JOULE HEATING IN NEUTRON STARS UNDER STRONG GRAVITATION

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ABSTRACT

Considering Joule heating caused by the dissipation of the magnetic field in the neutron star crust to be an efficient mechanism in maintaining a relatively high surface temperature in very old neutron stars, the role of general relativity is investigated. It is found that, although the effect of spacetime curvature produced by the intense gravitational field of the star slows down the decay rate of the magnetic field, modification of the initial magnetic field configuration and the initial field strength by the spacetime curvature results in increasing the rate of Joule heating. Hence the spacetime curvature supports Joule heating in maintaining a relatively high surface temperature which is consistent with the observational detection.

Subject headings: relativity — stars: magnetic fields — stars: neutron

1. INTRODUCTION

Neutron stars cool down mainly by neutrino emission from the inner layers during the first million years and subsequently by photon emission from the surface. The rate of photon emission depends on the physical properties of matter inside the star and on the magnetic field.

Observational inference of the thermal radiation of several old neutron stars in the X-ray band (Becker & Trumper 1997) and in the UV range (Pavlov, Stringfellow, & Cordova 1996; Mignani, Caraveo, & Bignami 1997) indicates much higher surface temperature as compared to the predictions of the standard cooling models. Therefore, additional heating mechanisms are needed in order to remove the discrepancy between the detected temperature and the theoretical models.

One of the two possible mechanisms is the heat generated by the frictional energy of neutron superfluid with normal matter in the inner crust (Shibazaki & Lamb 1989; Umeda et al. 1993), which is independent of the magnetic field. The other mechanism of additional heating is associated with the ohmic dissipation of currents (Miralles, Urpin, & Konenkov 1998), which is strongly sensitive to the configuration and strength of the magnetic field. These authors found that the ohmic dissipation produces enough heat to change the thermal evolution of neutron stars substantially at the late stage, although observational data on the magnetic field evolution of isolated pulsars support a slow decay rate.

Considering magnetic field configurations, which are initially confined to a small part of the crust and which vanish in the stellar core, several authors (Chanmugam & Sang 1989; Urpin & Muslimov 1992; Urpin & Van Riper 1993) found that the relatively low electrical conductivity of the crustal matter causes the decay times too short to be of observational interest if the impurity content is high. Sengupta (1997, 1998) investigated the contribution of spacetime curvature on the decay rate of the crustal magnetic fields in isolated neutron star by assuming a spherically symmetric stationary gravitational field. It was demonstrated clearly by Sengupta (1998) that the role of impurity content which increases the decay rate is suppressed by the effect of spacetime curvature. As a result, even with high impurity content, the decay rate is signifi-

cantly less at the late stage of evolution if general relativistic effects are taken into consideration.

In the present paper, I investigate the effect of spacetime curvature produced by the intense gravitational field of the star on the rate of Joule heating caused by ohmic dissipation. The basic equations that describe the magnetic field evolution and the rate of Joule heating under general relativistic framework are presented in the next section. In § 3 the model adopted in this investigation is described. The results are discussed in § 4, and the conclusions are drawn in § 5.

2. EQUATIONS FOR MAGNETIC FIELD EVOLUTION AND JOULE HEATING

Assuming hydrodynamic motions to be negligible and the anisotropy of the electrical conductivity of the crustal material is small, the induction equation in flat spacetime can be written as

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{\nabla} \times \left(\frac{c^2}{4\pi\sigma} \, \boldsymbol{\nabla} \times \boldsymbol{B} \right), \tag{1}$$

where σ is the electrical conductivity.

If a stationary gravitational field is taken into account, then using the covariant form of Maxwell equations and the generalized Ohm's law (Sengupta 1998) (neglecting the displacement current and taking $u^i = 0$), the corresponding induction equation in curved spacetime can be derived as

$$\frac{\partial F_{kj}}{\partial x^0} = \frac{\partial}{\partial x^k} \left[\frac{c}{4\pi} \frac{1}{\sqrt{-g}} \, g_{\mu j} \frac{1}{\sigma u^0} \frac{\partial}{\partial x^i} \left(\sqrt{-g} F^{\mu i} \right) \right]$$
$$- \frac{\partial}{\partial x^j} \left[\frac{c}{4\pi} \frac{1}{\sqrt{-g}} \, g_{\mu k} \frac{1}{\sigma u^0} \frac{\partial}{\partial x^i} \left(\sqrt{-g} F^{\mu i} \right) \right], \tag{2}$$

where $F_{\mu\nu}$ are the components of the electromagnetic field tensor, J^μ are the components of the four-current density, u^μ are the components of the four velocity of the fluid, $g_{\mu\nu}$ are the components of spacetime metric that describes the background geometry and $g = \det |g_{\mu\nu}|$. Here and afterwards Latin indices run over spatial coordinates only,

whereas Greek indices run over both time and space coordinates.

For the description of the background geometry I consider the exterior Schwarzschild metric which is given by

$$ds^2 = \left(1 - \frac{2m}{r}\right)c^2\,dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2$$
$$- r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\,, \qquad (3)$$

where $m = MG/c^2$, M being the total gravitational mass of the core. The justification for adopting the exterior Schwarzschild metric is provided in Sengupta (1998). Since the crust consists of less than a few percent of the total gravitational mass, M can be regarded as the total mass of the star.

Using the metric given in equation (3), equation (2) can be reduced to

$$\frac{\partial F_{kj}}{\partial x^0} = \frac{c}{4\pi}\left[\frac{\partial}{\partial x^k}\left\{\frac{1}{r^2\sin\theta}\,g_{lj}\frac{1}{\sigma u^0}\frac{\partial}{\partial x^i}\,(r^2\sin\theta F^{li})\right\}\right.$$
$$\left. - \frac{\partial}{\partial x^j}\left\{\frac{1}{r^2\sin\theta}\,g_{lk}\frac{1}{\sigma u^0}\frac{\partial}{\partial x^i}\,(r^2\sin\theta F^{li})\right\}\right]. \quad (4)$$

Following the convention, I consider the decay of a dipolar magnetic field which has axial symmetry so that the vector potential $\boldsymbol{A}$ may be written as $(0, 0, A_\phi)$ in spherical polar coordinates where $A_\phi = A(r, \theta, t)$. Since the hydrodynamic motion is negligible so $u^i = dx^i/ds = 0$ and the metric gives

$$u^0 = \left(1 - \frac{2m}{r}\right)^{-1/2}.$$

Therefore, from equation (4) we obtain using the definition $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$,

$$\frac{\partial A_\phi}{\partial t} = \frac{c^2}{4\pi\sigma}\left(1 - \frac{2m}{r}\right)^{1/2}\sin\theta$$

$$\times \left\{\frac{\partial}{\partial r}\left[\left(1 - \frac{2m}{r}\right)\frac{1}{\sin\theta}\frac{\partial A_\phi}{\partial r}\right] + \frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\theta}\right)\right\}. \quad (5)$$

Choosing

$$A_\phi = \frac{f(r, t)}{r}\sin\theta \qquad (6)$$

for the flat spacetime and

$$A_\phi = -g(r, t)\sin^2\theta \qquad (7)$$

for the curved spacetime, where r and θ are the spherical radius and polar angle, respectively, one gets from equation (1) and equation (5)

$$\frac{\partial^2 f(r, t)}{\partial r^2} - \frac{2}{r^2}f(r, t) = \frac{4\pi\sigma}{c^2}\frac{\partial f(r, t)}{\partial t} \qquad (8)$$

and

$$\left(1 - \frac{2m}{r}\right)^{1/2}\left[\left(1 - \frac{2m}{r}\right)\frac{\partial^2 g(r, t)}{\partial r^2} + \frac{2m}{r^2}\frac{\partial g(r, t)}{\partial r} - \frac{2}{r^2}g(r, t)\right]$$
$$= \frac{4\pi\sigma}{c^2}\frac{\partial g(r, t)}{\partial t} \qquad (9)$$

respectively.

The ϕ component of the electric current maintaining the dipolar magnetic field configuration for flat spacetime is given by

$$j_\phi = -\frac{c}{4\pi}\frac{\sin\theta}{r}\left(\frac{\partial^2 f}{\partial r^2} - \frac{2f}{r^2}\right). \qquad (10)$$

In Schwarzschild spacetime geometry the above quantity can be written as

$$j_\phi = -\frac{c\sin^2\theta}{4\pi}$$
$$\times\left[\left(1 - \frac{2m}{r}\right)\frac{\partial^2 g(r, t)}{\partial r^2} + \frac{2m}{r^2}\frac{\partial g(r, t)}{\partial r} - \frac{2}{r^2}g(r, t)\right]. \quad (11)$$

When transferred to a locally Lorentz frame the above equation takes the following form:

$$j_\phi = -\frac{c}{4\pi}\frac{\sin\theta}{r}\left[\left(1 - \frac{2m}{r}\right)\right.$$
$$\left.\times\frac{\partial^2 g(r, t)}{\partial r^2} + \frac{2m}{r^2}\frac{\partial g(r, t)}{\partial r} - \frac{2}{r^2}g(r, t)\right]. \quad (12)$$

Following the approach of Miralles, Urpin, & Konenkov (1998), I neglect nonsphericity in cooling calculations and use the angle-averaged expression for Joule heating. The rate of Joule heating $\dot{q} = j^2/\sigma$ in flat spacetime is written as

$$\dot{q} = \frac{c^2}{24\pi^2 r^2\sigma}\left(\frac{\partial^2 f}{\partial r^2} - \frac{2f}{r^2}\right)^2. \qquad (13)$$

Normalizing the function $f(r, t)$ to its initial value at the surface $f(R, 0)$ which is related to the initial field strength at the magnetic equator B_e by the relation $f(R, 0) = R^2 B_e$, the expression for the rate of Joule heating in flat spacetime can be written as

$$\dot{q} = \frac{c^2 R^4 B_e^2}{24\pi^2 r^2\sigma}\left(\frac{\partial^2 F}{\partial r^2} - \frac{2F}{r^2}\right)^2, \qquad (14)$$

where $F(r, t) = f(r, t)/f(R, 0)$.

Similarly in curved spacetime,

$$\dot{q} = \frac{c^2}{24\pi^2\sigma r^2}\left[\left(1 - \frac{2m}{r}\right)\right.$$
$$\left.\times\frac{\partial^2 g(r, t)}{\partial r^2} + \frac{2m}{r^2}\frac{\partial g(r, t)}{\partial r} - \frac{2}{r^2}g(r, t)\right]^2. \quad (15)$$

The function $g(r, t)$ is normalized to its initial value at the surface by $G(r, t) = g(r, t)/g(R, 0)$ and $g(R, 0)$ is related to the initial field strength at the magnetic equator by the expression

$$g(R, 0) = -\frac{B_e R}{2m}\left[R^2\ln\left(1 - \frac{2m}{r}\right) + 2mR + 2m^2\right]$$
$$\times\left[\frac{R}{m}\ln\left(1 - \frac{2m}{R}\right) + \left(1 - \frac{2m}{R}\right)^{-1} + 1\right]^{-1}$$
$$\times\left(1 - \frac{2m}{R}\right)^{-1/2}. \qquad (16)$$

In the present work I have assumed that the spacetime metric within the crustal region and exterior to the stellar surface is the same. Since there exists a plasma-filled magnetosphere outside the surface of the star, one should have a time-varying dipole magnetic field outside the boundary.

However, the electric conductivity of the surrounding plasma is such that the time variation of the magnetic field is effectively negligible as compared to that within the crust. Hence for both relativistic and nonrelativistic cases, I impose the usual boundary conditions as given in Urpin & Muslimov (1992).

3. THE MODEL

The formalisms adopted in the present work have been discussed in detail by Miralles, Urpin, & Konenkov (1998). The evolution of the magnetic field in flat spacetime is discussed extensively by Urpin & Muslimov (1992) and in curved spacetime by Sengupta (1998). In the present investigation, the gravitational mass of the star is taken to be 1.4 $M_\odot$. However, two different configurations for the neutron star model are adopted: one with a radius 7.35 km and the other with a radius 11 km. The first one is obtained if the equation of state of the matter inside the star is soft, and the second one is obtained if it is intermediate or stiff.

If one assumes the initial value of $f(r, t) = f(r)$ at $t = 0$ for flat spacetime, then for curved spacetime (Wasserman & Shapiro 1983; Sengupta 1995)

$$g(r, 0) = g(r) = \frac{3rf(r)}{8m^3}\left[r^2\ln\left(1 - \frac{2m}{r}\right) + 2mr + m^2\right]. \quad (17)$$

This is because of the fact that any given magnetic field configuration in flat spacetime is modified by the curvature of spacetime produced by the gravitational field of the central object. Asymptotically at a large distance $g(r)$ coincides with $f(r)$.

I have considered the decay of the magnetic field which initially occupies the surface layers of the crust up to a depth $x = 0.966$ where $x = r/R$, R being the radius of the star. The crustal region is considered to be extended up to $x = 0.875$. It should be noted that for the same value of x, the corresponding density for the two different mass-radius configurations is different. The main aim of the present work is to investigate the effect of spacetime curvature produced by the intense gravitational field of the star to the rate of Joule heating and hence to the thermal evolution of old neutron stars.

The electrical conductivity within the crust has been calculated following the approaches of Urpin & Van Riper (1993). The net conductivity of the crustal material at a given depth is computed as

$$\sigma = \left(\frac{1}{\sigma_{\rm ph}} + \frac{1}{\sigma_{\rm imp}}\right)^{-1}, \quad (18)$$

where $\sigma_{\rm ph}$ is the conductivity due to electron-phonon scattering and $\sigma_{\rm imp}$ is the conductivity due to electron-impurity scattering. The effect of electron-ion scattering has been neglected since the region where this effect could be important is sufficiently thin. $\sigma_{\rm imp}$ is inversely proportional to the impurity parameter ϵ defined as

$$\epsilon = \frac{1}{n}\sum_i n_i(Z - Z_i)^2, \quad (19)$$

where n and Z are the number density and electric charge of background ions in the crust lattice without impurity, Z_i and n_i are the charge and density of the ith impurity species. The summation is extended over all species of impurity. It is worth mentioning that the value of the impu-

rity parameter ϵ for neutron star crust is not known at present. Electron-impurity scattering becomes more important with increasing density and decreasing temperature. Hence at the late stage of evolution when the temperature becomes low, the conductivity is dominated by electron-impurity scattering. The impurity parameter ϵ has been taken as 0.01 and 0.1.

4. RESULTS AND DISCUSSIONS

The evolution of the surface magnetic field normalized to its initial value for both the general relativistic and the nonrelativistic cases with the standard cooling model and with the impurity parameter $\epsilon = 0.01$ and $\epsilon = 0.1$ are presented in Figure 1 and in Figure 2, respectively.

It is shown by Urpin & Muslimov (1992) that the field behavior is qualitatively independent of the forms of the initial configurations but the numerical results differ for various choices of the initial depth penetrated by the magnetic field. At a very early stage of evolution when the crustal matter is melted in the layers of a maximal current density, the magnetic field does not decay appreciably. After the outer crust solidifies, significant decay takes place and after 1 Myr no decay occurs if the impurity content is zero.

The above scenario for flat spacetime is not altered with the inclusion of general relativistic effects, but significant decrease in the numerical value at the late stage of evolution is found when the effect of spacetime curvature is incorporated. The impurity-electron scattering is dominant at the late stage of evolution when the crustal temperature is low. As a consequence the decay rate after $t > 10$ Myr changes appreciably. It should be mentioned here that at the late
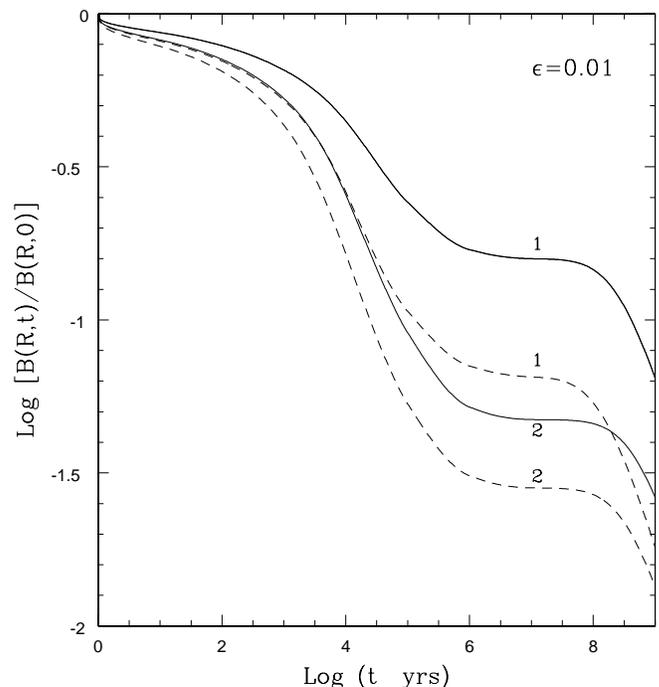

FIG. 1.—Evolution of surface magnetic field normalized to its initial value for flat and curved spacetimes with the impurity parameter $\epsilon = 0.01$. Solid line represents the results for curved spacetime while broken line represents that for flat spacetime. Curves labeled "1" represent the results for neutron star with mass 1.4 $M_\odot$ and radius 7.35 km, while curves labeled "2" represent results for neutron stars with mass 1.4 $M_\odot$ and radius 11 km. The results are obtained by using the standard cooling model without Joule heating.
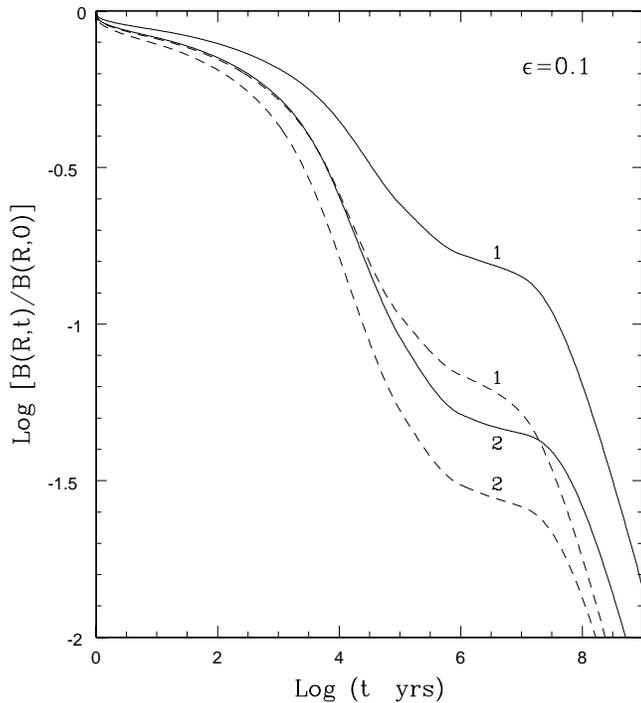
FIG. 2.—Same as Fig. 1 but with $\epsilon = 0.1$

FIG. 4.—Same as Fig. 3 but with $\epsilon = 0.1$

stage the electric conductivity becomes independent of temperature.

Figure 3 and Figure 4 show the rate of Joule heating for the flat spacetime and for the curved spacetime with $\epsilon = 0.01$ and $\epsilon = 0.1$, respectively. For both the cases we
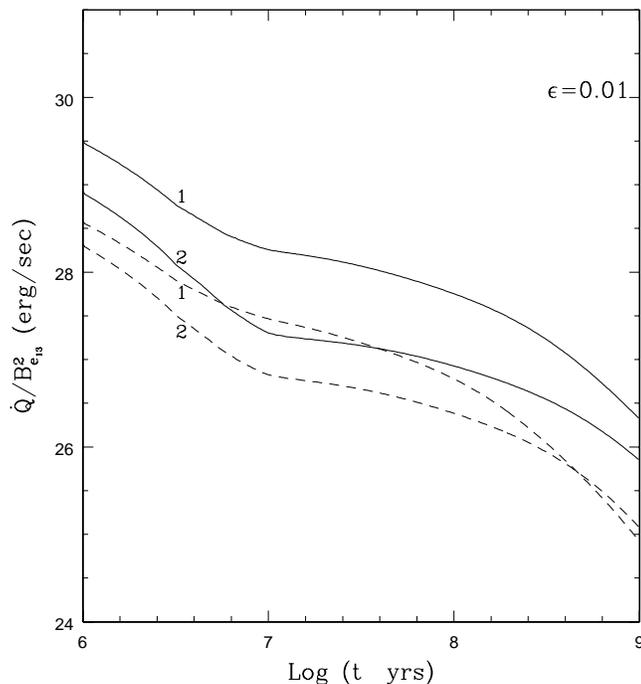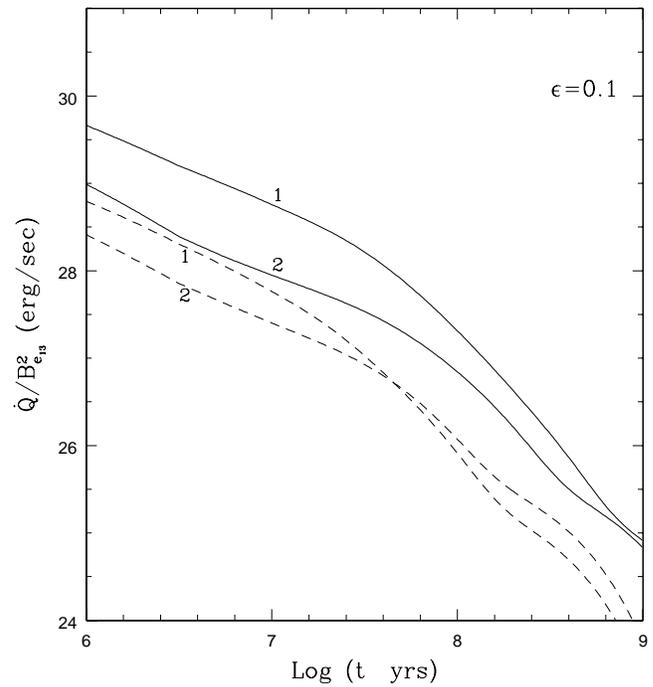

FIG. 3.—Temporal dependence of the rate of Joule heating integrated over the volume of the neutron star and normalized to $B_e/10^{13}$ G with $\epsilon = 0.01$. Solid line represents the results for curved spacetime, while broken line represents that for flat spacetime. Curves labeled "1" represent the results for neutron star with mass 1.4 $M_\odot$ and radius 7.35 km, while curves labeled "2" represent results for neutron stars with mass 1.4 $M_\odot$ and radius 11 km.

notice that the rate of Joule heating increases by almost 1 order of magnitude when the stellar radius is 7.35 km. In both Figure 3 and Figure 4, the results with the stellar radius $R = 11$ km are also presented in order to demonstrate the effect of compactness. The rate of heat production is equal to the rate of decrease of the magnetic energy which is proportional to the square of the magnetic field strength at any time. Figures 1 and 2 show that the rate of magnetic field decay decreases substantially when the effect of general relativity is incorporated. As a result one expects the rate of Joule heating to be less when the effect of general relativity is incorporated. However, the rate of Joule heating is very much sensitive to the configuration and strength of the initial magnetic field. The more the initial field strength, the more the heat generated. The spacetime curvature produced by the intense gravitational field of the star changes the geometry and increases the strength of the initial magnetic field substantially. The combination of the two phenomena, e.g., a slower decay rate but higher initial field strength, yields an overall increase in the rate of Joule heating by almost 1 order of magnitude. The whole purpose of this investigation is to understand the effect of strong gravitation of the neutron star. The effect of the different equation of states of matter and the effect due to the depth penetrated by the initial magnetic field have been investigated in detail by Miralles, Urpin, & Konenkov (1998).

Following the approach of Miralles, Urpin, & Konenkov (1998), I consider the surface temperature to follow the equation

$$\dot{Q} = 4\pi R^2 \sigma_{SB} T_s^4 , \qquad (20)$$

where σ_{SB} is the Stefan-Boltzmann constant. This is valid because of the fact that except for a short initial period of 3 to 10 Myr when the surface temperature is very high, approximately all heat released due to the magnetic field dissipation is emitted from the surface. At the earlier age the influence of Joule heating is not important. At the late stage

the surface temperature is determined by balancing the Joule heating integrated over the neutron star volume with the photon luminosity. It should also be mentioned that at the late stage when electron-impurity scattering becomes dominant, the electric conductivity becomes independent of the temperature of the crust.

The surface temperature with and without the effect of general relativity is presented in Figures 5 and 6 for $\epsilon = 0.01$ and $\epsilon = 0.1$ respectively with $R = 7.35$ km. Since the earlier evolution is not affected by Joule heating the results are shown for the age $t > 2.5 \times 10^6$ yr. For comparison the surface temperature without Joule heating is also presented. Figures 7 and 8 present the same results with $R = 11$ km. The results show that for both the cases, with or without the effect of general relativity, the surface temperature increases if the initial magnetic field strength is increased. Figures 5–8 clearly show that although general relativistic effect slows down the magnetic field decay rate substantially, the change in the initial magnetic field configuration and the strength of the initial field result into an increase in the surface temperature of the star as compared to the result obtained without the effect of general relativity. Although this increase is not very large, the results clearly indicate that general relativity supports Joule heating in maintaining a relatively high surface temperature of old neutron stars which is consistent with observational detection.

Pavlov, Stringfellow, & Cordova (1996) estimated the bolometric luminosities of three isolated pulsars, B0656 + 14, B0950 + 08, and B1929 + 10. Becker & Trumper (1997) presented the upper limit of the bolometric luminosities of several millisecond pulsars. The temperature of
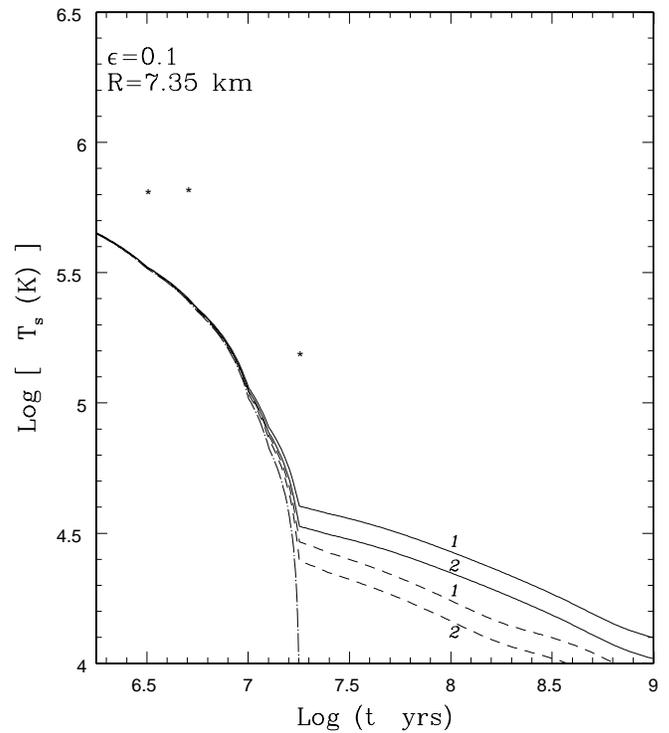

FIG. 6.—Same as Fig. 5 but with $\epsilon = 0.1$

a pulsar can be calculated from the observed flux by assuming blackbody radiation and keeping in mind the fact that the temperature as calculated in the surface is higher by a factor of $1 + z = (1 - 2GM/Rc^2)^{-1/2}$. The surface temperature of all the millisecond pulsars as pre-
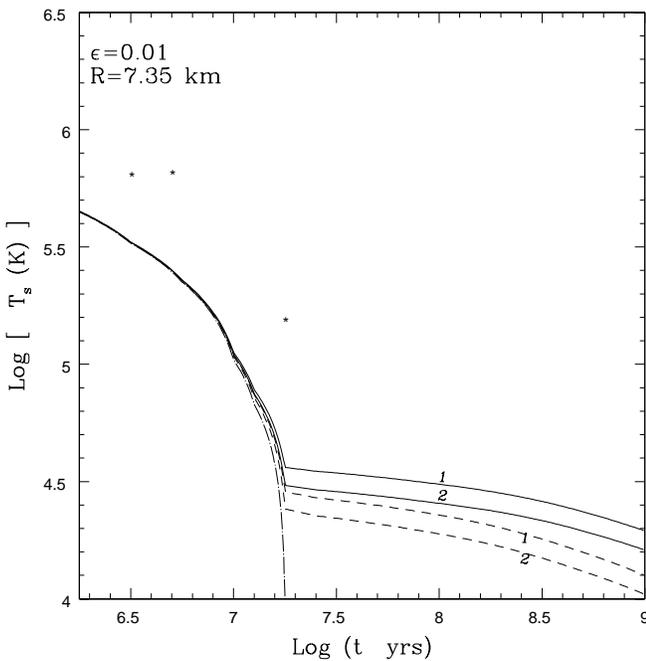

FIG. 5.—Surface temperature of neutron star with and without Joule heating with $\epsilon = 0.01$. Solid line represents the results for curved spacetime, dashed line represents that for flat spacetime, and dash-dotted line represents the surface temperature without Joule heating. Curves labeled "1" represent the surface temperature with $B_e = 3.0 \times 10^{13}$ G, and curves labeled "2" represent the surface temperature with $B_e = 1.5 \times 10^{13}$ G. Stars represent the surface temperature of isolated neutron stars inferred from observation.
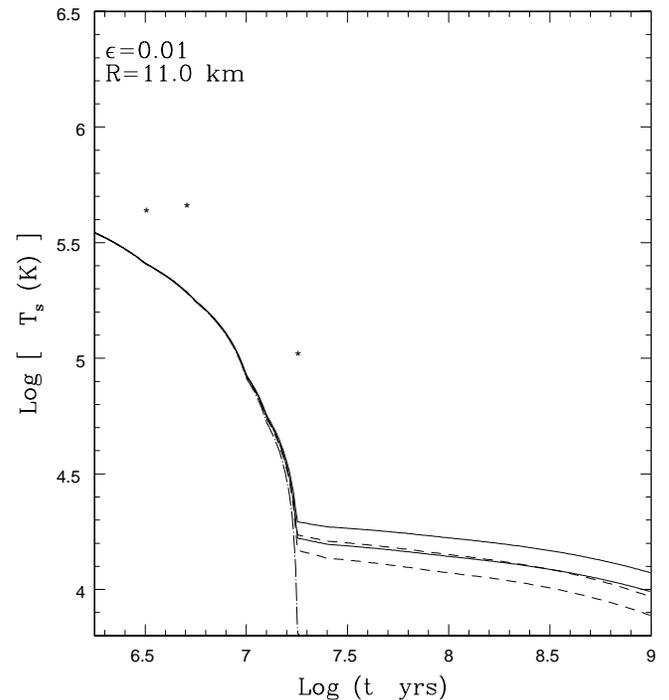

FIG. 7.—Same as Fig. 5 but with the stellar radius 11 km. The upper two curves (solid and dashed lines) represent the results with $B_e = 3.0 \times 10^{13}$ G, while the lower two curves represent the results with $B_e = 1.5 \times 10^{13}$ G.
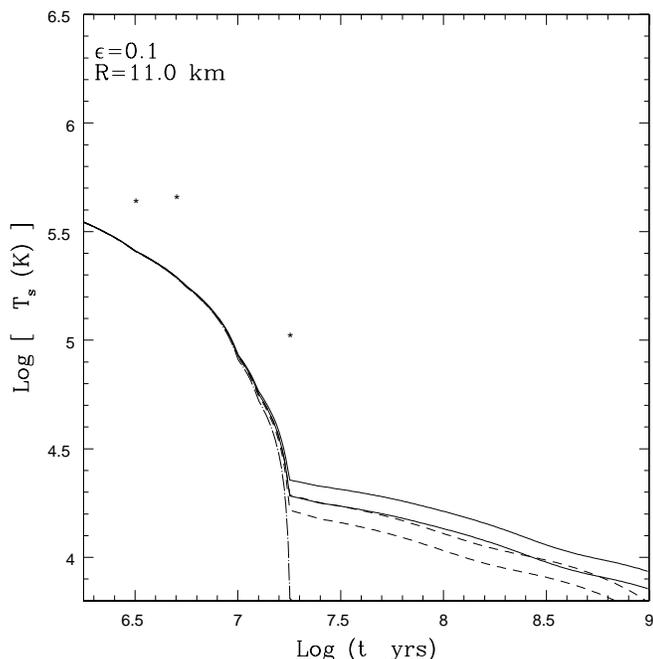
$\epsilon = 0.1$
$R = 11.0$ km

FIG. 8.—Same as Fig. 7 but with $\epsilon = 0.1$. The upper solid lines and the upper dashed lines represent the result with $B_e = 3.0 \times 10^{13}$ G, while the lower solid and dashed lines represent that with $B_e = 1.5 \times 10^{13}$ G.

sented by Becker & Trumper (1997) is several orders of magnitude higher than that predicted by the standard cooling models. However, it is believed that the millisecond pulsars are in accretion stage. Therefore, they should have much higher surface temperature than that of isolated pulsars, and their thermal evolution is different from that of isolated pulsars. It should be pointed out here that the impurity content does not have any significant role at a high temperature. Hence, unlike isolated pulsars, the general relativistic effects should not be significant for the

case of accreting neutron stars even at the late stage of evolution.

In Figures 5–8, I have presented the surface temperature of three isolated pulsars: B0823+26, B1929+10, and B0950+08, whose bolometric luminosities are estimated from observations (Pavlov, Stringfellow, & Cordova 1996; Becker & Trumper 1997). The surface temperature of three isolated pulsars which are older than 2.5×10^6 yr has been considered from the above references because Joule heating is important only after the age $t > 2.5 \times 10^6$ yr. Comparison with the theoretical models that incorporate Joule heating shows that the inclusion of general relativistic effects makes the results more consistent with the observational data. It is worth mentioning that the observational data provides only the upper limit of the bolometric luminosity owing to the uncertainties in the distance measure of the pulsars.

5. CONCLUSIONS

Considering Joule heating caused by the decay of the crustal magnetic field in neutron stars to be a potential mechanism in explaining the high surface temperature detected in many old neutron stars, the effect of spacetime curvature produced by the intense gravitational field of the star on the thermal evolution is investigated. In spite of the fact that general relativity slows down the magnetic field decay rate substantially, the rate of Joule heating increases almost by an order of magnitude because the initial field configuration and the field strength get modified by the spacetime curvature. As a result, general relativistic effects support Joule heating in maintaining a high surface temperature at a very late stage of evolution of isolated neutron stars.

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