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Electromagnetic properties of neutral and charged spin-1 particles

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The structure of the electromagnetic vertex of spin-1 particles is studied in a general way, for the diagonal as well as the off-diagonal couplings. In each case, we consider in detail the consequences of gauge invariance and the space-time discrete symmetries, paying particular attention to the implications for the couplings of the photon to neutral gauge bosons, which arise in higher orders. Several general results concerning the static electromagnetic properties of neutral bosons are derived, analogous to the results that are well known in the case of neutral spin-1/2 particles. [S0556-2821(97)07105-1]

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I. INTRODUCTION AND NOTATION

The properties of vector bosons have been the subject of intense studies recently [1]. This is motivated mainly by the fact that direct measurement of the trilinear vector boson couplings may be possible in the pair production of electroweak bosons at e^+e^- and hadron colliders. A possible way to test whether the predictions of the standard model are sustained is to parametrize the trilinear vector boson couplings in some generalized form, and then try to constrain the generalized couplings from the experimental results. A consistent implementation of such parametrization must satisfy certain requirements that follow from fundamental principles such as Lorentz and gauge invariance, the discrete space-time symmetries C, P, T, and their combinations, or any other symmetry that may be applicable. This has been done in great detail for the $W^+W^-\gamma$ vertex [2,3], which awaits experimental tests in the near future.

Although neutral gauge bosons cannot have a coupling with the photon at the tree level, they do develop such couplings in higher orders of perturbation theory. Thus, for example, the standard model admits a $ZZ\gamma$ coupling at loop levels. It is conceivable that vector bosons in addition to the standard W and Z exist in nature. In fact, it is not uncommon to find in the literature theories beyond the standard model that predict the existence of such additional bosons. In those theories, the trilinear couplings involving the photon can be more complicated than those of the standard model. In particular, for example, off-diagonal couplings $VV'\gamma$ involving different vector bosons can in principle exist. While such couplings arise only at loop levels, they may be important for discovering physics beyond the standard model. For example, a heavy gauge boson may be detected by its radiative decay into a lighter one. In order to prepare ourselves for these possibilities, it is useful to look in a more general way at the electromagnetic vertex of neutral spin-1 particles. This is our purpose in this paper. We study the structure of the electromagnetic vertex of charged as well as neutral spin-1 particles, for the diagonal and for the off-diagonal couplings. We consider in detail the consequences of gauge invariance and the space-time discrete symmetries, paying particular attention to the implications for the electromagnetic couplings of neutral gauge bosons. This kind of study can serve as a guide in the quest to find the physics that may lie beyond the standard model in a way that is general and model independent. Moreover, it may allow us to recognize possible deviations of the new physics from basic physical principles such as Lorentz and gauge invariance, and crossing symmetry.

We begin by introducing the notation for the electromagnetic vertex of spin-1 particles. The off-shell vertex function $\Gamma_{\alpha\alpha'\mu}(k,k')$ is defined such that the matrix element of the electromagnetic current is

$$\langle V'(k')|j_\mu^{(\mathrm{EM})}(0)|V(k)\rangle = \epsilon'^{*\alpha'}(k')\epsilon^\alpha(k)\Gamma_{\alpha\alpha'\mu}(k,k') \equiv j_\mu(Q,q), \tag{1.1}$$

corresponding to the vertex

$$V(k)\to V'(k')+\gamma(q). \tag{1.2}$$

For the sake of convenience, we represent the function j_μ in terms of the two independent momenta

$$Q\equiv k+k', \quad q\equiv k-k', \tag{1.3}$$

where q is the photon momentum as depicted in Eq. (1.2).

Before we write the most general form of the vertex consistent with Lorentz invariance, it is useful to make the following observations. (i) If V and V' are neutral, electromagnetic gauge invariance implies

$$q^\mu\Gamma_{\alpha\alpha'\mu}=0\,, \tag{1.4}$$

for arbitrary values of k and k', and hence of q. On the other hand, if V and V' are charged, the relation analogous to Eq. (1.4) contains some terms in the right-hand side involving the inverse propagators of V and V'. Therefore, what we get is the weaker condition

$$q^\mu j_\mu(Q,q)=0\,, \tag{1.5}$$

where $j_\mu(Q,q)$ is the on-shell vertex function as defined in Eq. (1.1), which follows simply by using the current conservation condition

$$q^\mu\langle V'(k')|j_\mu^{(\mathrm{EM})}(0)|V(k)\rangle=0 \tag{1.6}$$

in Eq. (1.1). (ii) For neutral V and V', $\Gamma_{\alpha\alpha'\mu}$ is well defined in the limit $q\to 0$. This property does not hold for V and V' charged. (iii) Ultimately, for the calculation of an amplitude for a physical process, the indices α',α of Γ will be contracted with the polarization vectors $\epsilon'^{*\alpha'}\epsilon^\alpha$ of V' and V, or with fermion currents, and similarly for the photon index μ.

II. PARAMETRIZATION OF THE VERTEX AND ITS PHYSICAL INTERPRETATION

Let us consider for the moment the amplitude for processes in which the vector particles are on-shell and/or the fermion currents to which they couple are conserved. From the transversality conditions

$$k'\cdot\epsilon'=k\cdot\epsilon=0, \tag{2.1}$$

we obtain

$$Q\cdot\epsilon=-q\cdot\epsilon,\quad Q\cdot\epsilon'=q\cdot\epsilon'. \tag{2.2}$$

Similar relations to Eq. (2.2) hold for the fermion currents if they are conserved, or in the limit in which the fermion masses can be neglected. Therefore the terms in $\Gamma_{\alpha\alpha'\mu}$ proportional to Q_α or $Q_{\alpha'}$ are not independent of similar terms with Q_α replaced by q_α and $Q_{\alpha'}$ replaced by $q_{\alpha'}$. In this case the most general form of $\Gamma_{\alpha\alpha'\mu}$ that is consistent with Lorentz invariance can be written in terms of 10 form factors as follows:

$$\begin{aligned}\Gamma^{(T)}_{\alpha\alpha'\mu}=&(a_1q_\mu+a_1'Q_\mu)g_{\alpha\alpha'}+(a_2q_\mu+a_2'Q_\mu)q_\alpha q_{\alpha'}\\&+a_3(g_{\mu\alpha'}q_\alpha-g_{\mu\alpha}q_{\alpha'})+a_4(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'})\\&+b_1\varepsilon_{\alpha\alpha'\mu\nu}q^\nu+b_1'\varepsilon_{\alpha\alpha'\mu\nu}Q^\nu+b_2q_\alpha[Qq]_{\mu\alpha'}\\&+b_3q_{\alpha'}[Qq]_{\mu\alpha}\end{aligned} \tag{2.3}$$

where we have defined

$$[Qq]_{\alpha\alpha'}\equiv\varepsilon_{\alpha\alpha'\beta\gamma}Q^\beta q^\gamma \tag{2.4}$$

for the sake of brevity. For future reference, we note here that our convention for $\varepsilon_{\alpha\alpha'\beta\gamma}$ is such that

$$\varepsilon^{0123}=+1. \tag{2.5}$$

In principle Eq. (2.3) can contain the terms

$$(h_1q_\mu+h_2Q_\mu)[Qq]_{\alpha\alpha'}, \tag{2.6}$$

but in fact these are redundant. To see this, recall the identity

$$\begin{aligned}g_{\lambda\mu}\varepsilon_{\alpha\alpha'\beta\gamma}-g_{\lambda\alpha}\varepsilon_{\mu\alpha'\beta\gamma}-g_{\lambda\alpha'}\varepsilon_{\alpha\mu\beta\gamma}-g_{\lambda\beta}\varepsilon_{\alpha\alpha'\mu\gamma}\\-g_{\lambda\gamma}\varepsilon_{\alpha\alpha'\beta\mu}=0\,,\end{aligned} \tag{2.7}$$

which follows from the fact that the left-hand side is a completely antisymmetric four-dimensional tensor in the indices $\mu\alpha\alpha'\beta\gamma$. Contracting Eq. (2.7) with $q^\lambda Q^\beta q^\gamma$, we obtain

$$\begin{aligned}q_\mu[Qq]_{\alpha\alpha'}=&-q^2\varepsilon_{\alpha\alpha'\mu\nu}Q^\nu+Q\cdot q\varepsilon_{\alpha\alpha'\mu\nu}q^\nu\\&-q_{\alpha'}[Qq]_{\mu\alpha}+q_\alpha[Qq]_{\mu\alpha'}.\end{aligned} \tag{2.8}$$

Similarly, by contracting Eq. (2.7) with $Q^\lambda Q^\beta q^\gamma$, we obtain

$$\begin{aligned}Q_\mu[Qq]_{\alpha\alpha'}=&Q^2\varepsilon_{\alpha\alpha'\mu\nu}q^\nu-Q\cdot q\varepsilon_{\alpha\alpha'\mu\nu}Q^\nu\\&-Q_{\alpha'}[Qq]_{\mu\alpha}+Q_\alpha[Qq]_{\mu\alpha'}.\end{aligned} \tag{2.9}$$

In the last two terms in this equation, $Q_{\alpha'}$ and Q_α can be replaced by $q_{\alpha'}$ and q_α due to Eq. (2.2), which finally justifies the omission of $h_{1,2}$ in Eq. (2.3).

On the other hand, for amplitudes in which the vector particles are not on-shell and the fermion currents to which they couple are not conserved, the simplification made by using Eq. (2.2) is not valid. In these cases, the general form of the vertex function contains in addition to the terms given in Eq. (2.3) the following:

$$\begin{aligned}\Gamma^{(L)}_{\alpha\alpha'\mu}=&(c_1q_\mu+c_1'Q_\mu)k_\alpha k'_{\alpha'}+(c_2q_\mu+c_2'Q_\mu)k_\alpha k_{\alpha'}\\&+(c_3q_\mu+c_3'Q_\mu)k'_\alpha k'_{\alpha'}+c_4g_{\mu\alpha'}k_\alpha+c_5g_{\mu\alpha}k'_{\alpha'}\\&+d_1k_\alpha[Qq]_{\mu\alpha'}+d_2k'_{\alpha'}[Qq]_{\mu\alpha}.\end{aligned} \tag{2.10}$$

In the most general case, the vertex function is the sum

$$\Gamma_{\alpha\alpha'\mu}=\Gamma^{(T)}_{\alpha\alpha'\mu}+\Gamma^{(L)}_{\alpha\alpha'\mu}. \tag{2.11}$$

For $V=V'$, the form factors in Eq. (2.3) have a definite physical interpretation in terms of the static electromagnetic properties of the particles. To deduce this correspondence we use the following procedure. Let us consider as an example the electric dipole moment d_E. The classical definition of the electric dipole moment of a charge distribution is

$$\vec{D}_E=\int d^3x\vec{x}\rho^{(\mathrm{EM})}(\vec{x}), \tag{2.12}$$

where we have denoted by $\rho^{(\mathrm{EM})}(\vec{x})$ the zeroth component of the electromagnetic current density $j^{(\mathrm{EM})}(\vec{x})$. In quantum mechanics $\vec{D}_E$ becomes an operator, and the electric dipole moment of the particle is given by its expectation value. For normalizable states, with $\langle ks'|ks\rangle=N_k\delta_{s,s'}$, we can define the electric dipole moment matrix elements as

$$\xi'^\dagger\vec{d}_E\xi=(\vec{d}_E)_{ss'}=\left.\frac{\langle ks'|\vec{D}_E|ks\rangle}{N_k}\right|_{\vec{k}\to 0}. \tag{2.13}$$

The limit $\vec{k}=0$ reflects the fact that the intrinsic static properties of the particle are determined in the limit of zero momentum or, equivalently, in the rest frame of the particle. In Eq. (2.13) the symbols ξ,ξ' stand for the space part of the polarization vectors

$$\epsilon^{\mu}(k,s)\equiv(\epsilon^0,\vec{\xi}),$$

$$\epsilon'^{\mu}(k',s')\equiv(\epsilon'^0,\vec{\xi}'),\qquad(2.14)$$

written as column matrices. In the limit $\vec{k}=0$, they are simply the spin wave functions of the particle at rest. However, since the plane wave states satisfy

$$\langle V(k's')|V(ks)\rangle=(2\pi)^3 2k^0\delta^{(3)}(\vec{k}-\vec{k}')\delta_{s,s'},\qquad(2.15)$$

the appropriate formula is

$$(\xi'^{\dagger}\vec{d}_E\xi)(2\pi)^3 2k^0\delta^{(3)}(\vec{k}-\vec{k}')=\langle V(k's')|\vec{D}_E|V(ks)\rangle|_{|\vec{k}'|=|\vec{k}|\to 0}.\qquad(2.16)$$

By translation invariance,

$$\langle V(k's')|\vec{D}_E|V(ks)\rangle=\int d^3x\vec{x}e^{i\vec{x}\cdot(\vec{k}-\vec{k}')}\langle V(k's')|\rho^{(\mathrm{EM})}(\vec{0})|V(ks)\rangle=(2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')\left[\frac{i\partial j^0(Q,q)}{\partial\vec{q}}\right],\qquad(2.17)$$

where, to arrive at the second equality, Eq. (1.1) has been used. Then, taking the limits indicated in Eq. (2.16),

$$(\xi'^{\dagger}\vec{d}_E\xi)=\frac{1}{2m_V}\left[\frac{i\partial J^0(\vec{q})}{\partial\vec{q}}\right]\Bigg|_{\vec{q}=0},\qquad(2.18)$$

where we have introduced

$$J_{\mu}(\vec{q})=j_{\mu}(Q,q)|_{q^0=0,\vec{Q}=0}=[\epsilon'^{*\alpha'}(k')\epsilon^{\alpha}(k)\Gamma_{\alpha\alpha'\mu}(k,k')]|_{q^0=0,\vec{Q}=0}.\qquad(2.19)$$

The expression for $\vec{d}_E$ can finally be obtained by substituting Eq. (2.3) into Eq. (2.18). The terms contained in $\Gamma^{(L)}_{\alpha\alpha'\mu}$ do not contribute because they vanish when the vertex function is multiplied by the polarization vectors, and it is important also to keep in mind that, in the limit $q^0=0$ and $\vec{Q}=0$,

$$\epsilon^0=\frac{\vec{q}\cdot\vec{\xi}}{2m_V},\quad \epsilon'^0=-\frac{\vec{q}\cdot\vec{\xi}'}{2m_V},\qquad(2.20)$$

which follow from Eq. (2.1). In this way we then obtain

$$\vec{d}_E=d_E\vec{S},\qquad(2.21)$$

where

$$d_E=\left(\frac{b_1(0)}{2m_V}\right),\qquad(2.22)$$

and we have introduced the spin-1 matrices $\vec{S}$ with elements

$$(S^i)_{jk}=-i\varepsilon^{ijk}.\qquad(2.23)$$

The notation $b_1(0)$ is meant to indicate that the form factor is to be evaluated by first putting k and k' on shell, and then taking the limit $q^0=0$, $\vec{Q}=0$ and $\vec{q}=0$. Since, in vacuum, the on-shell form factors are functions only of q^2, in order to evaluate $b_1(0)$ it is enough to put $q^2=0$. However, in more general situations, such as if the particle is propagating in a matter background, the form factors depend on the components of q^{μ} and Q^{μ} separately, and the static limit is meant to be as we have indicated, namely, $(q^0=0,\vec{q}\to 0)$ and $Q^{\mu}\to(2m_V,\vec{0})$. In what follows, we use the same notation for the other form factors.

We can proceed in similar fashion for the electric charge and the quadrupole moment. The formulas analagous to Eq. (2.16) are

$$(\xi'^{\dagger}q_V\xi)(2\pi)^3 2k^0\delta^{(3)}(\vec{k}-\vec{k}')=\langle V(k's')|q_E|V(ks)\rangle|_{|\vec{k}'|=|\vec{k}|\to 0},\qquad(2.24)$$

$$(\xi'^{\dagger}x_E^{ij}\xi)(2\pi)^3 2k^0\delta^{(3)}(\vec{k}-\vec{k}')=\langle V(k's')|X_E^{ij}|V(ks)\rangle|_{|\vec{k}'|=|\vec{k}|\to 0},\qquad(2.25)$$

where q_E is the charge operator

$$q_E=\int d^3x\rho^{(\mathrm{EM})}(\vec{x})\qquad(2.26)$$

and

$$X_E^{ij}=\int d^3x x^i x^j\rho^{(\mathrm{EM})}(\vec{x})\qquad(2.27)$$

are the elements of the quadrupole moment tensor operator. Following the steps leading to Eq. (2.18), we obtain, for q_V and x_E,

$$(\xi'^{\dagger}q_V\xi)=\frac{1}{2m_V}J^0(\vec{0}),\qquad(2.28)$$

$$(\xi'^{\dagger}x_E^{ij}\xi)=\frac{1}{2m_V}\left[\frac{i\partial}{\partial q^i}\frac{i\partial}{\partial q^j}J^0(\vec{q})\right]\Bigg|_{\vec{q}=0},\qquad(2.29)$$

from which we obtain

$$q_V=-a_1'(0),\qquad(2.30)$$

$$x_E^{ij}=\frac{1}{3}Q_E I^{ij}.\qquad(2.31)$$

In Eq. (2.31) the I^{ij} are a set of symmetric matrices with elements

$$(I^{ij})_{kl}=\delta^i_k\delta^j_l+\delta^j_k\delta^i_l\qquad(2.32)$$

and

$$Q_E=-3\left[a_2'(0)-\frac{a_1'(0)+2a_3(0)}{4m_V^2}\right].\qquad(2.33)$$

It is customary to define the quadrupole moment tensor as [4]

$$Q_E^{ij} \equiv 3x_E^{ij} - \delta_j^i \left(\sum_k x_E^{kk} \right) = Q_E \left(I^{ij} - \frac{2}{3} \delta_j^i \mathbf{1} \right), \quad (2.34)$$

where $\mathbf{1}$ stands for the 3×3 unit identity matrix.

At this point it is useful to consider the amplitude for the scattering of a slowly moving V particle in an external electromagnetic field. Introducing the potentials

$$\phi(\vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{x}} \phi(\vec{q}),$$

$$\vec{A}(\vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{x}} \vec{A}(\vec{q}), \quad (2.35)$$

the amplitude is

$$iM = -i[\phi(\vec{q}) J^0(\vec{q}) - \vec{A}(\vec{q}) \cdot \vec{J}(\vec{q})]. \quad (2.36)$$

If we expand $J^0(\vec{q})$ in powers of $\vec{q}$, the term in Eq. (2.36) that is independent of $\vec{q}$ is then identified with the charge, those proportional to $\vec{q}$ are identified with the dipole moments, while those proportional to two powers of $\vec{q}$ correspond to the quadrupole moments. More precisely, from Eqs. (2.18), (2.28), and (2.29) we deduce that

$$\phi(\vec{q}) J^0(\vec{q}) = \xi'^\dagger \left(q_V \phi - \vec{d}_E \cdot \vec{E} - \frac{1}{2} x_E^{ij} E^{ij} \right) \xi, \quad (2.37)$$

and, by analogy, we can determine the magnetic moments by writing the amplitude in the form

$$M = -2m_V \xi'^\dagger \left(q_V \phi - \vec{d}_E \cdot \vec{E} - \frac{1}{2} x_E^{ij} E^{ij} - \vec{d}_M \cdot \vec{B} - \frac{1}{2} x_M^{ij} B^{ij} \right) \xi. \quad (2.38)$$

Here we have introduced

$$\vec{E} = i\vec{q}\phi, \quad \vec{B} = -i\vec{q} \times \vec{A}, \quad (2.39)$$

which are the Fourier transforms of the external electric and magnetic field, and

$$E^{ij} = -iE^i q^j, \quad B^{ij} = -iB^i q^j \quad (2.40)$$

are the Fourier transforms of their space derivatives $\partial E^i(\vec{x})/\partial x^j$ and $\partial B^i(\vec{x})/\partial x^j$.

Thus, using Eq. (2.3) to evaluate the amplitude, and then comparing with the form given in Eq. (2.38), we obtain for the magnetic dipole and quadrupole moment matrices

$$\vec{d}_M = d_M \vec{S}, \quad (2.41)$$

$$x_M^{ij} = \frac{1}{3} Q_M I^{ij}, \quad (2.42)$$

where

$$d_M = -\left(\frac{a_3(0)}{2m_V} \right), \quad (2.43)$$

$$Q_M = -\frac{3}{2m_V} \left[\frac{b_1(0)}{m_V} - [b_2(0) + b_3(0)] \right]. \quad (2.44)$$

It is easy to verify that the above result for $\vec{d}_M$, for example, is the same as that obtained from the formula

$$\xi'^\dagger d_M^i \xi = \frac{1}{2m_V} \varepsilon^{ijk} \left[\frac{i\partial J^k(\vec{q})}{\partial q^j} \right] \Bigg|_{\vec{q}=0}, \quad (2.45)$$

which follows by using the definition

$$\xi'^\dagger \vec{d}_M \xi (2\pi)^3 2k^0 \delta^{(3)}(\vec{k} - \vec{k}') = \langle V(k's') | \vec{D}_M | V(ks) \rangle |_{|\vec{k}'| = |\vec{k}| \to 0}, \quad (2.46)$$

in terms of the magnetic dipole moment operator

$$\vec{D}_M = \frac{1}{2} \int d^3x \vec{x} \times \vec{j}^{(\mathrm{EM})}(\vec{x}). \quad (2.47)$$

We must mention that when $\vec{J}(\vec{q})$ is expanded in powers of $\vec{q}$ in Eq. (2.36), not all terms that are generated can be written in the form of Eq. (2.38). The amplitude contains additional terms such as, for example,

$$a_4 \xi^{*\prime j} \xi^i (A^i q^j + A^j q^i), \quad (2.48)$$

which do not fit in the form of Eq. (2.38). However, all such terms in fact vanish once the constraint of gauge invariance, which in the static limit is simply

$$\vec{q} \cdot \vec{J}(\vec{q}) = 0, \quad (2.49)$$

is imposed upon them. While we do not discuss this any further here, the implications of gauge invariance are considered in detail in Sec. IV.

III. IMPLICATIONS OF THE DISCRETE SYMMETRIES

In this section we deduce the constraints on the vertex function, and in turn on the form factors, that are imposed by several general requirements and/or symmetry principles. These are crossing symmetry, the Hermiticity of the Langrangian and, when applicable, the discrete space symmetries C, P, and T, or the appropriate combinations of them. We consider the diagonal and off-diagonal cases separately.

A. Diagonal case: $V = V'$

Let us consider the space-time symmetries first. Consider, for example, the parity symmetry. The effect of its transformation can be summarized by the statement

$$\Gamma_{\alpha\alpha'\mu}(k,k') \stackrel{P}{\rightarrow} \Gamma^P_{\alpha\alpha'\mu}(k,k'), \quad (3.1)$$

where $\Gamma^P_{\alpha\alpha'\mu}$ is obtained from $\Gamma_{\alpha\alpha'\mu}$ by multiplying every quantity that appears in $\Gamma_{\alpha\alpha'\mu}$ by its parity phase η_P tabulated in Table I. Using similar notation for the effect of the charge conjugation and time reversal transformations, we obtain

TABLE I. The transformation rules of different quantities appearing in $\Gamma_{\alpha\alpha'\mu}$ under the various discrete symmetries.

	η_P	η_T	η_C	η_{CP}	η_{CPT}
1	+	+	+	+	+
i	+	−	+	+	−
$\epsilon_{\alpha\beta\lambda\rho}$	−	−	+	−	+

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{C}{\rightarrow}-\Gamma^C_{\alpha'\alpha\mu}(-k',-k), \tag{3.2}$$

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{T}{\rightarrow}-\Gamma^T_{\alpha'\alpha\mu}(-k,-k'). \tag{3.3}$$

In general, the C transformation and the crossing relation, each one separately, give a relation that expresses the amplitude for the antiparticle process $(\bar{V}\rightarrow\bar{V}'+\gamma)$ in terms of $\Gamma_{\alpha\alpha'\mu}$. Strictly speaking, the C transformation rule written above is the combined effect of the charge-conjugation transformation and the crossing relation. As discussed below, in the case that V is self-conjugate the crossing relation gives by itself another independent constraint.

From these transformations we can deduce the effects of combined operations, for example,

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{CP}{\rightarrow}-\Gamma^{CP}_{\alpha'\alpha\mu}(-k',-k), \tag{3.4}$$

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{CPT}{\rightarrow}\Gamma^{CPT}_{\alpha'\alpha\mu}(k',k). \tag{3.5}$$

If the Lagrangian is symmetric under any of the discrete symmetry transformations, then the arrow in the corresponding relation in Eqs. (3.1)–(3.5) should be replaced by the equality sign. The resulting equations produce constraints on the form factors. For example, if CP is a symmetry of the Lagrangian, then from the above it follows that the vertex function satisfies

$$\Gamma_{\alpha\alpha'\mu}(k,k')=-\Gamma^{CP}_{\alpha'\alpha\mu}(-k',-k), \tag{3.6}$$

which implies that

$$a_1=a_2=a_4=0 \tag{3.7}$$

and

$$b_1=0,\quad b_3=-b_2. \tag{3.8}$$

From Eqs. (2.22) and (2.44), the relations in Eq. (3.8) imply that the electric dipole moment and the magnetic quadrupole moments are zero, which is a familiar result.

On top of all these relations, which may or may not be satisfied in a particular case, the following relation follows from the fact that the Lagrangian is Hermitian:

$$\Gamma_{\alpha\alpha'\mu}(k,k')=\Gamma^{*}_{\alpha'\alpha\mu}(k',k). \tag{3.9}$$

This relation, which is independent of whether the particle is neutral or charged, or of the status of the discrete symmetries, yields the following reality conditions on the form factors:

$$a_1',a_2',a_3,b_1=\text{ real},$$

$$a_1,a_2,a_4,b_1'=\text{ imaginary},$$

$$b_3=b_2^{*}, \tag{3.10}$$

which in particular imply that the electromagnetic moments, as we have identified them, are real. Notice also that once the Hermiticity condition is imposed, the CPT transformation in Eq. (3.5) becomes a trivial identity, as it should be.

The above discussion applies to both the charged and neutral cases. However, for the neutral case there is an additional independent constraint if the particle is not just neutral but also self-conjugate. If the particle is self-conjugate, then the vertex obeys the crossing relation corresponding to the exchange of the external V lines. The resulting condition is simply

$$\Gamma_{\alpha\alpha'\mu}(k,k')=\Gamma_{\alpha'\alpha\mu}(-k',-k). \tag{3.11}$$

Since all the form factors are now functions of q^2 only, they remain invariant under the substitution $k\leftrightarrow -k'$. Equation (3.11) applied to Eq. (2.3) implies that the form factors a_1',a_2',a_3,b_1 all vanish, while $b_3=-b_2$. Thus the symmetry reduces $\Gamma^{(T)}_{\alpha\alpha'\mu}$ to the form

$$\Gamma_{\alpha\alpha'\mu}=a_1q_\mu g_{\alpha\alpha'}+a_2q_\mu q_\alpha q_{\alpha'}+a_4(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'})+b_1'\epsilon_{\alpha\alpha'\mu\nu}Q^\nu+b_2(q_\alpha[Qq]_{\mu\alpha'}-q_{\alpha'}[Qq]_{\mu\alpha}). \tag{3.12}$$

Notice that this result implies that the charge form factor as well as the dipole moment form factors are zero for all q^2 and not just for $q^2=0$. Moreover, it implies in particular that a neutral (self-conjugate) vector particle can have neither an electric nor a magnetic dipole moment, in complete analogy with the result for Majorana fermions [5]. Furthermore, since $a_1'=a_2'=a_3=0$ the electric quadrupole moment is zero, and since $b_1=0$ and $b_3=-b_2$ the magnetic quadrupole moment is also zero. In short, a self-conjugate vector particle cannot have any static electromagnetic moments. Nevertheless, it is interesting to note that even self-conjugate particles can have an electromagnetic vertex. However, as will be shown in Sec. IV, gauge invariance places further restrictions on the other form factors a_1, a_2, a_4, b_1', and b_2, and in the end only b_2 and a linear combination of a_2 and a_4 can be nonzero.

In the standard model, the Z falls in this class; it is self-conjugate (which implies its charge neutrality). But in principle, there can exist vector particles that are electrically neutral, but which have a nonzero quantum number with respect to the charge of some other (global or local) symmetry which is at the moment unknown to us. For such a particle, which is neutral but not self-conjugate, the constraint of Eq. (3.11) does not apply. The distinction between the neutral and self-conjugate vector particles is analogous to the one between Dirac and Majorana neutrinos. For particles that are not self-conjugate, the crossing symmetry relations do not give us constraints on the form factors, but instead allow us to relate the form factors of the antiparticle (the conjugate particle) to those of the particle.

One interesting case that appears as a special one is the off-shell coupling of three photons. If C invariance holds, we can easily see that the vertex vanishes. This follows from Eqs. (3.2) and (3.11). From Table I, we see that η_C is $+1$ for all quantities of interest, so the quantity Γ^C appearing in Eq. (3.2) is really equal to Γ. If C invariance is valid, the two expressions in Eq. (3.2) should be equal, which directly contradicts Eq. (3.11) unless the entire vertex vanishes. This result is known as Furry's theorem [6]. However, note that this argument shows that in order that the vertex vanishes, V and V' need not be photons; C invariance and $V=V'$ are sufficient conditions. In other words, the off-shell vertex $VV\gamma$ vanishes for any self-conjugate V provided C invariance holds.

On the other hand, since we know that C invariance does not hold when the weak interactions are taken into account, the $VV\gamma$ vertex need not vanish in general. This will be discussed after introducing the constraints from gauge invariance in Sec. IV.

B. Off-diagonal case: $V=V'$

In analogy with Eq. (1.1) we define the vertex function for the process

$$V'(k')\to V(k)+\gamma(q) \tag{3.13}$$

by writing

$$\langle V(k)|j_\mu^{\rm EM}(0)|V'(k')\rangle=\epsilon'^{\alpha'}(k')\epsilon^{*\alpha}(k)\Gamma^{(V'\to V)}_{\alpha'\alpha\mu}(k',k). \tag{3.14}$$

Using a notation similar to before, the effect of the various transformations on the vertex function can be summarized by the following rules. For P and T we have

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{P}{\to}\delta_P'^*\delta_P\Gamma^P_{\alpha\alpha'\mu}(k,k'), \tag{3.15}$$

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{T}{\to}-\delta_T'^*\delta_T\Gamma^T_{\alpha'\alpha\mu}(-k,-k'), \tag{3.16}$$

where $\delta_{P,T}$ and $\delta'_{P,T}$ are the phases that appear in the P,T transformation rules of the V,V' fields. Under charge conjugation,

$$\Gamma_{\alpha\alpha'\mu}(k,k')\overset{C}{\to}-\delta'_C\delta^*_C\Gamma^{(V'\to V)}_{\alpha'\alpha\mu}(-k',-k), \tag{3.17}$$

while the Hermiticity condition becomes the statement that

$$\Gamma^{(V'\to V)}_{\alpha'\alpha\mu}(k',k)=\Gamma^*_{\alpha\alpha'\mu}(k,k'). \tag{3.18}$$

The comment made after Eq. (3.3) applies here also. In particular, for the case of self-conjugate particles the crossing relation gives the additional condition

$$\Gamma^{(V'\to V)}_{\alpha'\alpha\mu}(k',k)=\Gamma_{\alpha\alpha'\mu}(-k,-k'). \tag{3.19}$$

Notice that taking $V=V'$ as a particular case in Eq. (3.19), the condition expressed in Eq. (3.11) is reproduced.

As an example of the kind of relation that we can deduce from these results, suppose that CP is conserved. From Eqs. (3.15) and (3.17) it then follows that

$$\Gamma^{(V'\to V)}_{\alpha'\alpha\mu}(k',k)=-\delta'^*_{CP}\delta_{CP}\Gamma^P_{\alpha\alpha'\mu}(-k,-k'). \tag{3.20}$$

Comparing this with Eq. (3.18) we then obtain the condition

$$\Gamma^*_{\alpha\alpha'\mu}(k,k')=-\delta'^*_{CP}\delta_{CP}\Gamma^P_{\alpha\alpha'\mu}(-k,-k'), \tag{3.21}$$

which implies that $a_{1,2,3,4}$ and $a'_{1,2}$ have the same phase $e^{i\phi}$, while $b_{1,2,3}$ and b'_1 all have the same phase $ie^{i\phi}$, with the same ϕ. It is also easy to verify that the results that were derived previously for the diagonal case are reproduced here if we specialize these formulas to that case by setting $V'=V$.

The case of self-conjugate particles is more interesting. As already commented, the new feature is that Eq. (3.19) is valid independently of whether the discrete symmetries are conserved or not. That condition together with the Hermiticity condition of Eq. (3.18) give

$$\Gamma_{\alpha\alpha'\mu}(k,k')=\Gamma^*_{\alpha\alpha'\mu}(-k,-k'), \tag{3.22}$$

which implies that all the form factors are purely imaginary. Now, if CP is conserved, we can combine this with what we concluded after Eq. (3.21). There are two possibilities depending on the relative sign of the CP phases of V and V', which we denoted by δ_{CP} and δ'_{CP} in Eq. (3.20).

Same CP phase of V' and V. In this case we have

$$a_{1,2,3,4}=a'_{1,2}=0\ . \tag{3.23}$$

Only the b coefficients survive (and they are purely imaginary), so that the transition moments d_M,Q_E are zero while d_E,Q_M are nonzero.

Oppposite CP phases of V' and V. In this case the opposite occurs: the b coefficients are zero while the a coefficients are nonzero and purely imaginary. Accordingly, the transition moments d_E,Q_M are zero while d_M,Q_E are allowed. This is, once more, in complete analogy with the situation for Majorana fermions [5].

The discussion above immediately brings the question of whether the relative CP phase of all self-conjugate vector bosons is positive. For the photon and the Z it is true, but can one construct examples where it is not? The following examples illustrate the various possibilities that can arise.

Suppose that V and V' have the following couplings to a pair of neutrinos, $\nu_{Le,\mu}$:

$$\begin{aligned}L'=aV'^\mu[\overline{\nu_{Le}}\gamma_\mu\nu_{L\mu}+\overline{\nu_{L\mu}}\gamma_\mu\nu_{Le}]\\+ibV^\mu[\overline{\nu_{Le}}\gamma_\mu\nu_{L\mu}-\overline{\nu_{L\mu}}\gamma_\mu\nu_{Le}].\end{aligned} \tag{3.24}$$

It is clear from this that the CP phase of V and V' must be opposite in order to keep L' invariant under CP. A generalization of this interaction is to promote it to an SU(2) gauge interaction by writing

$$L'=-g\,\overline{\eta_L}\gamma_\mu\frac{\vec\tau}{2}\eta_L\cdot\vec V^\mu, \tag{3.25}$$

where η_L stands for a doublet formed by $\nu_{Le,\mu}$. In this model, under CP,

$$V^{1,3}\to-V^{1,3}, \tag{3.26}$$

just like the Z and the photon, but

$$V^2 \to V^2. \tag{3.27}$$

Interactions of a similar type generally appear in models that have been considered in various contexts, such as grand unified theories, family or horizontal symmetries, and superstring inspired models.

In the examples considered in Eqs. (3.24) and (3.25), the vector bosons are self-conjugate. On the other hand, if the theory is such that the individual lepton numbers L_e and L_μ are separately conserved, then it is more convenient to work with

$$V_\mu \equiv \frac{1}{\sqrt{2}}(V_\mu^1 + iV_\mu^2) \tag{3.28}$$

and its complex conjugate, because they are the eigenstates of the conserved $L_{e,\mu}$ operators. The couplings of V to the neutrinos is of the form $V_\mu \overline{\nu_{Le}} \gamma^\mu \nu_{L\mu} + \text{H.c.}$, and in such a theory V would carry both L_e and L_μ quantum numbers. While electrically neutral, V is not self-conjugate. If there is another such vector particle V', one of them can decay radiatively into the other.

We would like to point out that, in the examples given in this section, we have focused the attention on the relations for the a and b coefficients, which are contained in $\Gamma^{(T)}_{\alpha\alpha'\mu}$. We have chosen that only for illustrative purposes, motivated by the fact that those are the form factors that contribute when the vector bosons are on-shell, and therefore the ones that have a direct physical interpretation in terms of the electromagnetic moments of the particles. However, analogous relations can be similarly derived for the c and d coefficients contained in $\Gamma^{(L)}_{\alpha\alpha'\mu}$, by applying the conditions that the vertex function must satisfy, such as Eqs. (3.6) or (3.9), to Eq. (2.10).

IV. GAUGE INVARIANCE

Now we explore what are the constraints imposed by the electromagnetic gauge invariance on the vertex function. The analysis depends on whether the two gauge bosons V and V' are charged or neutral; therefore we consider them separately.

A. V, V' are neutral

The condition is expressed in Eq. (1.4), and applying it to Eq. (2.11) we get the relations

$$\begin{gathered}
a_1 q^2 + a_1' Q\cdot q = 0,\\
a_2 q^2 + a_2' Q\cdot q + 2a_4 = 0,\\
b_1' = 0,\\
c_1 q^2 + c_1' Q\cdot q - c_4 + c_5 = 0,\\
c_2 q^2 + c_2' Q\cdot q + c_4 = 0,\\
c_3 q^2 + c_3' Q\cdot q - c_5 = 0\,.
\end{gathered} \tag{4.1}$$

The nontrivial ones of these relations are solved, without introducing artificial singularities, by writing

$$\begin{gathered}
a_1' = q^2 a_0,\\
a_1 = -Q\cdot q a_0,\\
a_4 = -\frac{1}{2}(q^2 a_2 + Q\cdot q a_2'),\\
c_4 = -c_2 q^2 - c_2' Q\cdot q,\\
c_5 = c_3 q^2 + c_3' Q\cdot q,\\
c_1 + c_2 + c_3 = -c_0 Q\cdot q,\\
c_1' + c_2' + c_3' = c_0 q^2.
\end{gathered} \tag{4.2}$$

Therefore,

$$\begin{aligned}
\Gamma^{(T)}_{\alpha\alpha'\mu} = {} & (q^2 Q_\mu - Q\cdot q q_\mu) a_0 g_{\alpha\alpha'} + a_2\left(q_\mu q_\alpha q_{\alpha'} - \frac{1}{2}q^2(g_{\mu\alpha'}q_\alpha + g_{\mu\alpha}q_{\alpha'})\right) + a_2'\left(Q_\mu q_\alpha q_{\alpha'} - \frac{1}{2}Q\cdot q(g_{\mu\alpha'}q_\alpha + g_{\mu\alpha}q_{\alpha'})\right)\\
& + a_3(g_{\mu\alpha'}q_\alpha - g_{\mu\alpha}q_{\alpha'}) + b_1 \epsilon_{\alpha\alpha'\mu\nu}q^\nu + b_2 q_\alpha [Qq]_{\mu\alpha'} + b_3 q_{\alpha'}[Qq]_{\mu\alpha}
\end{aligned} \tag{4.3}$$

and

$$\begin{aligned}
\Gamma^{(L)}_{\alpha\alpha'\mu} = {} & c_0(q^2 Q_\mu - Q\cdot q q_\mu)k_\alpha k'_{\alpha'} + c_2 k_\alpha(q_\mu q_{\alpha'} - q^2 g_{\mu\alpha'}) + c_2' k_\alpha(Q_\mu q_{\alpha'} - Q\cdot q g_{\mu\alpha'}) - c_3 k'_{\alpha'}(q_\mu q_\alpha - q^2 g_{\mu\alpha})\\
& - c_3' k'_{\alpha'}(Q_\mu q_\alpha - Q\cdot q g_{\mu\alpha}) + d_1 k_\alpha [Qq]_{\mu\alpha'} + d_2 k'_{\alpha'}[Qq]_{\mu\alpha}\,.
\end{aligned} \tag{4.4}$$

In the most general case, the vertex cannot be simplified further. Simplification occurs in special cases as we discuss below.

1. $V \neq V'$, all particles on-shell

Physically, this represents a decay process where one vector boson decays radiatively to another one. In this case we can reduce the vertex of Eq. (4.3) further by using the on-shell conditions for the photon,

$$\epsilon^{(\gamma)}\cdot q=0, \quad q^2=0. \tag{4.5}$$

This gives the form

$$\Gamma^{(T)}_{\alpha\alpha'\mu}=a_2'\left[Q_\mu q_\alpha q_{\alpha'}-\frac{1}{2}Q\cdot q(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'})\right] +a_3(g_{\mu\alpha'}q_\alpha-g_{\mu\alpha}q_{\alpha'})+b_1\epsilon_{\alpha\alpha'\mu\nu}q^\nu +b_2q_\alpha[Qq]_{\mu\alpha'}+b_3q_{\alpha'}[Qq]_{\mu\alpha}, \tag{4.6}$$

while $\Gamma^{(L)}$ does not contribute. Notice that Eq. (4.6) involves a combination of dipole and quadrupole moment terms only and, in particular, the term with the charge form factor a_1' vanishes in this case. The reason behind this is that in this configuration (all particles on-shell) the a_1' form factor is the matrix element of the charge operator between the V and V' states, which is zero if the particles are different.

It is interesting to consider what occurs in this case when the particle V' also is the photon, which corresponds to the possibility of a vector boson decaying into two photons. In this case, the vertex function should satisfy the additional constraint

$$\Gamma_{\alpha\alpha'\mu}(k,k')=\Gamma_{\alpha\mu\alpha'}(k,q), \tag{4.7}$$

which corresponds to the condition that the two photons in the final state obey Bose symmetry. In addition, gauge invariance of the second photon requires

$$k'^{\alpha'}\Gamma_{\alpha\alpha'\mu}=0, \tag{4.8}$$

but this is satisfied automatically if Eq. (4.7) is imposed upon Eq. (4.6). Equation (4.7) translates to

$$\Gamma_{\alpha\alpha'\mu}=\Gamma_{\alpha\mu\alpha'}|_{Q\to\tilde{Q},q\to\tilde{q}}, \tag{4.9}$$

where

$$\tilde{Q}=k+q=(Q+3q)/2, \tag{4.10}$$

$$\tilde{q}=k-q=(Q-q)/2. \tag{4.11}$$

After substituting Eq. (4.6) into Eq. (4.9), the right-hand side of Eq. (4.9) can be reduced with the help of Eq. (2.9) and the relations in Eqs. (2.2) plus additional relations such as

$$[\tilde{Q}\tilde{q}]_{\mu\nu}=-[Qq]_{\mu\nu}. \tag{4.12}$$

It then follows easily from Eq. (4.9) that the vertex in fact vanishes. In other words, a spin-1 particle cannot decay to two photons. This is the well-known Yang theorem [7].

2. $V=V'$, both on shell

In this case the photon is not on-shell ($q^2\neq 0$) but, since both V lines are on shell, $Q\cdot q=k^2-k'^2=0$. The vertex function then reduces to

$$\Gamma_{\alpha\alpha'\mu}=q^2Q_\mu a_0 g_{\alpha\alpha'}+a_2\left(q_\mu q_\alpha q_{\alpha'}-\frac{1}{2}q^2(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'})\right)+a_2'Q_\mu q_\alpha q_{\alpha'}+a_3(g_{\mu\alpha'}q_\alpha-g_{\mu\alpha}q_{\alpha'}) +b_1\epsilon_{\alpha\alpha'\mu\nu}q^\nu+b_2q_\alpha[Qq]_{\mu\alpha'}+b_3q_{\alpha'}[Qq]_{\mu\alpha}. \tag{4.13}$$

As already discussed in Sec. III, if the particle is self-conjugate then the vertex obeys the crossing relation corresponding to the exchange of the external V lines. In this case, Eq. (3.11) applied to Eq. (4.13) implies that the form factors a_0, a_2', a_3, b_1 all vanish, while $b_3=-b_2$, using the fact that in this case all form factors must be functions of q^2 only. Thus Eq. (4.13) reduces to the form

$$\Gamma_{\alpha\alpha'\mu}=a_2\left\{q_\mu q_\alpha q_{\alpha'}-\frac{1}{2}q^2(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'})\right\} +b_2(q_\alpha[Qq]_{\mu\alpha'}-q_{\alpha'}[Qq]_{\mu\alpha}). \tag{4.14}$$

Therefore, while a self-conjugate particle cannot have any static electromagnetic moments, in the most general case it can have an electromagnetic vertex characterized by a_2 and b_2 as above.

3. $V=V'$, all particles off-shell

This corresponds, for example, to the off-shell $ZZ\gamma$ vertex in the standard model or beyond. We should now use the vertex given in Eqs. (4.3) and (4.4), remembering that the crossing symmetry of Eq. (3.11) applies here provided the particle V is self-conjugate. The difference with the previous case is that now the form factors are functions of the Lorentz invariants k^2, k'^2, and q^2, so the form factors a_0, a_2', a_3, b_1 do not vanish, but rather satisfy the constraints

$$a_0(k^2,k'^2,q^2)=-a_0(k'^2,k^2,q^2) \tag{4.15}$$

and similarly for the other three. Similarly, the condition $b_3=-b_2$ obtained for the previous case should be replaced in this case by

$$b_2(k^2,k'^2,q^2)=-b_3(k'^2,k^2,q^2). \tag{4.16}$$

Moreover, the terms in $\Gamma^{(L)}$ also should be present in this case, and the form factors in this part should satisfy the relations

$$c_0(k^2,k'^2,q^2)=-c_0(k'^2,k^2,q^2),$$
$$c_2(k^2,k'^2,q^2)=-c_3(k'^2,k^2,q^2),$$
$$c_2'(k^2,k'^2,q^2)=c_3'(k'^2,k^2,q^2),$$
$$d_1(k^2,k'^2,q^2)=d_2(k'^2,k^2,q^2). \tag{4.17}$$

4. $V'=\gamma$, both photons on-shell

When V' corresponds also to the photon, the vertex function, given by the sum of Eqs. (4.3) and (4.4), satisfies the additional constraint

$$k'^{\alpha'}\Gamma_{\alpha\alpha'\mu}=0. \tag{4.18}$$

Moreover, since two of the bosons are photons, the vertex must satisfy the symmetry condition given in Eq. (4.7). Let us consider Eq. (4.18) first. The contraction on the left-hand side produces six different kinds of tensor structures. Accordingly, we get six equations, but it turns out that only four of these are independent. Out of those four, one connects the b- and the d-type coefficients by

$$\frac{1}{2}b_1+b_3k'\cdot q+d_2k'^2=0, \tag{4.19}$$

which can be trivially solved for b_1. The other three can be written as

$$(c_3+c_3')k'^2=(2a_0+a_2+a_2')k'\cdot q,$$

$$(c_3-c_3')k'^2=2k\cdot qa_0+\frac{1}{2}(k^2-k'^2)a_2+\frac{1}{2}q^2a_2'-a_3,$$

$$c_2+c_2'=2a_0+2c_0k'^2. \tag{4.20}$$

These can be solved, without introducing artificial singularities, by writing

$$c_2=a_0+c_0k'^2+C_2,\quad c_2'=a_0+c_0k'^2-C_2,$$

$$c_3=k'\cdot qA_3+C_3,\quad c_3'=k'\cdot qA_3-C_3,$$

$$2a_0+a_2+a_2'=2k'^2A_3,$$

$$a_3=2k\cdot qa_0+\frac{1}{2}(k^2-k'^2)a_2+\frac{1}{2}q^2a_2'-2C_3k'^2, \tag{4.21}$$

so that a_0, a_2, c_0, C_2, C_3, and A_3 can be taken as the independent form factors in the a-c sector. Using these form factors we can then write

$$\begin{aligned}\Gamma_{\alpha\alpha'\mu}=&2a_0\{g_{\alpha\alpha'}(k\cdot qk_\mu'-k'\cdot qk_\mu)+q_{\alpha'}(k_\mu k_\alpha-Q_\mu q_\alpha)+g_{\mu\alpha'}(Q\cdot qq_\alpha-k\cdot qk_\alpha)\}+2(a_2q_\alpha+C_2k_\alpha)(g_{\mu\alpha'}k'\cdot q-k_\mu'q_{\alpha'})\\&+2A_3\{k\cdot qg_{\mu\alpha}(k'\cdot qk_{\alpha'}'-k'^2q_{\alpha'})-k'\cdot qk'^2g_{\mu\alpha'}q_\alpha+q_\alpha(k'^2Q_\mu q_{\alpha'}-k'\cdot qk_\mu k_{\alpha'}')\}\\&+c_0k_\alpha\{k_{\alpha'}'(q^2Q_\mu-Q\cdot qq_\mu)+2k'^2(k_\mu q_{\alpha'}-k\cdot qg_{\mu\alpha'})\}+2C_3\{k_{\alpha'}'(k_\mu'q_\alpha-k'\cdot qg_{\mu\alpha})-k'^2(g_{\mu\alpha'}q_\alpha-g_{\mu\alpha}q_{\alpha'})\}\\&+b_2q_\alpha[Qq]_{\mu\alpha'}+b_3\{q_{\alpha'}[Qq]_{\mu\alpha}-2k'\cdot q\epsilon_{\alpha\alpha'\mu\nu}q^\nu\}+d_1k_\alpha[Qq]_{\mu\alpha'}+d_2\{k_{\alpha'}'[Qq]_{\mu\alpha}-2k'^2\epsilon_{\alpha\alpha'\mu\nu}q^\nu\}.\end{aligned} \tag{4.22}$$

Up to this point we have not used the fact that the two particles in the final state are both photons. Therefore, the above expression is appropriate even in the case that V' is not the photon, provided that it couples in the Lagrangian to a conserved current. On the other hand, if V' is indeed the photon also, then the condition in Eq. (4.7) applies. In the case that we are considering, in which both photons are on-shell, this condition implies that all form factors, except C_2 and d_1, vanish. Thus the general form of the vertex function in this case is

$$\Gamma_{\alpha\alpha'\mu}=2C_2k_\alpha(g_{\mu\alpha'}k'\cdot q-k_\mu'q_{\alpha'})+2d_1k_\alpha[k'q]_{\mu\alpha'}. \tag{4.23}$$

Notice that if the particle V is on-shell, the vertex vanishes upon contracting with the polarization vector of the particle V, reproducing once more Yang's theorem. On the other hand, for an off-shell V, the amplitude is not zero as long as the V line in the corresponding Feynman diagram is not attached to a conserved current. This is the situation if, for example, V is attached to a neutrino current $\overline{\nu_L'}\gamma_\mu\nu_L$ and at least one of the neutrinos has a nonzero mass. Carrying this argument a little further, it explains why the amplitude for the process $\nu'\rightarrow\nu\gamma\gamma$ is proportional to the neutrino masses in the local limit of the W-boson propagator (leading order in M_W^2), which is a well-known general result [8] and is also corroborated by explicit calculations [9].

5. Off-shell V, V' which couple to conserved currents

If both V and V' couple to conserve currents like the photon, then the vertex function, given by the sum of Eqs. (4.3) and (4.4), satisfies the condition

$$k^\alpha\Gamma_{\alpha\alpha'\mu}=0 \tag{4.24}$$

in addition to the condition given in Eq. (4.18). This now gives some extra constraints on the form factors. For the b-type and d-type coefficients, one obtains

$$-\frac{1}{2}b_1+b_2k\cdot q+d_1k^2=0 \tag{4.25}$$

in addition to Eq. (4.19). These two equations imply that, in the b-d sector, there are three independent form factors. To express the form factors appearing in these equations in terms of three independent ones without introducing any artificial kinematic singularity, we first add the two equations to eliminate b_1, which gives

$$k^2(d_1+b_2)+k'^2(d_2-b_3)+k\cdot k'(b_3-b_2)=0. \tag{4.26}$$

This equation can be written as $\mathbf{B}\cdot\mathbf{K}=\mathbf{0}$, where

$$\mathbf{B}\equiv\begin{pmatrix} d_1+b_2 \\ d_2-b_3 \\ b_3-b_2 \end{pmatrix},\quad \mathbf{K}\equiv\begin{pmatrix} k^2 \\ k'^2 \\ k\cdot k' \end{pmatrix}. \tag{4.27}$$

We can then try to find two column matrices $\mathbf{K_1}$ and $\mathbf{K_2}$ which are both orthogonal to $\mathbf{K}$, and then the most general form for $\mathbf{B}$ would be $B_1\mathbf{K}_1+B_2\mathbf{K}_2$, in terms of two new form factors B_1 and B_2. Choosing

$$\mathbf{K}_2\equiv\begin{pmatrix} -k'^2 \\ k^2 \\ 0 \end{pmatrix},\quad \mathbf{K}_1\equiv 2\begin{pmatrix} k^2 k\cdot k' \\ k'^2 k\cdot k' \\ -(k^4+k'^4) \end{pmatrix}, \tag{4.28}$$

we thus get[1]

$$\begin{aligned} d_1+b_2&=2k^2k\cdot k'B_1-k'^2B_2,\\ d_2-b_3&=2k'^2k\cdot k'B_1+k^2B_2,\\ b_3-b_2&=-2(k^4+k'^4)B_1. \end{aligned} \tag{4.29}$$

The other form factor can be chosen by defining

$$b_3+b_2\equiv 2B_0. \tag{4.30}$$

Together with Eqs. (4.19) and (4.25), these definitions give

$$\begin{aligned} b_2&=B_0+(k^4+k'^4)B_1,\quad b_3=B_0-(k^4+k'^4)B_1,\\ d_1&=-B_0+(2k^2k\cdot k'-k^4-k'^4)B_1-k'^2B_2,\\ d_2&=B_0+(2k^2k\cdot k'-k^4-k'^4)B_1+k^2B_2,\\ b_1&=-2k\cdot k'B_0+2k\cdot k'(k^4-k'^4)B_1-k^2k'^2B_2. \end{aligned} \tag{4.31}$$

For the a- and c-type coefficients, in addition to the relations given in Eq. (4.20), we get two more independent conditions, which are

$$\begin{aligned} (c_2-c_2')k^2&=(2a_0-a_2+a_2')k\cdot q,\\ c_3-c_3'&=-2a_0-2c_0k^2. \end{aligned} \tag{4.32}$$

Thus, for example, the form factor C_3 introduced in Eq. (4.21) is identified in terms of a_0 and c_0 here, and similarly we can eliminate C_2 by introducing a new form factor A_2 through the relations

$$c_2-c_2'=2k\cdot qA_2,\quad 2a_0-a_2+a_2'=2k^2A_2. \tag{4.33}$$

So finally four form factors remain independent, which can be taken as a_0, c_0, A_2, and A_3. The other ones, in terms of these four independent ones, are identified by these relations:

$$\begin{aligned} a_0&=-A_0,\quad c_0=A_1,\\ a_2&=k'^2A_3-k^2A_2,\quad a_2'=2A_0+k'^2A_3+k^2A_2,\\ a_3&=2k^2k'^2A_1-(k^2+k'^2)A_0+k'^2k\cdot qA_3-k^2k'\cdot qA_2,\\ c_2&=-A_0+k\cdot qA_2+k'^2A_1,\\ c_2'&=-A_0-k\cdot qA_2+k'^2A_1,\\ c_3&=A_0+k'\cdot qA_3-k^2A_1,\\ c_3'&=-A_0+k'\cdot qA_3+k^2A_1. \end{aligned} \tag{4.34}$$

Putting these into Eqs. (4.3) and (4.4), we thus obtain the most general form of the vertex for this case:

$$\begin{aligned} \Gamma_{\alpha\alpha'\mu}=&2A_0\{g_{\alpha\alpha'}(k'\cdot qk_\mu-k\cdot qk_\mu')+g_{\mu\alpha}(k\cdot k'q_{\alpha'}-k'\cdot qk_{\alpha'})+g_{\mu\alpha'}(k\cdot qk_\alpha'-k\cdot k'q_\alpha)+k_{\alpha'}k_\mu'q_\alpha-k_\mu k_\alpha'q_{\alpha'}\}\\ &+2A_1\{k'^2[g_{\mu\alpha'}(k^2q_\alpha-k\cdot qk_\alpha)+k_\alpha k_{\alpha'}k_\mu]-k^2[g_{\mu\alpha}(k'^2q_{\alpha'}-k'\cdot qk_{\alpha'}')-k_\alpha'k_{\alpha'}'k_\mu']-k\cdot k'(k_\mu+k_\mu')k_\alpha k_{\alpha'}'\}\\ &+2A_2(k^2q_\alpha-k\cdot qk_\alpha)(k_\mu'q_{\alpha'}-k'\cdot qg_{\mu\alpha'})+2A_3(k'^2q_{\alpha'}-k'\cdot qk_{\alpha'}')(k_\mu q_\alpha-k\cdot qg_{\mu\alpha})-B_0\{2k\cdot k'\varepsilon_{\alpha\alpha'\mu\nu}q^\nu\\ &+k_\alpha'[Qq]_{\mu\alpha'}-k_{\alpha'}[Qq]_{\mu\alpha}\}+2k\cdot k'B_1\{(k^4-k'^4)\varepsilon_{\alpha\alpha'\mu\nu}q^\nu+k^2k_\alpha[Qq]_{\mu\alpha'}-k'^2k_{\alpha'}'[Qq]_{\mu\alpha}\}\\ &+(k^4+k'^4)B_1\{k_{\alpha'}[Qq]_{\mu\alpha}-k_\alpha'[Qq]_{\mu\alpha'}\}+B_2\{k^2k'^2\varepsilon_{\alpha\alpha'\mu\nu}q^\nu-k'^2k_\alpha[Qq]_{\mu\alpha'}+k^2k_{\alpha'}'[Qq]_{\mu\alpha}\}. \end{aligned} \tag{4.35}$$

For the particular case $V=V'$, there are additional restrictions which can be easily obtained from the conditions on the form factors expressed in Eqs. (4.15) – (4.17). These conditions imply the following relations for the new form factors appearing in Eq. (4.35):

$$\begin{aligned} A_{0,1}(k^2,k'^2,q^2)&=-A_{0,1}(k'^2,k^2,q^2),\\ A_{2,3}(k^2,k'^2,q^2)&=-A_{3,2}(k'^2,k^2,q^2),\\ B_0(k^2,k'^2,q^2)&=-B_0(k'^2,k^2,q^2),\\ B_{1,2}(k^2,k'^2,q^2)&=-B_{1,2}(k'^2,k^2,q^2), \end{aligned} \tag{4.36}$$

which we represent schematically by saying that, under the interchange $k^2\leftrightarrow k'^2$,

$$\begin{aligned} &A_0\to -A_0,\quad A_1\to -A_1,\quad A_2\leftrightarrow -A_3,\\ &B_0\to -B_0,\quad B_1\leftrightarrow -B_1,\quad B_2\leftrightarrow -B_2. \end{aligned} \tag{4.37}$$

[1]If we are working in the kinematic region that includes the point $k^2=k'^2=0$, then we cannot use $\mathbf{K}_1$ and $\mathbf{K}_2$ as defined in Eq. (4.28), since they become null vectors in this case. The analysis below has to be modified accordingly.

Finally, if $V=V'$ is the photon itself then Eq. (4.35) gives the three-photon vertex. However, in this case the vertex must satisfy also the condition expressed in Eq. (4.7) and the additional crossing relation

$$\Gamma_{\alpha\alpha'\mu}(k,k')=\Gamma_{\mu\alpha'\alpha}(-q,k'). \tag{4.38}$$

Applying them to Eq. (4.35), it follows that the relations given in Eq. (4.37) should in fact be valid if any two external momenta are interchanged. Using these extra crossing relations we then obtain

$$A_1=A_2=A_3=B_1=B_2=0. \tag{4.39}$$

Thus, in general, the off-shell three-photon vertex does not vanish and it is characterized by the two form factors A_0 and B_0. It can be easily seen that the effective interaction in this case can be rewritten as

$$2A_0F^{\lambda}{}_{\nu}(k)F^{\nu}{}_{\rho}(k')F^{\rho}{}_{\lambda}(q)-2B_0\widetilde{F}^{\lambda}{}_{\nu}(k)\widetilde{F}^{\nu}{}_{\rho}(k')\widetilde{F}^{\rho}{}_{\lambda}(q), \tag{4.40}$$

where

$$F_{\lambda\nu}(k)=-i[k_\lambda A_\nu(k)-k_\nu A_\lambda(k)],$$
$$\widetilde{F}_{\lambda\nu}(k)=\frac{1}{2}\varepsilon_{\lambda\nu\rho\sigma}F^{\rho\sigma}(k). \tag{4.41}$$

One may wonder, why not also interactions of the types $FF\widetilde{F}$ and $\widetilde{F}\widetilde{F}F$? It can be shown that the first of these has precisely the form of the interaction with three factors of $\widetilde{F}$, whereas the second is equal to the interaction with three factors of F.

B. V,V' charged

When V and V' are charged, the condition on the vertex function due to gauge invariance is expressed in Eq. (1.5). Thus, what we get are conditions on the form factors evaluated for $k^2=m_V^2$ and $k'^2=m_{V'}^2$. Since the implications depend on whether $m_V=m_{V'}$, we consider several cases separately.

1. $V\neq V'$, both on-shell

Using Eq. (2.3) in Eq. (1.5), we get the relations

$$a_1q^2+a_1'Q\cdot q=0,$$
$$a_2q^2+a_2'Q\cdot q+2a_4=0, \quad b_1'=0, \tag{4.42}$$

remembering that these are the form factors evaluated with k,k' on shell. Since in this case $k^2=k'^2$, then $Q\cdot q=0$. Thus, we then have for $a_{1,4}$ and a_1' a set of relations analogous to Eq. (4.2), and we finally arrive at an expression for the vertex function that is identical in form to $\Gamma^{(T)}_{\alpha\alpha'\mu}$ given in Eq. (4.3).

If the photon is also on shell, corrresponding to the decay process $V\rightarrow V'\gamma$, then vertex function reduces to exactly the same form as given in Eq. (4.6). The comments made after Eq. (4.6) are applicable in this case also.

2. $V=V'$, both on-shell

In this case $k^2=k'^2$, and therefore $Q\cdot q=0$. Instead of Eq. (4.42) we now have, for k,k' on-shell,

$$a_1=b_1'=0, \quad a_2q^2+2a_4=0\ , \tag{4.43}$$

and

$$\Gamma_{\alpha\alpha'\mu}=a_1'Q_\mu g_{\alpha\alpha'}+a_2'Q_\mu q_\alpha q_{\alpha'}+a_3(g_{\mu\alpha'}q_\alpha-g_{\mu\alpha}q_{\alpha'})$$
$$+a_4\left(g_{\mu\alpha'}q_\alpha+g_{\mu\alpha}q_{\alpha'}-2\frac{q_\mu q_\alpha q_{\alpha'}}{q^2}\right)+b_1\epsilon_{\alpha\alpha'\mu\nu}q^\nu$$
$$+b_2q_\alpha[Qq]_{\mu\alpha'}+b_3q_{\alpha'}[Qq]_{\mu\alpha}. \tag{4.44}$$

The main difference between this case and the previous one is that here the a_1' term does not vanish at $q^2=0$. This is how it should be since, as shown in Eq. (2.30), in this limit the a_1' term corresponds to the electric charge of the particle. Notice also that, in spite of appearances, the a_4 term is well defined for on-shell photons because the apparently troublesome term q^μ/q^2 vanishes when it is contracted with the photon polarization vector. In fact, that term does not contribute whenever the photon line is connected to a conserved current, such as one generated by a pair of fermions on-shell. However, in a more complicated diagram in which the photon line connects to an off-shell charged particle propagator, the contribution from the q^μ term is not zero and must be retained. The apparent singularity at $q^2=0$ is eliminated by the integration over the internal loop momenta.

It is useful at this point to compare our expression in Eq. (4.44) with the expression given by Hagiwara, Peccei, Zeppenfeld, and Hikasa [2], which is much in use by other authors working in the field and therefore serves as a good reference point. Equation (2.4) of Ref. [2] gives the vertex function $\Gamma_{\alpha\beta\mu}$ for the process

$$\gamma_\mu(P)\rightarrow W_\alpha(q)+\overline{W_\beta}(\overline{q}). \tag{4.45}$$

The vertex function for the process $\overline{W_\alpha}(q)\rightarrow\gamma_\mu(P)+W_\beta(\overline{q})$ is then given by making the substitution $P\rightarrow -P$, $Q\rightarrow -Q$ in their Eq. (2.4). Finally, by setting $\beta\rightarrow\alpha'$ and making a trivial relabeling of the momentum vectors in the resulting expression (which in the end amounts to simply set $P\rightarrow q$), we obtain the vertex function for the process $W_\alpha(k)\rightarrow\gamma_\mu(q)+W_\beta(k')$, which corresponds to the process we are considering with the identification $V=V'=W$. Thus we obtain that the expression that must be compared with our Eq. (4.44) is

$$\Gamma^{(\mathrm{HPZH})}_{\alpha\alpha'\mu}=-f_1Q_\mu g_{\alpha\alpha'}+\frac{f_2}{M_W^2}Q_\mu q_\alpha q_{\alpha'}-f_3(q_\alpha g_{\mu\alpha'}-q_{\alpha'}g_{\mu\alpha})$$
$$-if_4\left(q_\alpha g_{\mu\alpha'}+q_{\alpha'}g_{\mu\alpha}-2\frac{q_\mu q_\alpha q_{\alpha'}}{q^2}\right)$$
$$-if_5\left(\epsilon_{\mu\alpha\alpha'\rho}Q^\rho-\frac{q^\mu}{q^2}[qQ]_{\alpha\alpha'}\right)$$
$$+f_6\epsilon_{\mu\alpha\alpha'\rho}q^\rho+\frac{f_7}{M_W^2}Q_\mu[qQ]_{\alpha\alpha'}. \tag{4.46}$$

In Eq. (4.46) we have included a term proportional to q^μ in the factors of the $f_{4,5}$ terms. Such terms were implicit in Ref. [2] but they were omitted under the assumption that the photon coupled to a conserved fermion current and therefore do not contribute to the amplitude, as it is the case for the process $e^-e^+\to W\bar{W}$ considered there. We have restored them here for the purpose of our comparison since we have not made that assumption in the corresponding Eq. (4.44).

Simple inspection of Eqs. (4.44) and (4.46) reveals the following direct correspondence between the form factors:

$$a_1' = -f_1, \quad a_4 = -if_4,$$

$$a_2' = \frac{f_2}{M_W^2}, \quad a_3 = -f_3. \tag{4.47}$$

The correspondence between the remaining ones is not immediately obvious, but follows straightforwardly upon using the identities

$$q^2\varepsilon_{\alpha\alpha'\mu\nu}Q^\nu - q_\mu[qQ]_{\alpha\alpha'} = -q_{\alpha'}[Qq]_{\mu\alpha} + q_\alpha[Qq]_{\mu\alpha'}, \tag{4.48}$$

$$Q_\mu[Qq]_{\alpha\alpha'} = Q^2\varepsilon_{\alpha\alpha'\mu\nu}q^\nu - q_{\alpha'}[Qq]_{\mu\alpha} - q_\alpha[Qq]_{\mu\alpha'}, \tag{4.49}$$

which follow from Eqs. (2.8) and (2.9) by specializing them to the present situation ($V=V'$, both on-shell). In this way we then obtain

$$b_1 = f_6 - \frac{Q^2}{M_W^2}f_7, \quad b_2 = -i\frac{f_5}{q^2} + \frac{f_7}{M_W^2},$$

$$b_3 = i\frac{f_5}{q^2} + \frac{f_7}{M_W^2}. \tag{4.50}$$

To take this comparison one step further, suppose that the effects of P and C violation are negligible in the $VV\gamma$ coupling. It then follows from Eqs. (3.1) and (3.2) that the only nonvanishing terms in Eq. (4.44) are a_1', a_2', and a_3. Apart from a slight change of notation, this is the form adopted, for example, in Refs. [10,11].

V. CONCLUSIONS

We have studied in this article the structure of the couplings of spin-1 particles to the photon. In Sec. II we considered the general form that the electromagnetic vertex can have, consistent with Lorentz invariance, and we established the physical interpretation of the various form factors that parametrize the vertex in terms of the static electromagnetic moments. In Sec. III we derived the consequences of the various discrete space-time symmetries on the form factors, paying special attention to the case of neutral bosons and in particular to the case of self-conjugate bosons. In the latter case, we derived some results that are analogous to similar results that are known to hold regarding the electromagnetic couplings of Majorana fermions. Finally, in Sec. IV we analyzed in detail the implications due to gauge invariance for the structure of the vertex function. In particular, several results concerning the electromagnetic properties of self-conjugate bosons were obtained there. For example, it was shown that while a self-conjugate particle cannot have any static electromagnetic moments, in the most general case it can have an electromagnetic vertex characterized by two form factors. This result is analogous to the corresponding one for Majorana fermions, which cannot have static electromagnetic moments either, but it can have an electromagnetic vertex characterized by an axial charge radius form factor. For the three-photon vertex, which vanishes in pure QED due to Furry's theorem, we obtained a general form which need not vanish due to the breaking of the charge conjugation symmetry by the weak interactions.

We have been motivated by the fact that experimental studies of this kind of coupling will be feasible in the future. In this context, the analysis that we have presented can be useful in at least two ways. On one hand, it can serve as a guide to parametrize any possible deviation of the couplings from the values predicted by the standard model, in a way that is general and model-independent. On the other hand, whenever the study of a new kind of phenomena is accessible to us, it is useful to keep in mind that our present knowledge may be shaken by new discoveries in a more unexpected way than simply just a deviation from the detailed values predicted by the standard model for a given physical quantity. The results of our analysis can be used to test deviations from fundamental physical principles, such as gauge invariance and crossing symmetry, in the context of the processes described by the electromagnetic couplings of vector bosons.

Note added in proof. After this paper was submitted for publication we became aware of the papers cited in Refs. [12,13]. In Ref. [12], the authors considered the on-shell electromagnetic coupling of a self-conjugate particle of any spin. As far as the spin-1 case is concerned, this is the very special case that we have considered in Eq. (4.14) of case 2 in Sec. IV; i.e., $V=V'$, both on shell and self-conjugate. For this case, our results agree with theirs. However, we go further than this since the cited references do not consider the off-shell couplings, nor the case of non-self-conjugate particles nor the off-diagonal case, all of which is contained in the present work. Reference [13] presents explicit calculations of the $ZZ\gamma$ vertex in the standard model, for various configurations, with results that exhibit the general features that are described in the present article. We would like to thank F. Boudjema for bringing these references to our attention.

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