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On the estimation of the amount of heating for solar coronal loops

Suresh Chandra *Institut für Theoretische Physik der Universität Frankfurt/Main Robert-Mayer Strasse 8-10, D-6000 Frankfurt/Main, W. Germany*

Udit Narain *Astrophysics Research Group, Meerut College, Meerut 250 001,*

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Abstract. In the present investigation the amount of heating for some coronal magnetic loops and kernels has been investigated in a general way in line-dipole and point-dipole geometries. This investigation differs from that of Elwert & Narain (1980) in the sense that the electron density has been taken to be finite at the base of the flux-tube in point-dipole geometry as well as in line-dipole geometry and the plane parallel geometry case has been excluded. Results show that the total amount of heating for the loop is larger than that for the kernels. It is larger for line-dipole geometry than for point-dipole geometry. Further, a wider flux-tube requires more heating than a narrower one, as expected.

Key words : heating—line-dipole geometry—point-dipole geometry—corornal loops

1. Introduction

A simple model of the loop structure of a hot post-flare plasma has been presented by Antiochos & Sturrock (1976) on the basis of the following observations and assumptions : (i) The evaporation phenomenon occurs in the initial stage in the development of flare plasma. After the plasma is evaporated there should be a second stage (initial cooling phase of post-flare plasma) when conduction dominates the radiation mechanism. After some time however radiation will be the dominant mechanism. (ii) For the initial cooling phase of post-flare plasma, where the conduction is dominant, the magnetic field geometry plays an important role and the conduction across the magnetic field lines is negligible. Hence, the cooling of a small flux tube is independent of all other tubes, and the energy conducts along the magnetic field lines only. (iii) The magnetic field is current-free (potential-field). This assumption simplifies the structure of the magnetic field, which is along the loop. (iv) Observationally it is found that the magnetic field on the base of the loop is much larger than that on the top of the loop. For the conservation of the magnetic flux, it shows that the cross-section of the flux tube changes along the length, and the plane parallel model of the loop is quite misleading.

Recent investigations (Neupert *et al.*, 1974; Cheng & Widing, 1975; Pallavicini *et al.*, 1975; Widing & Cheng, 1974; Vorpahl *et al.*, 1977) show that the observed cooling times of the solar coronal features are longer than the conductive cooling times. The statement (iv) is one of the proposals put forward to explain this discrepancy (Neupert *et al.*, 1974). Another proposal is that the observed feature is, somehow, continuously heated (Cheng & Widing, 1975; Pallavicini *et al.*, 1975; Vorpahl *et al.*, 1977). Using these ideas recently Elwert & Narain (1980) estimated the required amount of heating with geometrical inhibition (line-dipole geometry) and without (plane-parallel geometry). In their investigation they assumed that the electron density, which is a function of the length along the tube, on the base of the tube is infinite. This assumption, for the constant pressure along the tube, leads to zero temperature on the base of the tube, which is not physically possible. This leads to the idea that the electron density on the base must be finite. Further, the plane-parallel model of the loop is quite misleading.

In the present investigation the amount of heating for some coronal magnetic loops and kernels is investigated. The line-dipole and the point-dipole geometries (Antiochos & Sturrock, 1976) are considered for the loop structure. Present investigation differs basically from that of Elwert & Narain (1980) in the sense that the electron density is being taken to be finite on the base of the tube and the point-dipole geometry also has been taken into account, while the plane parallel geometry has been excluded.

2. Mathematical formulation

According to the loop model presentcd by Antiochos & Sturrock (1976) a current-free magnetic field above the chromosphere is produced either by a horizontal line-dipole (L) or by a horizontal point dipole (P) situated at a depth D below the chromosphere. The height of the flux tube above the chromosphere is H and the flux tube is symmetrical about the point $s = 0$, taken on the top of the tube. The distance s along the tube is related to the angle θ, between the vertical line and the line joining the point to the dipole, by

$$\frac{s}{R}\theta, = \qquad \ldots(\mathrm{iL})$$

$$\frac{s}{R} = \tfrac{1}{2}\sin\theta(1 + 3\sin^2\theta)^{1/2} + \frac{1}{2\sqrt{3}}\ln\left[\sqrt{3}\sin\theta + (1 + 3\sin^2\theta)^{1/2}\right]$$
$$= f(\theta), \qquad \ldots(\mathrm{IP})$$

where

$$R = H + D. \qquad \ldots(2)$$

The symbols have their usual meaning (Antiochos & Sturrock 1976; Elwert & Narain 1980).
The area of the tube A (θ), normalised to unity at $\theta = 0$, is given by

$$A(\theta) = \cos^2 \theta, \qquad \text{...(3L)}$$

$$A(\theta) = \cos^6 \theta(1 + 3 \sin^2 \theta)^{-1/2}. \qquad \text{...(3P)}$$

Since the height of the loop is smaller than the pressure scale height, the gravitational effect may be neglected in the present investigation. Now with the assumption (ii) and considering a source of continuous heating, the equation of energy transfer is given by (Elwert & Narain 1980)

$$\frac{\partial}{\partial t} (3nkT) = \frac{1}{A(s)} \frac{\partial}{\partial s}\left[A(s) \; \kappa \frac{\partial T}{\partial s} \right] + Q(0, t) \exp. \left(- \frac{s^2}{\gamma^2 R^2}\right), \qquad \text{...(4)}$$

where $\kappa(\alpha T^{5/2}, \alpha \approx 10^{-6})$ is the coefficient of thermal conductivity, γ a measure of the breadth of the source, and $Q(0, t)$ is a function of time. The electron density $n = n(s)$ and the temperature $T = T(s, t)$ are such that the hydrogen plasma pressure is given by

$$p(t) = 2knT,$$

where k is the Boltzmann constant. ...(5)
Using equations (4) and (5) one obtains

$$1.5 \; p(t)^{-3 \cdot 5} \frac{dp}{dt} = \frac{\alpha}{A} \frac{d}{ds} \left(A \frac{dG}{ds} \right) + Q(0, t) \, p(t)^{-3 \cdot 5} \exp \left(- \frac{s^2}{\gamma^2 R^2}\right), \qquad \text{...(6)}$$

where

$$G(s) = \frac{2}{7} [2kn(s)]^{-7/2}. \qquad \text{...(7)}$$

The nature of $Q(0, t)$ is unknown and a convenient choice may be

$$Q(0, t) = Cp(t)^{3 \cdot 5} \qquad \text{...(8)}$$

where C is a constant. With this particular choice the right-hand side (RHS) of equation (6) becomes independent of time t, and the equation (6) may be solved by separating the variables. By putting each side of equation (6) equal to K_s, with the particular choice

$$K_s = - 0.6 \, p_0^{-2 \cdot 5} \, \tau^{-1}, \qquad \text{...(9)}$$

the solution of left-hand side of equation (6) is

$$p(t) = p_0 \left(1 + \frac{t}{\tau}\right)^{-0 \cdot 4}. \qquad \text{...(10)}$$

Here, τ is the characteristic time of decay and p_0 is the initial pressure at $t = 0$. The solution of RHS of equation (6), under the condition $dG/ds = dG/d\theta = 0$ at $s = 0$, is

$$G(\theta) = \frac{2}{7}\left(\frac{p_0}{T_{00}}\right)^{-3.5} - \frac{R^2}{\alpha} CI_2(\theta) + \frac{R^2}{\alpha} K_s F(\theta), \qquad \text{...(11)}$$

where

$$F(\theta) = \tfrac{1}{2}\, \theta \tan \theta, \qquad \text{...(12L)}$$

$$F(\theta) = \frac{1}{70} [32 \sec^4 \theta - 16 \sec^2 \theta - 2 \cos^2 \theta + 7.5 \cos^4 \theta - 21.5], \qquad \text{...(12P)}$$

$$I_2(\theta) = \int_0^\theta I_1(\theta) \sec^2 \theta \, d\theta; \quad I_1(\theta) = \int_0^\theta \cos^2 \theta \exp\left(-\frac{\theta^2}{\gamma^2}\right) d\theta, \qquad \text{...(13L)}$$

$$\left.\begin{aligned} I_2(\theta) &= \int_0^\theta I_1(\theta) \left(\frac{1 + 3 \sin^2 \theta}{\cos^5 \theta}\right) d\theta; \\ I_1(\theta) &= \int_0^\theta \cos^7 \theta \exp\left(-\frac{f^2(\theta)}{\gamma^2}\right) d\theta, \end{aligned}\right\} \qquad \text{...(13P)}$$

and T_{00} is the temperature at $s=0$ and $t=0$. If the foot point of the flux tube is situated at a distance s_b from the top, and the corresponding angle is θ_b, then for the finite electron density, such that $G(\theta_b) = \beta G(0)$, $n(\theta_b) = \beta^{-2/7} n_0$, one can easily calculate the unknown constant C :

$$C = \frac{(1 - \beta)\, \tau\alpha\, T_{00}^{3.5} - 2.1 p_0 R^2 F(\theta_b)}{3.5\tau\, p_0^{3.5}\, R^2 I_2(\theta_b)}, \qquad \text{...(14)}$$

where β is an adjustable parameter.

Following Antiochos & Sturrock (1976) the compression factor, defined by

$$\Gamma = \frac{A(\theta = 0)}{A(\theta)} = \frac{1}{A(\theta)} = \frac{B(\theta)}{B(\theta = 0)}, \qquad \text{...(15)}$$

where B is the magnetic field, has the maximum value

$$\Gamma_{\max} = \frac{1}{A(\theta_b)} = \frac{B(\theta_b)}{B(\theta = 0)}. \qquad \text{...(16)}$$

Using equation (3)

$$\Gamma_{\max} = \sec^2 \theta_b, \qquad \text{...(17L)}$$

$$\Gamma_{\max} = \sec^6 \theta_b (1 + 3 \sin^2 \theta_b)^{1/2}. \qquad \text{...(17P)}$$

With the knowledge of the magnetic field, calculated at the top and the base of the loop, the value of $\Gamma_{\max}$ can be calculated and the value of θ_b can be easily obtained with the help of equation (17). By using the length of the loop $l\,(= 2s_b)$ and the angle θ_b, the value of R can be calculated by equation (1). The heating function, with the help of equations (4), (8) and (10), is given by

$$Q(\theta, t) = C\, p_0^{3\cdot5} \left(1 + \frac{t}{\tau}\right)^{-1\cdot4} \exp\left(-\frac{s^2}{\gamma^2 R^2}\right). \qquad \ldots(18)$$

For calculating the total amount of heating of the source, Elwert & Narain (1980) assumed that the radius of the flux tube at the top is one tenth of its length. But, due to the lack of sufficient information, we assume that the radius of the tube r is related to the length through $r = \delta l$, where $\delta < 1$; then the area of cross section at the top of the flux tube is

$$A(0) = 4\pi\, \delta^2 s_b^2, \qquad \ldots(19)$$

and the total amount of heating is obtained as

$$Q_T = C p_0^{3\cdot5} \int_0^{\tau} \left(1 + \frac{t}{\tau}\right)^{-1\cdot4} dt \int_{-s_b}^{s_b} \exp\left(-\frac{s^2}{\gamma^2 R^2}\, A(0)\right) A(s)\, ds, \qquad \ldots(20)$$

or, using equations (19) and (3),

$$Q_T = 15.214\, C p_0^{3\cdot5}\, \tau\, \delta^2 R s_b^2\, I_1(\theta_b). \qquad \ldots(21)$$

Again, for the known values of the parameters n_0, T_{00}, l and τ one can easily calculate from equation (11) the value of the function $G(\theta)$. The electron density and temperature are given by

$$n(\theta) = [3.5\, G(\theta)]^{-2/7}/2k, \qquad \ldots(22)$$

$$T(\theta, t) = \frac{n_0 T_{00}}{n(\theta)} \left(1 + \frac{t}{\tau}\right)^{-0\cdot4}. \qquad \ldots(23)$$

3. Results and discussion

For the numerical calculations one needs the values of the parameters n_0, T_{00}, l, τ, and the magnetic field strength at the top and the base of the event. Since much information about the events is not known, for calculating the value of the maximum compression factor, $\Gamma_{\max}$, Krieger (1978) followed the suggestion of Stenflo (1973) that the magnetic flux is concentrated into the elements of field strength ≈ 2000 gauss. Further, for getting the magnetic field strength at the top of the loop he conjectured that the magnetic confinement of the plasma would be difficult if the magnetic pressure falls below the gas pressure. Therefore, for the hydrogen plasma

$$\frac{B_{\min}^2}{8\pi} = 2nkT \qquad \ldots(24)$$

and

$$\Gamma_{\max} = \frac{B_{\max}}{B_{\min}} = \frac{2000}{(16\pi nkt)^{1/2}} = 7.6 \times 10^2\, (n_9 T_6)^{-1/2}, \qquad \ldots(25)$$

where n_9 and T_6 are the electron density and the temperature in units of 10^9 cm^{-3} and 10^6 K, respectively. Then, by using the observed values of the parameters n and

T Krieger (1978) calculated the values of Γ_{max} for all the five events reported in Table 1. The observed values of the parameters l, n_0, τ are also given in table 1. Although these values are averaged values of the parameters, due to the lack of detailed information these are assumed to be on the top of the events at $t = 0$ (Elwert & Narain 1980). By using the values of Γ_{max} in equation (17) the values of θ_b are calculated. Values of R are calculated with the help of equation (1) by using these values of θ_b and the length $l(= 2s_b)$. These values of R and θ_b are also given in table 1.

Table 1. Values of the parameters

Event (1973)	l (10^9 cm)	T_{0e} (10^6 K)	n_0 (10^9 cm^{-3})	Γ_{max}	τ (s)	R (cm) L	R (cm) P	θ_b (Degrees) L	θ_b (Degrees) P
Filament disappearance (Aug. 21)	14	2.5	1.2	439	2.7(4)*	4.6(9)	5.8(9)	87.26	66.25
Flare loop (June 15)	3.1	6.5	15	77	6.4(2)	1.1(9)	1.4(9)	83.46	57.76
Flare loop (Nov. 26)	2	5.8	13	88	2.1(3)	6.8(8)	9.1(8)	83.88	58.52
Flare kernel (Aug. 9)	0.36	9.5	400	12	1.3(2)	1.4(8)	2.1(8)	73.22	44.54
Flare kernel (Sep. 1)	0.15	8	200	19	4.5(2)	5.6(7)	8.1(7)	76.74	48.36

*The numbers in the brackets are the powers of ten; *e.g.* 2.7 (4) = 2.7×10^4.

Table 1 shows that the value of R calculated for the first two events—filament disappearance of 1973 August 21 and flare loop of 1973 June 15, is comparable to the pressure scale height ($\approx 10^{10}$ cm) and we cannot neglect the gravitational effect for them. Therefore, these two events are not taken into account. For the remaining three events—flare loop of 1973 November 26, flare kernel of 1973 August 9, and flare kernel of 1973 September 1—the values of the parameter β are taken to be 10^{-2}, 10^{-3}, 10^{-4} and of γ 1.0, 0.5, 0.1. For these three events in both line-dipole and point-dipole geometries the values of the constant C and the integral $I_1(\theta_b)$ are calculated and listed in table 2. The integral $I_2(\theta_b)$ from equation (13) can be calculated numerically and by using these values, the values of the function $G(\theta)$ can be calculated easily [equation (11)]. With the knowledge of $G(\theta)$ the distribution of the electron density and the temperature can be calculated with the help of equations (22) and (23).

The general idea about the size of loops is that these are of typical lengths of $10^4 - 10^5$ km and width $2 - 8 \times 10^3$ km (Chiuderi *et al.* 1977). In the present investigation the total amount of heating (for the three events with $\beta \doteq 10^{-2}$ and $\gamma = 1.0$) is calculated and plotted in figure 1 as a function of $1/\delta$ (δ being the ratio of the radius of cross section at the top of event to the length).

Table 2. Values of the constant C and the parameters I_1 (θ_b)

β	$\gamma = 1.0$		$\gamma = 0.5$		$\gamma = 0.1$	
	L	P	L	P	L	P
Flare loop (1973 Nov. 26)						
10^{-2}	1.11 (−6)*	7.16 (−7)	1.67 (−6)	9.09 (−7)	7.27 (−6)	3.23 (−6)
10^{-3}	1.12 (−6)	7.24 (−7)	1.68 (−6)	9.19 (−7)	7.35 (−6)	3.27 (−6)
10^{-4}	1.12 (−6)	7.25 (−7)	1.68 (−6)	9.20 (−7)	7.35 (−6)	3.27 (−6)
$I_1(\theta_b)$	=0.604	0.404	0.394	0.314	8.82 (−2)	8.67 (−2)
Flare kernel (1973 Aug. 9)						
10^{-2}	4.24 (− 10)	3.30 (−10)	6.11 (−10)	4.02 (−10)	2.55 (−9)	1.35 (−9)
10^{-3}	4.29 (− 10)	3.34 (−10)	6.19 (−10)	4.07 (−10)	2.59 (−9)	1.36 (−9)
10^{-4}	4.30 (− 10)	3.34 (−10)	6.20 (−10)	4.07 (−10)	2.59 (−9)	1.37 (−9)
$I_1(\theta_b)$	= 0.603	0.400	0.394	0.314	8.82 (−2)	8.67 (−2)
1973 Flare kernel (Sep. 1)						
10^{-2}	3.11 (−8)	2.42 (−8)	4.55 (−8)	3.00 (−8)	1.93 (−7)	1.03 (−7)
10^{-3}	3.14 (−8)	2.44 (−8)	4.59 (−8)	3.02 (−8)	1.95 (−7)	1.04 (−7)
10^{-4}	3.14 (−8)	2.45 (−8)	4.60 (−8)	3.03 (−8)	1.95 (−7)	1.04 (−7)
$I_1(\theta_b)$	= 0.603	0.402	0.394	0.314	8.82 (−2)	8.67 (−2)

*The numbers in the brackets are the powers of ten; *e.g.* 1.11 (−6) = 1.11 × 10^{-6}.

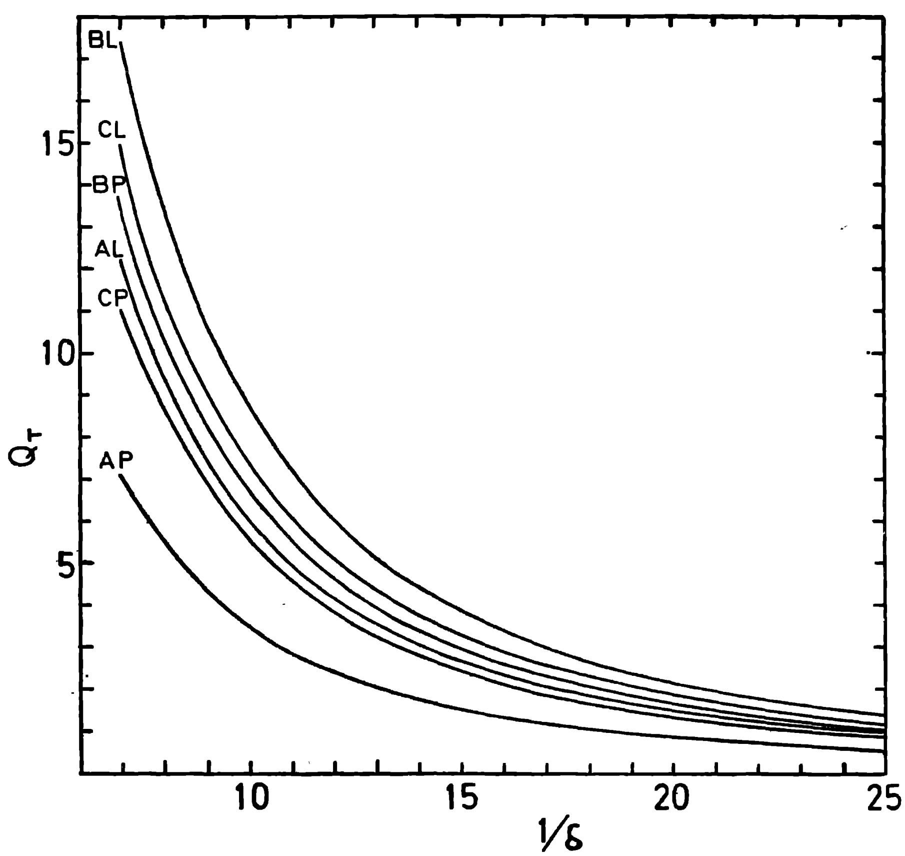

Figure 1. Total amount of heating vs δ^{-1} (δ being the ratio of the radius of cross-section at the top of the event to the length). Plots AL, AP are for the flare loop of 1973 November 26; plots BL, BP for the flare kernel of 1973 August 9; and plots CL, CP for the flare kernel of 1973 September 1. The values of the parameters are $\beta = 10^{-2}$ and $\gamma = 1.0$. The letter L denotes line-dipole geometry, and the letter P the point-dipole geometry. For the flare loop Q_T is in units of 10^{27} erg, while for flare kernels it is in units of 10^{26} erg.

It is obvious from the figure that if the flux tube is wide (δ large) heating required would be large and if it is narrow (δ small) the heating required would be small. This seems quite sensible. Figure 1 also clearly shows that the heating for a line-dipole geometry would be larger than that for a point-dipole geometry. Q_T for flare loop is in units of 10^{27} erg whereas it is in units of 10^{26} erg for flare kernels. Thus heating for a flare loop is larger than that for flare kernels. Hopefully this is a general trend and is quite expected in view of the fact that a flare kernel occupies a smaller volume whereas a flare loop occupies a larger volume.

It may be concluded that the present approach is more general and close to reality than the previous one (Elwert & Narain 1980).

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