Cowsik *et al.* Reply: Let us first respond to the Comments by Gates *et al.* (GKT) [1] in our Letter [2]. Among the various issues raised by GKT, the most crucial one is their claim that our result disagrees with the observations; this is not true. Their claim stems from (i) confusing model-dependent results (obtained on the basis of certain mass models) with actual observational constraints, (ii) confusing the classical mass models of the halo with models that probe its phase space structure, and (iii) using the notion of superposition not allowed by the self-consistent Boltzmann-Poisson equations which involve a nonlinear coupling among the various components. Indeed, the very purpose of our Letter [2] was to present a method which would sensitively probe the density and dispersion velocity of dark-matter particles in the solar neighborhood, circumventing some of the problems encountered in previous analyses.

Let us first address the claim of GKT that the velocity dispersion of the dark-matter particles in the solar neighborhood should be $\langle v^2\rangle^{1/2}_{\mathrm{DM},\odot} = 270\ \mathrm{km\,s^{-1}}$, based on the formula $\langle v^2\rangle^{1/2}_{\mathrm{DM}} = \sqrt{\frac{3}{2}}\,V_{c,\infty}$. For this claim to be valid, two conditions need to be satisfied: (1) The dynamics has to be that of a single component isothermal sphere so that the formula is applicable, and (2) the asymptotic circular speed $V_{c,\infty}$ should be $\sim$220 $\mathrm{km\,s^{-1}}$. The first of these conditions is violated in the problem at hand; in the central regions of the galaxy the density of the visible matter exceeds that of the dark matter by factors $\sim$1000. Even the integrated mass of the dark matter within a sphere of radius $R_\odot \sim 8.5$ kpc is smaller than that of the visible matter. In other words, the dark matter is not the dominant component within the solar circle. Thus the above asymptotic relation between $\langle v^2\rangle^{1/2}_{\mathrm{DM}}$ and V_c is established only at much larger distances, as shown in Fig. 1 below in our response to the Comments by Evans [3]. Second, there is no observational basis for the claim that $V_{c,\infty} = V_{c,\odot} = 220\ \mathrm{km\,s^{-1}}$. After extensive review Fich and Tremaine (cited in [2]) concluded that the rotation curve continues to rise beyond $R_\odot$. Indeed all available rotation curve data up to $R \sim 20$ kpc have been incorporated into Fig. 1 of Ref. [2] which forms the basis of our results.

The observations of halo stars and globular clusters are also not in conflict with our results. Frenk and White, as well as Norris and Hawkins (Ref. [2] of GKT), have discussed both the limitations and the uncertainties involved in the analysis of the problem. It is to be emphasized that all previous analyses of the problem, including those in Ref. [2] of GKT, were concerned with the *mass distribution* in the halo, rather than its phase space structure. Even for the simpler problem of determining the density distribution, it is necessary to measure the size variables $(\vec{r}, \vec{v})$ for each of the objects under study, and furthermore, a large statistical sample of these objects is needed. GKT's Comment [1] seems to foster the impression that the results of our analysis are inconsistent with observation. However, the so-called observations are in fact no more than model-dependent inferences. This is clear from the fact that out of the six variables only four, namely, $(\vec{r}, v_r)$, have been measured, leaving the models highly underconstrained. Also contrary to the assertion of GKT, inspection of Fig. 5 of Frenk and White shows that the parameters of the halo mass models have a wide dispersion, but they are not inconsistent with our results. In this context, we do not know how GKT obtained the value of $\langle v^2\rangle^{1/2}_{\mathrm{DM},\odot} \sim 200\ \mathrm{km\,s^{-1}}$, as this value does not appear in any of the papers cited in Ref. [2] of GKT.

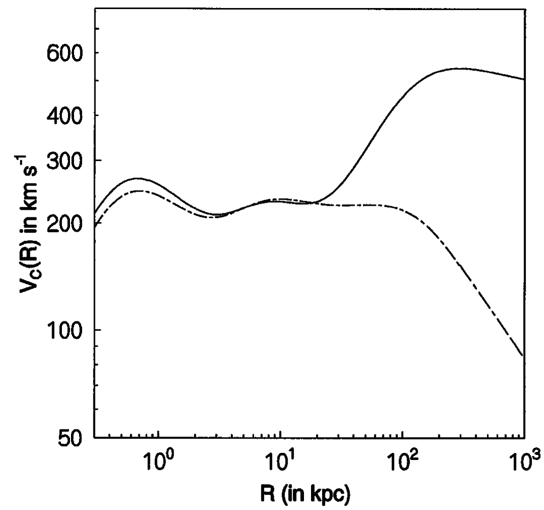

FIG. 1. Rotation curves of the Galaxy for the infinite isothermal halo model (solid line) with $\langle v^2\rangle^{1/2}_{\mathrm{DM}} = 600\ \mathrm{km\,s^{-1}}$ and for the lowered isothermal halo model (dashed line) for $r_t = 300$ kpc and $\sigma = 330\ \mathrm{km\,s^{-1}}$ which corresponds to $\langle v^2\rangle^{1/2}_{\mathrm{DM},\odot} \sim 570\ \mathrm{km\,s^{-1}}$.

The analysis in Ref. [3] of GKT also does not probe the phase space structure of the halo. Different components of the mass density distribution cannot simply be superposed because of nonlinear couplings among the components. Moreover the value $\langle v^2\rangle^{1/2} \sim 30\ \mathrm{km\,s^{-1}}$ that they quote for the disk stars has little to do with the problem at hand, because the disk stars are supported against the galactic gravity mainly by their circular motion about the center of the Galaxy. Finally, the work in Ref. [4] of GKT does not uniquely determine the value of $\langle v^2\rangle^{1/2}_{\mathrm{DM},\odot}$ as no attempt is made to fit the rotation curves to the actual data.

In contrast with all earlier analyses, we have formulated the problem to directly address the phase-space structure. There are two adjustable parameters in our model, the central density $\rho_{\mathrm{DM}}(r = 0)$ and the velocity dispersion $\langle v^2\rangle^{1/2}_{\mathrm{DM}}$ of the dark matter particles. By fitting the rotation curve of the Galaxy up to $R \sim 20$ kpc, both the parameters were determined, even though we placed particular emphasis on the value of $\langle v^2\rangle^{1/2}_{\mathrm{DM}}$. The χ^2 for $\langle v^2\rangle^{1/2}_{\mathrm{DM}} = 300\ \mathrm{km\,s^{-1}}$ is more than four times the value

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for $\langle v^2\rangle_{\rm DM}^{1/2} = 600\ {\rm km\,s^{-1}}$, ruling out smaller dispersion velocities.

Let us now consider Evans' Comments [3]. Evans has expressed concern that the value of $\langle v^2\rangle_{\rm DM}^{1/2} = 600\ {\rm km\,s^{-1}}$ derived by us may not be valid in general because of our assumption of the Maxwellian form for the distribution function (DF). Below, we argue that our results are quite general and are only weakly sensitive to the precise form of the DF assumed in the analysis. In addition we argue that Evans' worries about the mass estimates of the Galaxy are unfounded.

We have based our analysis on two of the simplest and most widely used DFs, namely, (a) the Maxwellian and (b) the "lowered isothermal" or "King model" [4]. The latter has the property that both the spatial density and the velocity dispersion vanish at the "tidal radius" r_t, and $\langle v^2\rangle_{\rm DM}^{1/2}$ depends on the galactocentric distance (R); it decreases from $\langle v^2\rangle_{\rm DM}^{1/2} \sim \sqrt{3\sigma^2}$ at $R = 0$ to $\langle v^2\rangle_{\rm DM}^{1/2} = 0$ at $R = r_t$, where σ is the velocity parameter of the model [4].

Figure 1 shows the rotation curves for the two DFs. With the King model, the best fit to the rotation curve is obtained for $\sigma = 330\ {\rm km\,s^{-1}}$ which corresponds to the solar neighborhood value of dark matter velocity dispersion, $\langle v^2\rangle_{\rm DM,\odot}^{1/2} \sim 570\ {\rm km\,s^{-1}}$, not significantly different from $600\ {\rm km\,s^{-1}}$ for the Maxwellian DF (hence our comment in Ref. [18] of [2]). The fact that our calculated rotation curves have the correct asymptotic behavior gives us full confidence in the correctness of our iterative numerical algorithm. Note from Fig. 1 that while both Maxwellian and King forms of the DF predict essentially identical $V_c(R)$ up to $\sim$20 kpc, the curves are very different at large R. This underscores the need to measure with greater precision V_c as a function of R, especially at large galactocentric distances, to fix the parameters describing the dark matter halo more exactly. Nevertheless, our analysis shows that the existing rotation curve data up to $R \sim 20$ kpc already yield an estimate of $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$ which is about twice as large as the value usually assumed, and which is roughly the same for the two different DF's of the particles constituting the dark halo. In this sense, our result is quite robust. There is another way in which the robustness of the result can be understood: In the absence of streaming ($\langle v\rangle = 0$), the leading moment $\langle v^2\rangle$ appears as the pressure term in the Jeans equations [see Eqs. (4)–(27) of [4] and the discussions following it]. Thus for all pressure supported halos, corresponding to *a given rotation curve*, the value of $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$ will be roughly the same as the value $\sim 600\ {\rm km\,s^{-1}}$ derived by us [2]. In this respect, we disagree with Evans' interpretation of the Jeans equation. Contrary to what Evans mentions, we have *not* "simply taken a much deeper central potential than is reasonable or warranted," As already mentioned above in response to GKT's Comments, we have in fact *determined* the central potential (equivalently central density) of the dark halo, in addition to the velocity dispersion $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$, by requiring that the calculated rotation curve fit the available data. This is the correct way of finding out what is "reasonable or warranted."

Coming to the question of mass estimate of the Galaxy, it may not be reasonable to assume that the isothermal model provides a valid description of the Galaxy's dark halo at distances extending to $\sim$1 Mpc. Evans' mass estimate of the Galaxy obtained by integrating the isothermal halo density profile to a distance as large as 1 Mpc is, therefore, specific to this assumption. The actual DF for the dark halo describing a finite system such as the Galaxy could very well be something like the truncated isothermal model or the King model for which the density at large distances falls off much faster than that for the isothermal model and the corresponding mass within 1 Mpc can be much smaller. We display the rotation curve in Fig. 1 above up to 1 Mpc only to demonstrate that our numerical algorithm yields the correct and expected asymptotic behavior. Since the correct DF that describes the dark halo of the Galaxy is not known, we have used two of the well-known DF's as trial ones, and in each case, we have self-consistently determined the corresponding $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$ and the central density of the dark halo by fitting to the available rotation curve data. The fact that we obtain roughly the same value of $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$ for the two DF's is indeed very appealing. The point is that different behavior of the rotation curves for the two DF's at distances much beyond $\sim$20 kpc is not all that relevant when we are concerned with the value of $\langle v^2\rangle_{\rm DM}^{1/2}$ at the solar neighborhood ($R \sim 8.5$ kpc).

In summary, we believe the conclusions reached in our Letter [2] are correct, robust, and not in conflict with any established observational facts. We wish to end this Reply by emphasizing that the value of $\langle v^2\rangle_{\rm DM,\odot}^{1/2}$ had only been *assumed* thus far; in contrast, we believe, we have made a useful beginning to a new approach in which this value can be determined on the basis of observational data.

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