THE CHROMOSPHERIC NETWORK

(A) *Network Evolution Viewed as a Diffusion Process*

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Abstract. Autocorrelation and cross-correlation techniques have been applied to obtain quantitative information about the dynamics of magnetic flux on the solar surface. The speed of network magnetic elements and the diffusion coefficient associated with their random motion is derived. The speed is found to be about 0.1 km s^{-1}, independent of activity level. However, the diffusion coefficient shows a strong activity dependence: it is about 370–500 km^2 s^{-1} in the quiet network and 135–210 km^2 s^{-1} in the enhanced network. It is found that the lifetime of the enhanced network relative to the quiet network is compatible with that suggested by a comparison of their respective diffusion coefficients. This supports the proposition that a diffusion-like dispersion of magnetic flux is the dominant factor in the large-scale, long-term evolution of the network.

1. Introduction

The structure and dynamics of the features on the solar surface are a result of magnetoconvective processes. Convective flow in the solar photosphere restricts magnetic flux to move along supergranular edges (Spruit, Nordlund and Title, 1990). This is believed to result in the preferential heating of the overlying chromosphere and thus in the chromospheric network (Simon and Leighton, 1964). Leighton (1964) suggested that the dispersal of magnetic flux can be modelled as a dispersion-like process driven by supergranular evolution. Under its influence the network magnetic elements execute random motions, which causes cells to eventually break-up, disappear or merge.

Hence, a study of the diffusion coefficient of the motion of network magnetic elements is relevant to the understanding of the evolution of the network. Marsh (1978) quoted a value of 800 km^2 s^{-1} for the diffusion coefficient D. On the basis of autocorrelation (AC) and cross-correlation (CC) data obtained from a time-series of magnetograms, Wang (1988a) deduced a value of 151 km^2 s^{-1}. Schrijver and Martin (1990) derived a value of about 110 km^2 s^{-1} in the core region of magnetic plage, and 230–290 km^2 s^{-1} in the area surrounding the core. On the basis of a model of the distribution of magnetic flux over the solar surface, Wang, Nash, and Sheeley (1989) found a best-fit value of $D \approx 600$ km^2 s^{-1}. Simon, Title, and Weiss (1995) have derived coefficients in the range 500–700 km s^{-1}, in agreement with Wang *et al.*

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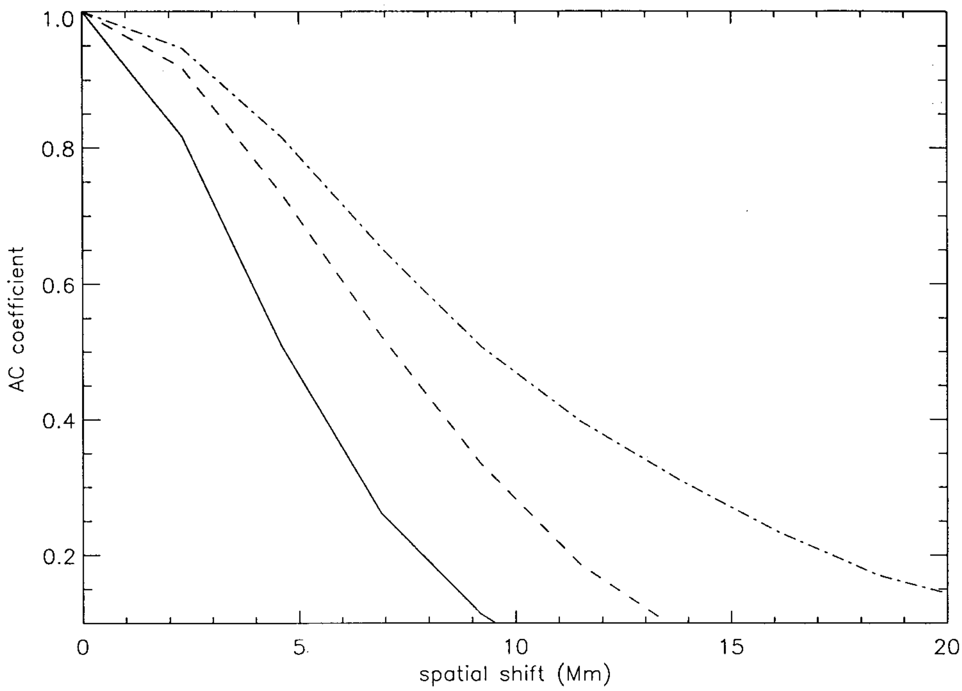

Figure 1. AC curves for quiet, enhanced network and active region, for a maximum spatial lag of 20 Mm, given respectively, by bold, dashed and dash-dotted lines. The required value is Δ (AC $=$ 0.75). The slope of the quiet AC curve is steepest, implying smallest network features. The results are summarized in Table I.

In this paper we study the relation between lifetimes and activity level in terms of the diffusion model for magnetic flux. The study is based on the active region associated with the spotgroup NOAA/USAF 5874 (CMP 10 January 1990) and the surrounding quiet region. The term enhanced network, as used in the present manuscript, covers the active and semi-active networks described in our earlier paper (Raju, Srikanth, and Singh, 1998a). The active network is one that shows cell structure at the periphery of a plage; the semi-active network is one detached from a plage, but still show signs of activity (Figure 1 in the above reference).

2. Observations and Analysis

The data consisted of about 106 hrs of almost uninterrupted sequence of Ca II K filtergrams, spaced at about 10 min, and obtained at Antarctica during the local summer of 1989–1990 (Singh *et al.*, 1994). Details regarding reduction of data are given in our previous paper (Raju, Srikanth, and Singh, 1998a). The present analysis uses the correlation method to derive statistical information about the network cells. The correlation function was computed for windows of size $160'' \times 160''$.

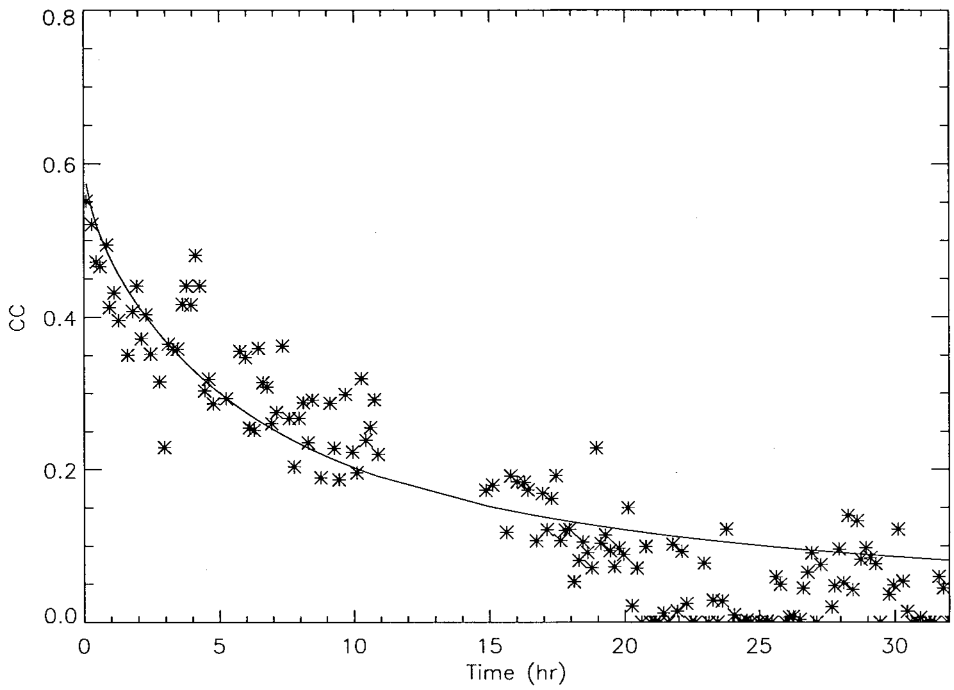

Figure 2. Cross-correlation curve for the quiet region using a $160'' \times 160''$ window. The solid line represents a least-squares fit using Equation (2) modified by the factor y_0, and corresponds to $y_0 = 0.57$ and $R^2/D = 19.8$ hr.

Length-scales of the network cells were derived using AC technique (Rogers, 1970; Sýkora, 1970; Singh and Bappu, 1982; Wang 1988b; Raju, Srikanth, and Singh, 1998a). Typical AC plots for different activity levels are given in Figure 1. Cell lifetimes were derived from CC data (Rogers, 1970; Worden, 1975; Worden and Simon, 1976; Duvall, 1980; Raju, Srikanth, and Singh, 1998a). Typical CC plots for quiet and enhanced network regions are given in Figures 2 and 3.

In quiet regions, the CC function tends to abruptly fall to about 0.55–0.65 within the first 10 min (Rogers, 1970; Raju, Srikanth, and Singh, 1998a). The fall is specific to quiet regions, thereby suggesting that it is an intrinsic property to these regions and not an artefact due to image degradation. It occurs because of short term processes such as 5 min oscillations, seeing and the disappearance and appearance of bright mottles, whose lifetime is of the order of 10 min (Rogers, 1970).

3. Speed of Network Magnetic Elements

The fall in $CC_{(\tau)}$ can be attributed to one or both of: (a) intrinsic changes in the network, which is characterized by the appearance, disappearance, splitting and

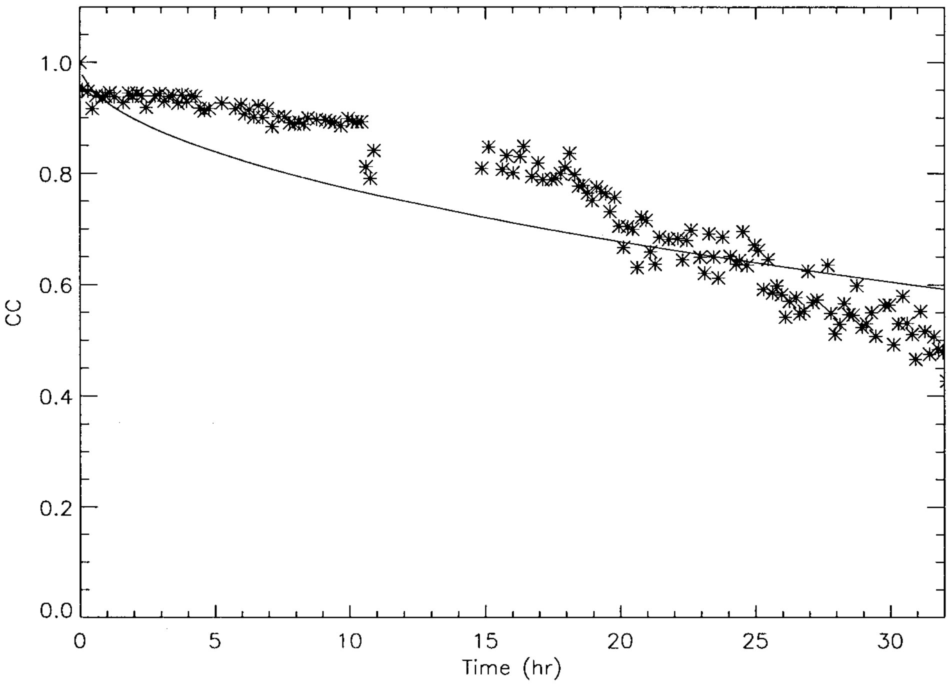

Figure 3. Cross-correlation curve for the enhanced network using a $160'' \times 160''$ window. The solid line represents a least-squares fit using Equation (2) modified by the factor y_0, and corresponds to $y_0 = 1.0$ and $R^2/D = 206.0$ hr.

merging of flux (Zirin, 1987; Wang, 1988a; Schrijver *et al.*, 1997); (b) movement of flux tubes, which consists of a random component, and a systematic component which responds to a superposition of large scale flow patterns on the solar surface like supergranulation, meridional flows and differential rotation. If the fall in CC is wholly due to intrinsic change, when the CC has fallen to a value of about 0.75 about a quarter of the features have changed. If the fall is wholly due to movement of magnetic flux tubes, then an upper limit on the speed of the movement of the structures can be calculated using the correlation data. We consider two similar flux tubes moving past each other. If relative motion alone causes the fall in CC, then the feature has moved in the time τ (CC $= 0.75$) taken by the CC to fall to 0.75 through the distance Δ (AC $= 0.75$), which is the space shift over which AC drops to 0.75 (Wang, 1988a). The choice of correlation 0.75, rather than for example 0.5, is to enable a smooth interpolation of the AC curve. In practice, the CC limit was taken as $0.75y_0$ rather than 0.75, where y_0 is the y-axis intercept of a least-squares straight-line fit to the CC data (Raju, Srikanth, and Singh, 1998a), so that the effect of short-term processes on the CC is excluded. The speed of the network element is estimated as the ratio $v = \Delta$ (AC $= 0.75)/\tau$ (CC $= 0.75y_0$).

This method assumes that all flux tubes within a given image window for which the AC and CC were derived are of similar size and shape, and move at the same

TABLE I

Typical time lapse for CC $= 0.75y_0$ and spatial shift for AC $= 0.75$ for different activity levels, used to estimate the speed of motion of K line flux tubes

No	Region	Δ (AC $= 0.75$) (km)	τ (CC $= 0.75y_0$) (hr)	Speed Δ/τ (km/s^{-1})
1	Quiet	3150	9.5	0.092
2	Enhanced network	4340	12.5	0.096

y_0 is the y-axis intercept of the linear fit to the CC data.

speed. In reality, they come in various sizes and shapes, as shown by magnetogram studies. For example, a range of velocities derived from an inspection of magnetograms is given by Schrijver and Martin (1990).

In Figure 1, the AC curve for three regions of varying activity level is given for spatial lags up to 20 Mm. The bold curve is obtained from quiet region data, the dashed one from enhanced network, and the dash-dotted curve from the active region. The correlation falls steepest in case of the quiet region. The steepness in the fall of the AC curve is an inverse measure of the size of network magnetic elements.

The results of speed estimates are summarized in Table I. We find that in both quiet and enhanced network regions, the speeds of the network magnetic elements are approximately the same, namely about 0.1 km s^{-1}. Our result for quiet regions confirms that of Zirin (1987), who quotes the same value, and of Wang (1988a), who estimates a value of 0.15 km s^{-1} on basis of a method similar to ours. Schrijver and Martin (1990) come to a similar conclusion after studying over 700 flux concentrations in magnetogram time-sequences in and around the core of a magnetic plage. A reason for uniformity in speed could be that the speeds are determined by the opposing effects of convective motions and magnetic inhibition of motion. Both these factors are stronger in more active regions, thereby leading to similar speeds in all regimes.

4. Diffusion Coefficients

Mosher (1977) constructed a diffusion model of the solar surface magnetic fields based on AC and CC data. Assuming that the magnetic network elements are circular and of average radius R, he estimated that:

$$R = \mathrm{HWHM}_{\mathrm{AC}}/0.8\ , \tag{1}$$

where $\mathrm{HWHM}_{\mathrm{AC}}$ is the half width at half maximum of the AC curve. According to Mosher's model, the diffusion coefficient D is related to the CC function through the relation:

$$\mathrm{CC}(\tau) = 1 - \left(\frac{\sqrt{\pi D \tau}}{1.7R}\right) \mathrm{erf}\left(\frac{1.7R}{\sqrt{4D\tau}}\right) , \tag{2}$$

where erf is the error function. The model assumes that the network magnetic elements are of the same size and speed. It considers only the effect of the random motion, but not the contribution of drifts along supergranular boundaries in the first few hours.

For consistency with $\mathrm{CC}(\tau)$ used in Table I, the right hand side of Equation (2) was multiplied by a factor y_0, so that $\mathrm{CC}(0) = y_0$ and not 1.0. This compensates for the fact that Equation (2) does not take into account short-term processes leading to the initial drop in CC, such as seeing and 5-min oscillations. In order to fit the thus modified Equation (2) to the CC, the variables y_0 and R^2/D were treated as free parameters. In the quiet region, the variable y_0 was varied in a narrow range about the observed CC value in the 10–20 min range (i.e, the value immediately after the initial fall). For the active region, the range was $0.95 \leq y_0 \leq 1.0$.

In Figure 2, the solid line gives a best fit to the quiet region CC data with a functional form given by Equation (2) modified by the y_0 factor. It corresponds to $R^2/D = 19.8$ h. To calculate D it was necessary to take into consideration the intrinsic spread in AC length scale, since D depends sensitively on it. Usually, HWHM values of the AC plot for quiet regions lie in the range 4100–4800 km. According to Equation (1), this corresponds to R ranging in 5125–6000 km. From Equation (2) and the result of Figure 1, we estimate $370 \leq D \leq 505$ km^2 s^{-1}. The estimates of D due to Marsh (1978), Wang, Nash, and Sheely (1989) and Simon, Title, and Weiss (1995), which cover the range 500–800 km^2 s^{-1}, agree with our estimate. Mosher (1977) obtained a value of $D = 400$ km^2 s^{-1} from fits to Hα data (Rogers, 1970). Schrijver and Martin (1990) derive a value of about 230–290 km^2 s^{-1} for the area surrounding the core of a magnetic plage. Applying the present technique to magnetograms, Wang (1988a) has obtained a somewhat smaller value of about 150 km^2 s^{-1}. The wide range of values found in the literature is probably a reflection of different methods used. The CC method as applied to Ca II K filtergrams sees larger concentrations of flux and combines the diffusions of a number of truely random walking elements, thereby giving an apparently larger value of D.

In Figure 3, the solid line gives a best fit to the enhanced network CC data with a functional form given by Equation (2) modified by the y_0 factor. It corresponds to $R^2/D = 206$ h. Typical HWHM values of the AC plot for enhanced network regions are 8000–10 000 km (the last two curves in Figure 1). According to Equation (1), this corresponds to R ranging in 10 000–12 500 km. From Equation (2) and the result of Figure 1, we estimate $135 \leq D \leq 210$ km s^{-1}.

The difference between the diffusion coefficients in the different regions could be caused by a difference in the speeds of network magnetic elements or in the step-length or both. Referring to Table I, since the speeds are the same – about 0.1 km s^{-1} – in all regions, step-lengths are expected to be lower in the high flux density active regions. This confirms a similar conclusion reached by Schrijver and Martin (1990).

5. Discussion

When random motions occur on a two-dimensional surface, the probability density of position at time t for a random walking element is given by

$$P(r,t) = \frac{2r}{4Dt} \exp\left(\frac{-r^2}{4Dt}\right) , \tag{3}$$

where r is the displacement from the origin. The expectation value for the square of displacement is

$$\langle r^2(t)\rangle = \int_0^\infty P(r,t) r^2 \, \mathrm{d}r = 4Dt . \tag{4}$$

To relate lifetime T of the cell to the diffusion coefficient, we assume that a random walking magnetic element is displaced (in a statistical sense) by the distance of a cell radius during one lifetime of the cell. In view of Equation (4), this can be expressed as

$$\frac{L^2}{4} = 4DT , \tag{5}$$

where L is the cell diameter. According to Equation (5), the ratio of the enhanced network (represented by the subscript 'e.n.') and the quiet network lifetimes is given by

$$\frac{T_{\mathrm{e.n.}}}{T_{\mathrm{quiet}}} = \frac{A_{\mathrm{e.n.}} D_{\mathrm{quiet}}}{D_{\mathrm{e.n.}} A_{\mathrm{quiet}}} , \tag{6}$$

where A stands for cell area, and the cells are assumed to be of the same shape. The above equation implies: (a) that larger cells in a region of given activity level will live longer, with lifetime increasing linearly with area (Singh *et al.*, 1994; Srikanth, Raji, and Singh, 1998); (b) that for a given size of cells, those in more active regions have longer lifetimes. This is qualitatively in agreement with observations (Wang, 1988b; Singh *et al.*, 1994; Raju, Srikanth, and Singh, 1998a).

The value of R inferred in general by using Equation (1), furthermore in the case of K line data, is usually much larger than the observed dimensions of flux tubes whose diameters are less than $\sim$1000 km. This suggests that the magnetic elements referred to in Equation (1) may not be true random walking flux tubes but possibly larger concentrations composed of flux tubes. This idea agrees with the fact that, in the literature, smaller elements accompany smaller estimates of D.

Here D may be interpreted as the flux expansion or dispersal rate of such elements as a result of continued random displacements of flux tubes within them.

Observational data regarding the dependence of cell size on activity are not unequivocal. Chandrasekhar (1961) showed from considerations of hydrodynamic stability that convective cells are smaller in more active regions. Singh and Bappu (1982) observed that the K line network diameters are about 5% smaller during the solar maximum than at minimum. Along these lines, Raju, Srikanth, and Singh (1998b) have suggested a reduction of size in higher flux density regions to explain observed cell size dependence on heliographic latitude. On the other hand, Sýkora (1970), Wang (1988b), and Münzer *et al.* (1989) report slightly larger cells in more active regions.

Assuming that cell size shows little dependence on activity level, if cells evolve by the process of diffusion alone, then according to Equation (6) one would expect cells to have lifetimes in the inverse ratio of their respective diffusion coefficients. The observed enhanced network lifetime is about 2 times larger than that of the quiet region lifetime (Raju, Srikanth, and Singh, 1998a; by grouping the lifetimes of the semi-active and active network cells, as defined in that paper). From the values of D derived in the last section, we infer that enhanced network diffusion coefficient in the range 135– 210 km^2 s^{-1} is compatible with the quoted quiet diffusion coefficient assuming $T_{\mathrm{e.n.}}/T_{\mathrm{quit}} \approx 2$. This supports the scenario that network evolution can be described as a diffusion-like processes whereby the network magnetic flux is dispersed in response to supergranular flows.

The evolution of the cell as deduced from the diffusion coefficients is of a large-scale, long-term character. Short-term changes indeed occur driven by non-diffusive processes like emergence, collision and merging or cancellation of flux (Zirin, 1987; Wang, 1988a), but their dynamic equilibrium ensures that the network preserves its overall reticular structure (Schrijver *et al.*, 1997).

6. Conclusions

We have studied the evolution and decay of supergranules in terms of the movement of magnetic elements of the network using the correlation technique. The speed of the network magnetic elements was found to be about 0.1 km s^{-1} irrespective of the activity level of the region. The diffusion coefficient for quiet regions was found to be to lie in the range 370–500 km^2 s^{-1}, while that for the enhanced network lies in the range 135–210 km^2 s^{-1}. It was found that the difference in lifetimes in the quiet and enhanced network regions can be explained in terms of their respective diffusion coefficients.

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