula to be modelled but also of its central star and is usually an elaborate procedure. Since all lines in a nebula are produced by two-body atomic processes, the observed intensity in an optically thin emission line can be written as

$$I_\lambda = \int j\lambda ds = \int n_e\, n_i\, \epsilon_\lambda\left(T_e\right) ds . \frac{hc}{\lambda}, \qquad \ldots(8)$$

where n_e, n_i are the electronic and ionic densities involved, $\epsilon_\lambda\ (T_e)$ is an emission coefficient which is a sensitive function of T_e and a weak function of n_e and ds is the differential path length in the nebula. For recombination lines of H°, He° and He^+, the emission coefficient is an effective recombination coefficient, written $\alpha_\lambda^{\mathrm{eff}}$, which has been calculated for a variety of temperatures and densities from the two-body recombination theory, The results are fitted, for convenience, to a power-law of the temperature of the form T^{-m} with $m \sim 1$ (see Brocklehurst 1971, 1972). Most of the other lines are collisionally excited by electrons and a general expression for their intensity is

$$I_\lambda = \int n_e n_1 \, q^1_{1k} \left(T_e \right) \frac{A_{kl}}{\sum\limits_{l<k} A_{kl}} ds. \, h\nu_{kl}, \qquad \text{...(9)}$$

where q^i_{1k} is the collisional excitation rate of the ground level of ion i to its kth level and $A_{kl}/\sum\limits_{l<k} A_{kl}$ is the fractional transitions from the level k to the level l resulting in the emission of line photons with frequency ν_{kl}. The collisional excitation rate between any two states is given by

$$q_{kl}(T_e) = \frac{8.63 \times 10^{-6}}{T_e^{1/2}} \frac{\Omega(k, l)}{g_k} \exp\left(-E_{kl}/T_e \right), \qquad \text{...(10)}$$

where $\Omega(k, l)$ is the dimensionless atomic collision strength, g_k the statistical weight of the level k and E_{kl} the excitation energy of the level l. It is the occurrence of the exponential in T_e in equation (10) that makes all collisionally excited lines extremely sensitive to the electron temperature and provides justification for the enormous efforts usually undertaken to determine accurate values of T_e. It is fairly straightforward, then, to use these equations to obtain n_i/n_p.

Peimbert (1967) proposed a slightly more sophisticated scheme to take into account the temperature fluctuations in the nebula and defined two parameters—a mean temperature T_0 and a mean-square temperature deviation t^2—which could be obtained from observations and used in a Taylor-series expansion of ϵ_λ (T_e) to derive better abundances. This procedure was used by Peimbert & Costero (1969) in their pioneering work on the abundance determination of galactic H II regions and later by Torres-Peimbert & Peimbert (1971, 1977) to derive abundances in planetary nebulae. Peimbert & Costero were also the first to apply the ionization-correction scheme to correct for the unobserved ionic states and to derive reliable total abundances of several elements. The scheme of ionization-correction factors (ICF) is based on the idea of ionization stratification and utilizes the fact that ions with similar ionization potentials coexist in the nebula. For example, oxygen and nitrogen whose first and second ionization potentials are similar should coexist as O^+ and N^+ in the outer regions of the nebula and as O^{++} and N^{++} in the higher excitation inner regions. The [O II] λ3727 and [O III] λ5007 are strong lines in the optical spectrum of any nebula and their intensities can be used to determine O^+/H^+ and O^{++}/H^+. However, N^{++} does not have any line in the visible region and only [N II] λλ6548, 6584 are seen in the red. To infer N^{++}/H^+, therefore, the near-coincidence in the ionization potentials are used to write

$$\frac{n(O^+) + n(O^{++})}{n(O^+)} = \frac{n(N^+) + n(N^{++})}{n(N^+)}.$$

Then,

$$\frac{n(\mathrm{N})}{n(\mathrm{H})} = \frac{n(\mathrm{N}^+) + n(\mathrm{N}^{++})}{n(\mathrm{H}^+)}$$

$$= \frac{n(\mathrm{O}^+) + n(\mathrm{O}^{++})}{n(\mathrm{O}^+)} \frac{n(\mathrm{N}^+)}{n(\mathrm{H}^+)}$$

$$= i_c \frac{n(\mathrm{N}^+)}{n(\mathrm{H}^+)}, \qquad \text{...(11)}$$

where the ICF is given by the observed ratio of $n(\mathrm{O}^+) + n(\mathrm{O}^{++})$ to $n(\mathrm{O}^+)$. Similar considerations led to ICFs for Ne and S. Argon has been recently included in this scheme by French & Grandi (1981).

The ICF scheme has been subjected to considerable analysis lately. Hawley & Miller (1977) made a detail study of NGC 6720 by measuring line intensities in six different positions and derived n_e, T_e and the abundances. The ionic abundances showed the expected variation with position while the total abundances of He, O and N showed reasonable constancy. The ICF for N varied by a factor of nearly 30, yet the derived N abundance was remarkably consistent. The situation for Ne was not so good. The total Ne abundance varied by a factor of 5 in certain positions of the nebula and indicated that the adopted ICF greatly overestimated the abundance of neon in low-ionization regions. Barker (1982) has reobserved NGC 6720 in the same positions but with IUE and has confirmed that the use of a naive ICF procedure does not work for neon particularly since the charge-transfer reactions $\mathrm{O}^{++} + \mathrm{H}^\circ \rightarrow \mathrm{O}^+ + \mathrm{H}^+$ and $\mathrm{Ne}^{++} + \mathrm{H}^\circ \rightarrow \mathrm{Ne}^+ + \mathrm{H}^+$ proceed at very different rates forcing more oxygen in the O^+ state than neon in Ne^+. Barker (1978) used the ICF formula of Peimbert & Costero to determine the abundance of sulphur in 20 planetary nebulae. According to this formula

$$\frac{n(\mathrm{S})}{n(\mathrm{H})} = \frac{n(\mathrm{O}^{++}) + n(\mathrm{O}^+)}{n(\mathrm{O}^+)} \; \frac{n(\mathrm{S}^{++}) + n(\mathrm{S}^+)}{n(\mathrm{H}^+)}$$

$$= i_c(\mathrm{S}) \; \frac{n(\mathrm{S}^{++}) + n(\mathrm{S}^+)}{n(\mathrm{H}^+)} . \qquad \text{...(12)}$$

Barker found that this formula did not work for the i_c (S) correlated with the total sulphur abundance showing an increase with the degree of ionization. Typically, the $\mathrm{S}^{+++}/\mathrm{S}^{++}$ ratio was overestimated by nearly an order of magnitude using this formula and this produced the spurious correlation.

Grandi & Hawley (1978) and French & Grandi (1981) explored the validity of the ICF scheme from a theoretical standpoint. By computing theoretical models from basic principles they derived integrated line strengths and from these obtained n_e, T_e and the abundances by the usual procedures followed in the analysis of observations. Further, the i_c were deduced and total abundances calculated. The conclusion from these exercises seems to be that the ICF procedure should work for N and S but not so well for Ne and He. In general, the procedure works better in high ionization situations (hotter stars as in planetaries) than in low ionization situations (as in H II regions) and breaks down entirely for Ne, He and Ar in the latter case. In the light of these analyses, the sulphur ionization remains a problem. The models, in spite of the inclusion of relevant charge-transfer reactions (e.g. $S^{+++} + H^{\circ} \rightarrow S^{++} + H^{+}$), fail to show why the ICF scheme overestimates S^{+++}/S^{++}. Natta *et al.* (1980) have offered a solution by looking into the various atomic parameters in the $O^{++} - O^{+}$ and $S^{+++} - S^{++}$ ionization equilibria. According to them, although S^{++} has the same ionization potential as O^{+}, it ionizes much less efficiently because of a lower value of the photoionization cross-section. They derive the following formula for the ICF :

$$i_c(S) = 1 + 0.10 \frac{n(O^{++})}{n(O^{+})} \times \frac{n(S^{++})}{n(S^{+}) + n(S^{++})} \qquad ...(13)$$

and based on this obtain sulphur abundances in 41 planetary nebulae. This formula brings down the ionization correction for S^{+++} considerably and the correlation of the total S abundance with $i_c(S)$ disappears. Thus it would seem that the use of a correct ICF procedure is capable of yielding reliable abundance estimates (to within 50 per cent) for many of the abundant elements and that in many cases optical data alone would suffice for the purpose.

The most important exception to the above is carbon for which the optical data are clearly insufficient to yield reliable abundance determination because the predominant ionic states of carbon in planetary nebulae are C^{++} and C^{+++}, neither of which has any lines in the visible region. On the other hand, the UV spectrum of planetary nebulae is rich in carbon lines, the most conspicuous of these being the C III] λ1909 and C IV λ1550 doublets. Until the advent of UV observations the determination of carbon abundance depended entirely on the weak C II λ4267 recombination line. Torres-Peimbert & Peimbert (1977, hereafter TPP) used λ4267 to derive C^{++}/H^{+} in a number of planetary nebulae and also obtained the total carbon abundance in these objects using an ICF scheme. When the early ANS observations on C IV λ1550 were analysed by Pottasch *et al.* (1978a), the carbon abundance, based on an ICF very similar to TPP, turned out to be very much lower. During the last few years, IUE has obtained UV spectroscopic data on many planetary nebulae and carbon abundances have been derived by combining data on C II] λ2326, C III] λ1909 and C IV λ1550 lines. The IUE data also showed a discrepancy between the collisionally excited C III] λ1909 and the recombination line λ4267 in the sense that the C^{++}/H^{+} ratio determined separately from these lines differed by nearly an order of magnitude. A general problem in using a collisionally excited line as opposed to a recombination line is the assignment of the correct T_e to derive the abundance. For C IV λ1550 this problem is acute since part of C IV emission surely originates in regions of a nebula where He is doubly ionized. Since the second ionization of He is accompanied by abundant production of He II Lyα whose absorption causes a large amount of heating and since the efficient cooling due to [O III] lines is absent, theoretical models (Hummer & Seaton 1963) had shown the He^{++} ionization zone to be significantly hotter than the He^{+} zone. The temperature of this zone is difficult to determine for lack of proper diagnostic tools. The problem is resolved in some cases by the use of O IV] λ1402 in conjunction with the He^{+} λ1640 to derive a T_e. Similarly, if there are significant temperature fluctuations, the emission in a collisionally excited line may get substantially modified while a recombination line is hardly affected. The disagreement in the calculated C^{++}/H^{+} ratios from λ1909 and λ4267 in IC 418 was thus explained away by Torres-Peimbert *et al.* (1980) by invoking temperature fluctuations. However, the reality of such temperature fluctuations is by no means clearly established (Harrington *et al.* 1982). Another puzzling feature of the UV data was the discrepancy between the C^{+}/C^{++} ratio estimated from C II] λ2326/C III] λ1909 and from the prediction of photoionization models. In IC 418 the observed ratio was 2.7 while the models predicted a value 0.5 (Harrington *et al.* 1980). This has now been solved following the work of Storey (1981) in which the importance of dielectronic recombination in determining C^{+}/C^{++} and C^{++}/C^{++} at nebular temperatures and densities has been recognised. Based on models which now include the process of dielectronic recombination Harrington *et al.* (1981) have successfully interpreted the carbon line spectrum of NGC 7009 and NGC 7662. Dielectronic recombination significantly enhances C III λ2297 and C II λ1335 but does not affect C II λ4267. Storey (1981) has suggested that some other high ionization lines of N^{++}, N^{+++} and O^{+++} may also show significant effect of dielectronic recombination. Although the models show good agreement for almost all the observed carbon lines in these spectra C IV λ1550 and C II λ4267 continue to be discrepant. λ1550 is always found weaker than its theoretically predicted value while λ4267 is always stronger It now appears that the early measurements of λ4267 were considerable overestimates and new observations indicate agreement with the UV data (Peimbert 1981a). The weakness of C IV λ1550 is attributed to the absorption by dust. Being a resonance line the scattering optical depth in λ1550 is very high in a nebula [$\tau_{\lambda 1548} \sim 3 \times 10^4$ in NGC 7662, Harrington *et al.* (1981)] and it is sensible to assume that eventually the photons are absorbed by dust. For abundance studies, therefore, both λ4267 and λ1550 should be avoided. A consistent set of abundances can be derived entirely from the UV data on C II] λ2326, C III λ2997 and C III] λ1909.

The total carbon abundance determined in this way shows that in a number of planetary nebulae C/O > 1 while for the sun C/O = 0.6 (Lambert 1978). IC 418, NGC 6302, NGC 7027 definitely show carbon enhancements while in NGC 6572 and 7662 the C/O ratio is 1.1 and 1.2 respectively. Our current ideas on the origin of planetary nebulae and the evolution of their precursor stars suggest that a fraction of the nebulae should be carbon-rich. The IUE data seem to confirm this.

Some of the common ionic states are more accessible in the IR than in the UV or optical due to the presence of fine-structure levels. Fine-structure lines of [S IV] 10.5 μm, [Ar III] 9.0 μm and [Ne II] 12.8 μm have been observed in 18 planetary nebulae by Beck *et al.* (1981) and used in conjunction with optical observations to derive abundances of He, S and Ar. Dinerstein (1980) used the optical and near infrared data on [S II] and [S III] lines with [S IV] 10.5 μm observations to derive model-independent total abundances of sulphur in several planetary nebulae. The two different sets of sulphur abundances agree reasonably well, although for some nebulae the data of Beck *et al.* show sulphur to be overabundant. No systematic effects or correlations are seen in these abundances which is consistent with the idea that planetary nebulae do not originate in massive stars.

The most extensive optical study of abundances in recent years is due to Barker (1978b, 1978c). From detail spectrophotometry of 37 planetary nebulae in the range 3400–7400 Å both physical conditions and chemical abundances were derived. An ICF scheme was used to correct for unseen ionic states. The helium abundance showed a scatter due mostly to the presence of He° in the outer parts of low-excitation objects or in condensations. This was further proven in a direct correlation of the measured He abundance with the abundance of S^+ which is known to exist in condensations. The nitrogen abundance showed enhanced values with respect to its observed abundance in Orion nebula. Moreover, the nitrogen abundance correlated with the He abundance, a result which is expected from theories of nucleosynthesis. A very important conclusion that emerged from Barker's studies is that He, O and S abundances show little variation amongst planetaries of widely different kinematical properties.

In table 4 we list the abundances of He, C, N, O, Ne and S in a sample of 24 nebulae. The last two rows quote the observed abundances in the sun and Orion nebula from Lambert (1978) and Peimbert (1981b) respectively. Starting with NGC 7027, Fe abundances have been obtained in a few planetary nebulae by Shields (1978a, 1978b) and Garstang *et al.* (1978). Fe seems to be heavily depleted. UV and IR data have now provided a means of also determining Si and Mg abundances in the nebulae by observing fluxes in the UV lines of Si III], Si IV, Mg II and [Mg V] and the infrared [Mg IV] 4.5 μm and [Mg V] 5.6 μm lines (Harrington & Marionni 1981; Pequignot & Stasinska 1980). The Mg abundance shows large variations while Si is consistently depleted by an order of magnitude with respect to its solar value. Moreover, the Mg abundance derived from [Mg V] lines indicates higher abundance (nearly solar values) than the values (less than solar) obtained from Mg II resonance lines. Since [Mg V] lines are expected to originate in the inner high-excitation regions while Mg II comes mostly from the outer parts of a nebula, this may indicate significant gas-phase depletion of Mg in the outer regions. Pequignot & Stasinska obtain a gradient in the Mg abundance in NGC 7027 and interpret the variation as due to selective destruction of metallic Mg grains by radiation heating in the inner parts.

5. Planetary nebulae and stellar evolution

(a) *Location on H-R diagram*

To discuss the evolution of planetary nebulae and their central stars, it is essential to know the position of these stars on the H-R diagram. The two important parameters determining their location on this diagram are the distance which determines the luminosity L and the effective temperature T_*. The effective temperatures of central stars are generally derived by the use of a method due originally to Zanstra (1931) and described in detail in Osterbrock (1974). A more recent and elaborate description is given by Helfer *et al.* (1981). This method was extensively used by O'Dell (1963) and by Harman & Seaton (1966). The latter authors derived Zanstra temperatures from the observed data on H_β, He I (λ4471) and He II (λ4686) and developed criteria for complete absorption of the respective ionizing radiations. They noted that for planetaries which are optically thin in Lyc the Zanstra temperature obtained from the He^+ λ4686 radiation, T_z (He^+), is a reliable estimate of T_*. These effective temperatures were used by Harman & Seaton (1964) and Seaton (1966), together with the distances calculated mainly from the application of the

Table 4. Abundances

	He/H	C	N	O	Ne	S	C/O	N/O	Source
NGC 6302	0.191	9.22	9.04	8.79	8.05	7.37	2.69	1.78	1
Hu 1–2	0.170	8.61		8.29			2.09		20,9
H 1–55	0.200		8.87	9.50				0.23	15
NGC 2440	0.105	9.10	7.97	8.66	8.02	7.10	2.75	0.20	16
	0.120	8.52	8.8	8.72	8.08	6.98	0.63	1.20	1
NGC 6741	0.110	9.10	8.70	8.84	8.27	7.20	1.82	0.72	1
						6.92			2
NGC 2371	0.130	8.0	8.10	8.41	8.17	0.39	0.39	0.49	20, 13
NGC 7009	0.132	8.60	8.47	8.72	7.98		0.76	0.56	6, 8, 12
NGC 7027	0.128	9.13	8.52	8.62			3.24	0.79	6,10
NGC 7662	0.105	8.78	7.96	8.72	7.95		1.15	1.74	6, 8, 3
					7.82	6.83			2
IC 2448	0.111	8.56	8.19	8.46			1.26	0.54	14
NGC 3918	0.113	8.96	8.39	8.78			1.51	0.41	14
NGC 6720	0.156	8.59	8.36	8.81	8.20		0.60	0.36	6, 4
NGC 6572	0.131	8.76		8.72	8.00	6.68	1.10		6, 5, 2
NGC 3242		8.7	8.2	8.9	7.85	6.69	0.63	0.20	11, 2
Me 2–1	0.102	8.91	8.12	8.86	8.36	7.30	1.12	0.18	1
					8.18	7.16			2
IC 418		8.85	7.9	8.7	7.88	6.79	1.41	0.16	7, 17, 2
NGC 2867	0.112	8.84	8.09	8.65	7.95	7.73	1.55	0.28	1
					7.90	6.73			2
NGC 2392	0.091	8.35	8.41	8.74	7.97	7.06	0.41	0.47	1
IC 2165		8.43	7.83	8.23	7.91	6.75	1.59	0.40	21, 2
IC 2003		8.60	8.00	8.52			1.20	0.30	21
NGC 6886	0.102	8.90	8.80	8.72	8.11	7.24	1.51	1.20	1
Halo Planetaries									
108−76° 1(BB1)	0.115		8.34	7.88	7.72			2.88	19
49+88°1(HA4 1)	0.107		7.80	8.34	6.72			0.29	19
K 648	0.100		6.07	7.65	6.40			0.03	19
Sun		8.67	7.99	8.92		7.23	0.56	0.12	
Orion	0.100	8.57	7.88	8.65	7.80	7.41	0.83	0.17	See text

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Shklovsky method, to obtain an H-R diagram for the central stars. The stars arranged themselves in a well-defined track to the left of the main-sequence. The nebular radii were used to indicate the sense of their evolution along this track. Starting from an initially cooler and less luminous location when their surrounding nebulae were generally optically thick in H I the stars increased in L and T_* reaching a maximum when the nebulae were also optically thin. Thereafter, the stars dropped sharply in luminosity and cooled somewhat. Thus entire evolution took place on a time-scale of less than 20,000 yr. This sequence has formed the basis of most discussions on the evolution of central stars of planetary nebulae (O'Dell 1968, 1974). However, there are both theoretical and observational uncertainties in the interpretation of the data and recent work has cast considerable doubt on the evolutionary picture conveyed by the Harman-Seaton sequence.

The early ANS results by Pottasch *et al.* (1978b) showed that the use of T_z (He$^+$) for T_*, in some of the cases where T_z (H I) $< T_z$ (He$^+$), was perhaps a misrepresentation. It has also been realised for some time that the presence of [O I] radiation in a nebula does not necessarily imply optical thickness in H I while this was one important criterion used by Harman & Seaton. Pottasch *et al.* (1978b) determined T_* by comparing the measured UV flux of these stars between 1500 Å–3300 Å with model black-body fluxes. T_* and T_z (He$^+$) were discrepant for many of the stars, the latter being always higher and predicting much larger UV fluxes than were actually seen. The agreement of T_* with T_z (H I) was considerably better. The new temperatures also affected the luminosities through bolometric corrections and produced a significant change in the positions of the central stars on the H-R diagram moving them to lower luminosities and lower temperatures. This is shown in figure 6 which is taken from Pottasch (1981) and where the open circles depict the central stars. The distances are from MP (see section 2). Theoretical evolutionary tracks for three different masses from the work of Paczyński (1971) have been superimposed. For comparison the older positions of the central stars from O'Dell (1974) are indicated by the shaded strip.

The discrepancy between T_*, determined from an examination of the visual spectra of the central stars, and T_z (He$^+$) was also noted by Heap (1977). Analysis of the visual line spectra indicated temperatures much lower than T_z (He$^+$) independent of whether blackbody models or model atmospheres were used to determine the latter. Secondly, although the Zanstra temperatures showed a correlation with the nebular radius, the visual line spectrum hardly correlated with such a change. The outstanding problem was exemplified by the central star of NGC 2392 whose T_z (He$^+$) = 92,000 K consistent with the appearance of [Ne V] lines in the nebular spectrum but whose visual line spectrum did not show the expected absorption lines from N V and instead showed lines from N III. A second curious case is NGC 6210 which, according to Seaton (1966), belongs to the class of nebulae optically thick in H I but Heap found that its T_z (H I) = 43,000 K while its T_z (He$^+$) = 70,000 K for blackbody models and 92,000 K for the Hummer-Mihalas model atmospheres. Heap pointed out the dangers of accepting these temperatures at their face value for interpretation in terms of evolution. If NGC 6210 were indeed optically thick in H I and if T_z (H I) were to be accepted as the correct present value of T_*, then, at a later time, when the nebula becomes optically thin in H I and T_z (He$^+$) becomes the correct measure for T_*, a spurious Harman-Seaton type evolution would

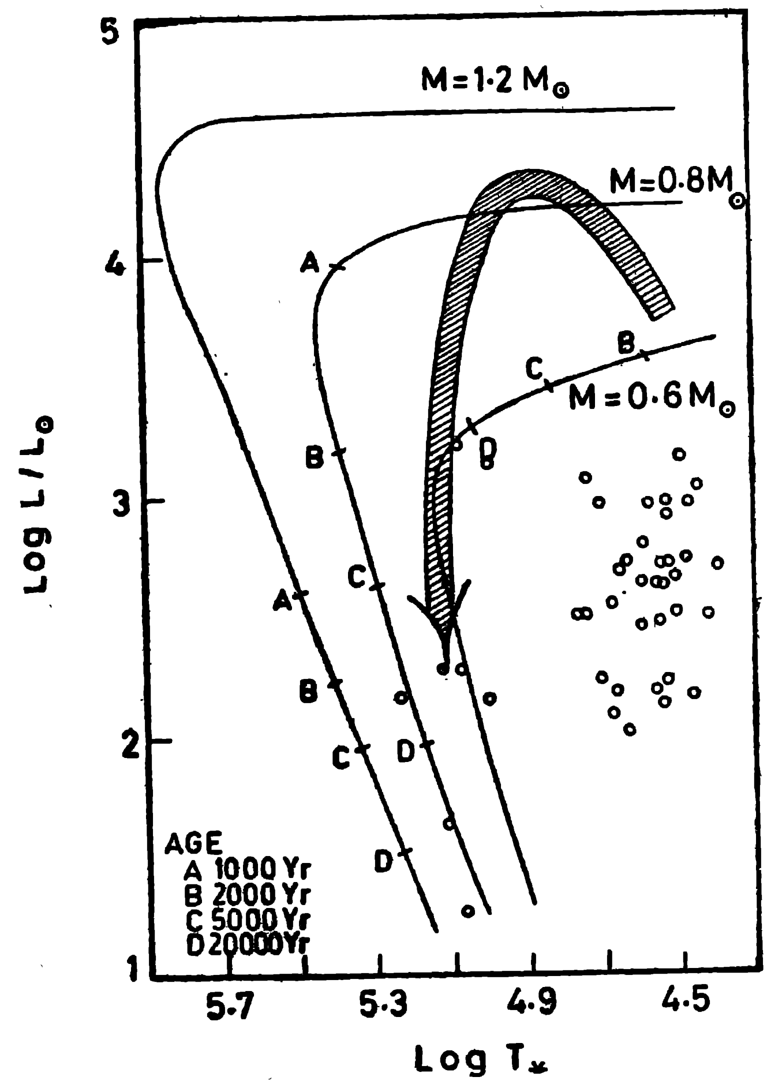

Figure 6. H-R diagram of the central stars of planetary nebulae. See text. (Courtesy : D. Reidel Publishing Co., Dordrecht.)

already have taken place, even without any change in the properties of the central star.

At present no consistent explanation has been given for the discrepancy between T_z (He^+) and other determinations of T_* although various possibilities have been suggested by Hummer (1978). If the hypothesis of the existence of a hot unseen companion in all these cases be confirmed, the effect on the Harman-Seaton sequence would be disastrous.

Mendez *et al.* (1981) have recently determined T_* from NLTE model atmosphere analysis of six planetary nuclei and have concluded that the use of blackbody fluxes to fit the observed energy distribution produces an overestimate, since compared to the NLTE models the blackbody distribution is flatter. They questioned some of the temperatures determined by Pottasch *et al.* (1978b) and argued that T_* for NGC 1360 and 7293 are actually lower. These authors also confirmed the Zanstra discrepancy and added NGC 4361 to the list of objects where this effect stands out.

Harrington *et al.* (1982) have criticized the temperature determinations based on the ANS data. They have shown that, in case of NGC 7662, the UV flux distribution obtained with IUE closely corresponds to an LTE model atmosphere at $T_* = 10^5$ K while T_z (He$^+$) $= 1.2 \times 10^5$ K. Pottasch *et al.* quoted a T_* of 48,000 K for this star. Moreover, the higher T_* reproduced correctly the ratio of the observed far-IR flux to the Lyc flux of this nebula (Moseley 1980) which a lower T_* ($\sim$50000 K) can by no means explain. In view of these conflicting trends, it is difficult to assess the real status of the Harman-Seaton sequence. Model-atmosphere analysis of UV flux distributions of central stars obtained with IUE will hopefully settle this question in near future.

(b) *Theoretical evolution*

There have been many theoretical attempts to explain the general features of the Harman-Seaton sequence. The location of the central stars on the H-R diagram clearly indicates an eventual evolution of these stars to the white dwarf state [see figure 1 in O'Dell (1968]. All theoretical discussions, therefore, have been addressed to the possible evolutionary courses followed by pre-white dwarf stars. Most of this work has been reviewed by Salpeter (1968), Osterbrock (1973), O'Dell (1974) and Paczyński (1978). Paczyński's (1970, 1971) models of contracting C-O cores with He- and H-burning shells supporting thin envelopes on top reproduce the observed evolution of the central stars quite clearly. However, at least three points of disagreement should be noted : (i) the early luminosity rise seen in O'Dell's figure is not reproduced by the theoretical models, (ii) the theoretical models evolve to very high T_* ($\gtrsim 10^{5.3}$ K) when their shell sources eventually burn out but the observed sequence does not extend to such high T_*s, (iii) the observed sequence cannot, in general, be reproduced by a single stellar mass evolving through, but by a variety of evolutionary tracks with masses spread over an interval of at least a few-tenths of a solar mass. All these features are clear in figure 6. While these disagreements may reflect certain inadequacies in the theory, the uncertainties in the observational data on both T_* and L indicate that the observed sequence may also require substantial modification.

Regarding the very high effective temperatures Helfer *et al.* (1981) have recently shown that if substantial amounts of dust are present, the Zanstra temperatures may show considerable underestimates, as far-UV photons may suffer absorption by the dust. In that case, stars with high T_* may systematically appear cooler. A second possibility has been discussed by Pottasch (1981b) who is also responsible for the discovery of a new class of very hot central stars. Since very hot stars are extremely faint in the visible and reasonably faint in UV Pottasch argued that the magnitude-limited samples may selectively miss them. Pottasch studied 17 nebulae from the list of Curtis (1918) whose central stars are of the 18th magnitude or fainter. When the data on these objects were analysed to derive T_z (H I) and T_z (He$^+$), it was found that all these stars are extremely hot ($T_* > 10^5$K). For half the cases T_z (H I) $\approx$ T_z (He$^+$) showing that the nebulae are perhaps optically thick in H I as well as in He II. These stars occupy a position in the H-R diagram which was unpopulated in Seaton (1966). In figure 7 these hot central stars are shown. They lie close to the evolutionary tracks with masses between 0.8–1.2 $M_\odot$ which is reassuring theoretically.

According to the currently accepted picture of stellar evolution (Iben 1974, 1977; Tinsley 1977; Wheeler 1978), all stars that develop degenerate C–O cores following core He exhaustion may, in principle, evolve to white dwarfs, if mass-loss processes ar eefficient enough to reduce their total mass below 1.4 $M_\odot$ before the point of carbon ignition is reached in the core. Since stars as massive as 8 $M_\odot$ or less degenerate develop C–O cores, the currently observed white dwarf population in the Galaxy may have progenitors with masses spread over as large an interval as 1–8 $M_\odot$. However, it is conceivable that mass loss is not very efficient and there exists a mass $M_{\rm crit}$, within this interval, above which carbon ignites in a degenerate core. In this case, stars up to only $M_{\rm crit}$ become white dwarfs. Stellar winds in the red giant and asymptomic giant branch (AGB) phases of the evolution and formation of planetary nebulae contribute to the loss of mass helping stars between 1.4 $M_\odot$ and $M_{\rm crit}$ to quiescently die as white dwarfs. Recent observations in UV (Pottasch *et al.* 1981; Castor *et al.* 1981; Benvenuti & Perinotto 1980) show that planetary nuclei do not lose mass at appreciable rates. Thus the mass loss process is essentially complete with the ejection of the nebula. The progenitors of planetary nebulae are thus stars in the mass range 1 $M_\odot$ $-M_{\rm crit}$. Masses of central stars are difficult to determine. Moreover, there is no unique way of relating the mass of a central star to its parent steller mass. Stellar wind theories show a large dispersion in the $M_{\rm f}$ ($M_{\rm i}$) relations and suffer from too many uncertainties to be a reliable guide to this problem. The observational clues are indirect and lean heavily on model-dependent interpretation of data (see Peimbert & Serrano 1980). However, it is clear that a spread in the masses of progenitors should be accompanied by a spread in the masses of central stars and also in the masses of the nebulae. All reasonable $M_{\rm f}(M_{\rm i})$ relations predict such a spread. If this be true, then it would seem meaningless to even try to explain the observed evolutionary sequence in terms of a single-mass evolutionary track. Although the steepness of any initial mass function guarantees that at any time the number of planetary nuclei originating from massive progenitors will be rather low; the spread in the masses of the nuclei due to the lower end of the progenitor mass-spectrum may not be insignificant. Since theoretical evolutionary tracks are quite sensitive to the mass, it is important to determine the mass-spectrum of the nuclei of planetary nebulae before hastening to an interpretation based on a single mass. If the observed sequence be now interpreted in terms of a spread in masses rather than a time evolution of a single mass very different conclusions are reached (Renzini 1981). The different portions of the Harman-Seaton sequence are then seen to be populated by central stars of different masses. This follows from a consideration of the evolutionary timescales of post-AGB stars of different masses. The age along an evolutionary track has been shown by tick marks in figures 6 and 7. Accordingly it is expected that the high-luminosity nuclei are less massive than the low-luminosity ones. Since a massive nucleus is likely to have a massive nebula around it, the latter may never get fully ionized. This may not contradict the observational data as nebulae on the final descending branch of the sequence are perhaps optically thick in H I although the reasons usually given for them to be so are quite different. All low mass nuclei on the other hand, will have nebulae that are generally optically thin. The lifetime of nebulae in the conventional sense ceases to have any meaning in this picture and estimates of birthrate are to be modified depending on the sample of nebulae considered.

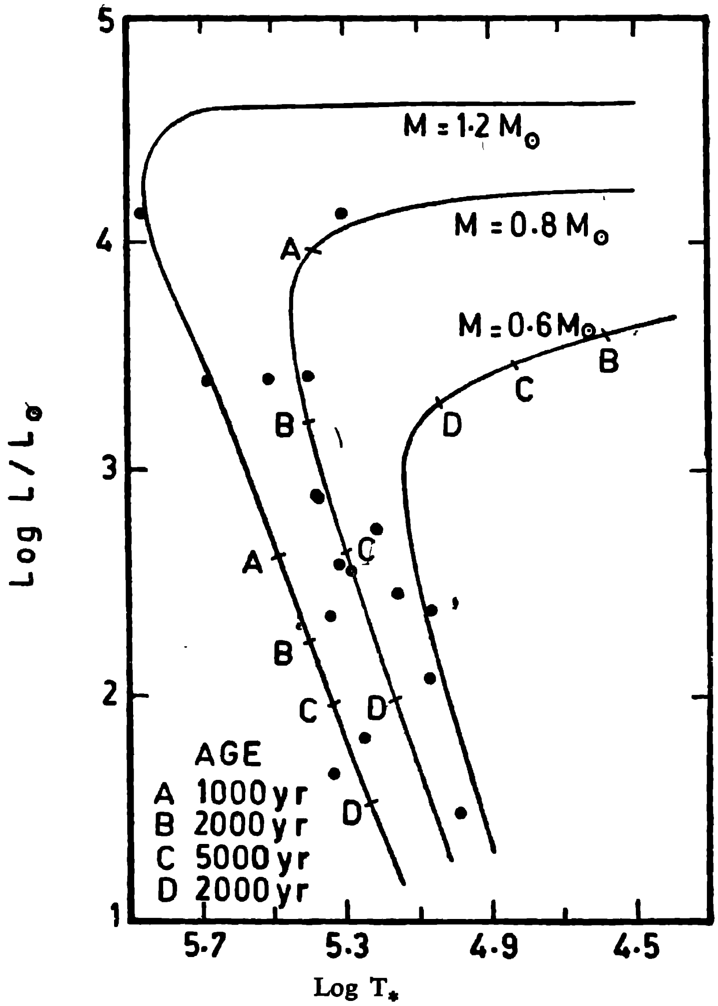

Figure 7. Hot central stars from Pottasch (1981b) on the H-R diagram. (Courtesy : D. Reidel Publishing Co., Dordrecht.)

This point of view has been challenged by Schönberner (1981) who has analysed data on 130 central stars with the help of new post-AGB evolutionary tracks. From this analysis Schönberner & Weidemann (1981) and Schönberner (1981) find that the mass distribution of planetary nuclei is very narrow and highly peaked at approximately 0.6 $M_{\odot}$ (figure 8). 98 per cent of all central stars sampled by them have masses in the range 0.55–0.64 $M_{\odot}$. This also confirms the earlier results by Koester & Weidemann (1980) who found a similar but not as narrow a mass distribution for the DA white dwarfs. The mean masses in both these distributions are nearly the same. According to these authors, the vast majority of planetary nebulae have central stars of approximately the same mass and their evolution should follow closely a single-mass track on the H-R diagram.

These data exclude the hot central stars discussed above. The positions of these stars on the H-R diagram do indicate higher masses and they do not fit into the mass distribution shown in figure 8. According to Pottasch (1981), 20 per cent of the nebulae in Curtis (1918) belong to this category. If they are as abundant or more, the mass distribution will be distorted. Since the progenitors of these nuclei

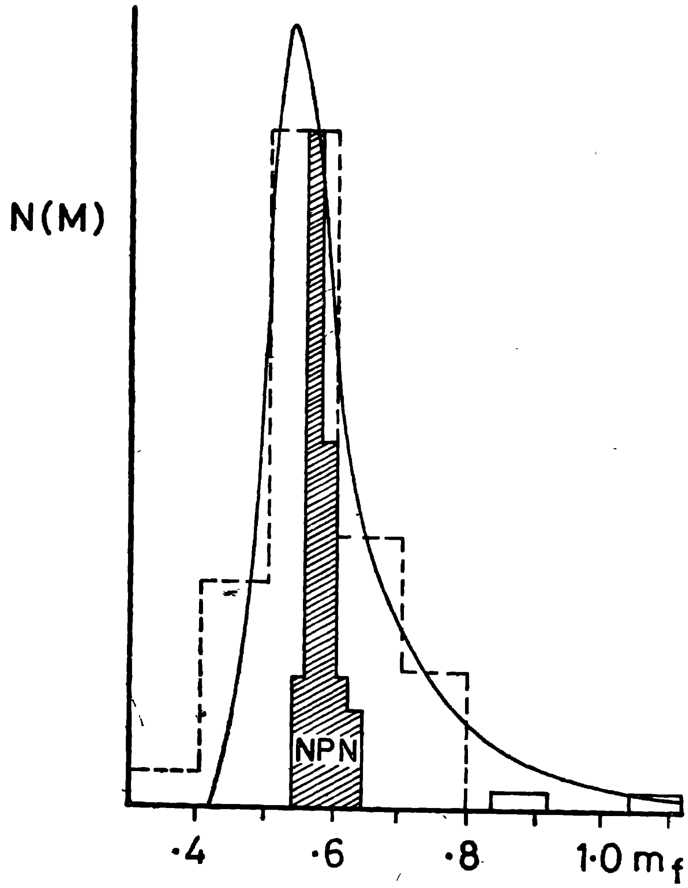

Figure 8. The mass distribution of nuclei of planetary nebulae (shaded histogram) and DA white dwarfs (dashed line) from Weidemann (1981). The solid curve is a theoretical distribution derived from galactic evolution models of Koester & Weidemann (1980) using the Miller-Scalo IMF. (Courtesy : D. Reidel Publishing Co., Dordrecht.)

are expected to be more massive, these nebulae should belong to a younger population and their chemical and kinematic properties may reflect this. They leave the interpretation of HS undisturbed because they were not included in the sequence.

The observed narrow mass distribution of both white dwarfs and nuclei of planetary nebulae imply an extremely flat M_f (M_i) relation and are difficult to understand in the light of the existing theories of mass loss. There are other observational indications that stars as massive as 5–6 $M_{\odot}$ become white dwarfs (Weidemann 1977; Romanishin & Angel 1980; Anthony-Twarog 1982). On the other hand, the canonical value of the pulsar birthrate requires stars as light as or lighter than this value to produce pulsars with all currently popular initial mass functions. Thus the value of M_{crit} is rather uncertain at the moment. Theoretical calculations of the planetary nebula birthrate cannot settle this question either, since these calculations show that the birthrate is extremely insensitive to the choice of the upper mass limit of integration which is the same as M_{crit}.

(c) *Precursors*

There seems to be little doubt that the immediate precursors of planetary nebulae are red giants. Calculations by Härm & Schwarzschild (1975) have convincingly shown that following a rapid mass ejection event a solar mass red giant makes a quick transit across the H-R diagram from its initial location to the region occupied by the nuclei of planetary nebulae. The transit time is a sensitive function of the mass of the thin hydrogenirich envelope that survives the mass ejection event. There are a number of theories on the mechanism of mass ejection, all having to do with some sort of dynamical or pulsational instability of an extended red giant envelope that ultimately results in its expulsion from the parent star (Wood 1981). Conflicting views are presented by some authors who insist that the quiescent stellar-wind type mass loss is sufficient to create a planetary nebula (Harpaz & Kovetz 1981).

The regions of dynamical instability on the H-R diagram originally delineated by Paczynski & Ziolkowski (1968) are occupied by Mira variables which are amongst the brightest of giant branch stars. Wood & Cahn (1977) modelled the observed pulsational properties of Mira variables based on existing theories of steller evolution including the effects of stellar winds. They were able to reproduce the period distribution of Mira variables near the galactic plane by considering a wedge shaped instability strip on the mass-luminosity diagram of double shell source burning stars. Assuming that the high-luminosity edge of the strip coincided with the ejection of the hydrogen-rich envelope they were able to derive, for a range of initial masses, the masses of the stellar remnants and of the ejected envelopes. Moreover, the lifetime of Mira pulsation was used to derive a birthrate of Mira variables to be compared with the birthrates of planetary nebulae and white dwarfs. Depending on the mass-loss rate, $M_{\rm crit}$ was 3.7 or 4.7 $M_{\odot}$. The derived white dwarf mass distribution was peaked between 0.6–0.8 $M_{\odot}$ and the envelope mass distribution (implying the mass distribution of planetary nebulae) was bimodal, with a low mass peak at 0.01 $M_{\odot}$ contributed by low-mass progenitors, whose entire envelopes were lost through stellar-wind type of mass loss and a high mass peak at 0.8 $M_{\odot}$ for stars whose envelopes were lost principally due to the rapid ejection event terminating the Mira phase. The birthrate of Mira variables was found to be an order of magnitude lower than the planetary nebula and white dwarf birthrates and these authors concluded that only a fraction of all stars evolving to white dwarfs passes through the Mira variability strip. A more theoretical approach was made by Tuchman *et al.* (1978, 1979) and Barkat & Tuchman (1980). These authors did not consider the stellar wind type mass loss to be important. They also delineated an instability strip on the mass luminosity diagram to correspond to Mira variability. Apart from the theoretical instability region they determined fact that was different from the semiempirical one due to Wood & Cahn, the derived period distribution showed a large overestimate in the relative number of low-period Miras, and the theoretical luminosity for a 1 $M_{\odot}$ star was much lower than the luminosities of Miras. The derived birthrate was again an order of magnitude lower than the birthrates of planetary nebulae or white dwarfs. Tuchman, Sack and Barkat suggested that as a consequence of He shell flashes, low-mass stars could perhaps be prevented from becoming Miras, in which case the difficulties in matching the observed characteristics would naturally be removed. This, of course, was in conflict with the idea that Miras are generally low-mass stars ($M < 2\ M_{\odot}$). Recent theoretical work by Willson (1981) on Mira pulsation which indicates a tenfold increase in the mass loss rate and recent downward revision of luminosities of Miras by Feast (1981) have now removed much of these difficulties. In Willson's work the envelope is removed in a steady superwind over 10^5 yr rather than in a specific ejection event and the lifetime of the Mira phase is correspondingly short. It thus appears that all stars evolving up the asymptotic giant branch will pass through a Mira variable phase which ends when mass ejection removes most of the envelope. Subsequent evolution of the system leads to a planetary nebula and a blue nucleus. The final mass distribution of the stellar remnants obtained in this last work compares favourably with the observed distribution of masses amongst the central stars of planetary nebulae (Weidemann 1981).

(d) *Chemical enrichment*

There are two different aspects to the problem of chemical enrichment in the context of the evolution of planetary nebulae. First, the nebulae themselves may show evidences of enrichment as a result of stellar nucleosynthesis. Second, by studying the chemical composition of these envelopes, it is possible to learn something about the spatial and temporal variation of abundances in the galactic interstellar medium and about processes contributing to its enrichment. Both these aspects have been studied in some detail in the last few years (Peimbert 1978, 1981a ; Tinsley 1978).

Peimbert (1978) classified planetary nebulae into four types depending on their chemical and kinematic properties. Peimbert & Serrano (1980) studied in detail the chemical composition of these various types and showed that type I planetary nebulae comprising about 20 per cent of the galactic population show significant enhancements of He and N in their envelopes while type II planetary nebulae reflect more closely the abundances in the interstellar medium from which the parent stars formed. Type II planetary nebulae thus display well defined abundance gradients across the galactic disk. Type III planetary nebulae are high-velocity objects whose peculiar radial velocities are less than 60 km s^{-1} and whose distances from the galactic plane are less than 1 kpc while planetary nebulae of type IV belong to the extreme population II with both peculiar radical velocities and distances from the plane exceeding these limits.

Since most low and intermediate mass stars eventually evolve through the planetary nebula phase and since the lifetime of stars over this mass interval spans 10^7–10^9 yr, it is not surprising that the present planetary nebula population represents a mixture of chemical and kinematic properties. The late stages of evolution of these stars have received a great deal of theoretical attention in the last few years (Gingold 1977; Iben & Truran 1978; Becker & Iben 1979, 1980; Renzini & Voli 1981). The dredge-up episodes which contribute to the changes in susface abundances of these stars have been incorporated in detail in these computations and the surface enrichment parameters as a function of parent stellar masses have become available. Therefore, it is now possible to test theories of stellar nucleosynthesis by comparing the observed chemical composition of planetary nebulae with these models. Alternatively, assuming the models to be correct, such comparisons may be used to obtain clues to the parent stellar masses of the individual nebulae studied. By using the same stellar evolution data it has also been possible to evaluate the contribution of low and intermediate mass stars to galactic chemical evolution (Lequeux *et al.* 1979; Mallik 1980; Serrano & Peimbert 1981; Chiosi & Mateucci 1982).

The comparison of observed abundances in planetary nebulae envelopes with theoretical predictions brings out a number of interesting results. Kaler *et al.* (1978)

had initially explained the positive correlation between He/H and N/O of the nebular envelopes (Kaler 1978, 1979) in terms of the first two dredge-up episodes. However, this implied that some of the nebulae originated in stars as massive as $6 M_{\odot}$. Perinotto & Renzini (1979) and Becker & Iben (1980) later showed that much larger changes in He/H occur during the third dredge up phase and in stars as light as 3 $M_{\odot}$. It was then found that in theory the largest He/H did not necessarily imply the largest N/O. Peimbert (1981b) showed that a large range in N/O values is observed even for moderate He enrichment and not only type I but also type II planetary nebulae show considerable range in N/O values. Secondly, the N/O vs He/H correlation for type II objects did not confirm the predictions of Kaler *et al.* (1978). If type II planetary nebulae are the more common and hence have progenitors less than 3 $M_{\odot}$ the agreement between the observed nitrogen abundances and the theoretical predictions in this regard is really poor.

The situation is much better with C/O ratios. Peimbert (1981a,b) has compared the C/O vs He/H correlation of the objects observed with IUE with predictions from the models of Renzini & Voli (1981). Most planetary nebulae cluster around the theoretical curve. However, this match implies that the progenitors of these planetaries are at least as massive as 2 $M_{\odot}$, if not 3 $M_{\odot}$. This is in conflict with other evidences according to which the mean progenitor mass of a planetary nebula is about 1.4 $M_{\odot}$. From such comparisons as well as from the observed narrow mass distribution of the nuclei, it is evident that the dredge-up processes during AGB evolution are important at even lower masses than what the theory predicts.

Galactic abundance gradients from planetary nebulae were originally derived by D'Odorico *et al.* (1976). Recent determinations are due to Barker (1978b) and Peimbert & Serrano (1980). Both He and N in type II planetary nebulae show radial abundance gradients. A gradient in oxygen may also be present but Barker remarks that it may be weak. The abundance analyses of three halo PNs, HA 4–1, BBI and K648 (Barker 1980, also Peimbert 1981a), indicate that their oxygen and neon abundances show a variation of over two orders of magnitude (from 1/100 solar to solar) while Ar is deficient in all of them by roughly a factor of 100 with respect to the sun. Argon is presumed to reflect more closely the heavy element abundance than O and Ne. Their He abundances are normal which may be interpreted as He enrichment through stellar processing if one adopts a pregalactic value of $Y = 0.25$ and assumes that the initial composition of the progenitors of these halo nebulae corresponded to this Y. The oxygen abundance, on the other hand, may suggest that some planetary nabulae originate from oxygen-rich stars or that oxygen enrichment in the Galaxy proceeded faster than Fe enrichment in the earlier epochs. We have already discussed the infrared observations which do imply that some planetaries are from oxygen-rich stars. There are some other observational indications that oxygen enrichment was indeed faster in earlier epochs. Its implications for the chemical evolution of the Galaxy have been discussed elsewhere (Mallik 1981).

In the 192 years since Herschel's first observation of "a singular phenomenon", our understanding and knowledge of planetary nebulae have matured a great deal. Yet some of the very basic problems still elude solution. Although observational data have registered an exponential increase in the last twenty years or so, we are yet to evolve a satisfactory method of obtaining distances to these objects, to decide whether they are optically thick or thin in Lyc radiation and to unambiguously place them on the H-R diagram to chart the course of their evolution. It also appears that planetary nebulae will continue to provide a testing ground for theories of nucleosynthesis and be of considerable importance in the study of the abundances in galaxies.

Acknowledgements

It is a pleasure to thank Sushma Mallik for all the help in the preparation of this review. I would like to take this opportunity to also thank R. B. Isaacman for sending me a copy of his thesis. Figures 1, 3, 6, 7 and 8 are reproduced with kind permission from the Springer International, Heidelberg, the Blackwell Scientific Publications, London and D. Reidel Publishing Co., Dordrecht.

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