Finite temperature neutron matter and rotating neutron stars in the quark meson coupling model

P. K. Panda,[1,*] R. Sahu,[2] and C. Das[2]

[1]*Indian Institute of Astrophysics, Koramangala, Bangalore-560034, India*

[2]*Department of Physics, Berhampur University, Berhampur-760007, Orissa, India*

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A mean-field description of nonoverlaping nucleon bags bound by the self-consistent exchange of σ, ω, and ρ mesons is used to investigate the properties of neutron matter at finite temperature. We use the equation of state of zero temperature to study the rotating neutron star in the Komatsu-Eriguchi-Hachisu method. The mass, radius, and angular velocity of the neutron star are calculated and compared with the Walecka model. [S0556-2813(99)04208-9]

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In the last 50 years, numerous calculations have been carried out to study the properties of dense nuclear and neutron matter with varying degrees of success. Most of these studies attempted to describe the nuclear matter properties using only the nucleonic degrees of freedom with different forms of realistic and semirealistic nuclear potentials. In recent years, there has been considerable interest in relativistic calculations. The simplest and probably the best known relativistic model is the Walecka model [1] which consists of structureless nucleons interacting through the exchange of σ and ω mesons in the mean-field approximation. Later Serot and Walecka [2] extended the model to incorporate the isovector meson ρ. The above meson theory [3] has been quite successful in describing the properties of nuclear matter as well as finite nuclei using only the hadronic degrees of freedom.

While descriptions of the nuclear phenomena have been efficiently formulated using hadronic degrees of freedom, there have been interesting observations such as the EMC effect [4], which reveal the medium modification of the internal structure of the nucleon. This necessitates models that incorporate quark-gluon degrees of freedom. Recently a quark meson coupling model has been proposed by Guichon [5] for the description of the nuclear matter incorporating the quark degrees of freedom. In this model, nuclear matter consists of nonoverlapping MIT bags bound by the self-consistent exchange of scalar (σ) and vector (ω) mesons. This model has been extensively used to study the properties of infinite nuclear and neutron matter [6–8] and finite nuclei [9]. It has also been used to study in-medium kaon and antikaon properties [10], hypernuclei [11], and structure functions of finite nuclei [12].

Recently this model has been generalized to finite temperature taking a medium-dependent bag parameter [13]. In this generalized model, only symmetric nuclar matter has been considered and hence the interaction of quarks through the exchange of scalar (σ) and vector (ω) mesons have been considered. We now extend this model to neutron matter at different temperatures. Now we have to include the contribution of the isovector meson ρ in addition to the σ and ω mesons. As described above, the neutron matter at zero temperature has already been considered by Saito and Thomas within the framework of the quark meson coupling model [6]. Here we study the effect of temperature on the equation of state of neutron matter with a density-dependent bag parameter.

Another motivation for studying the neutron matter is the possible extension to the study of the neutron star. A neutron star at the time of birth is composed of supernova matter at high temperature. Afterwards, this newborn neutron star is cooled down by the neutrino diffusion process. Within 10–20 s, it evolves into a usual cold neutron star. Hence it would be quite interesting to study the neutron star at zero temperature within the quark meson coupling model. Here the zero-temperature equation of state of pure neutron matter is used for neutron star calculation. One should also consider the rotation of the neutron star using the same.

The details of the QMC model have been given in Ref. [13]. Since we now include the ρ meson, we give here a few important steps for completeness.

In this model, the nucleon in nuclear matter is assumed to be described by a static MIT bag in which quarks interact with the scalar (σ), the vector (ω), and the isovector (ρ) mesons, which are treated as classical in a mean-field approximation. The quark field $\psi_q(\mathbf{r},t)$ inside the bag then satisfies the equation

$$\left[i\vec{\gamma}\cdot\vec{\partial}-(m_q^0-V_\sigma)-\gamma^0\left(V_\omega+\frac{1}{2}\tau_{3q}V_\rho\right)\right]\psi_q(\mathbf{r},t)=0, \quad (1)$$

where $V_\sigma=g_\sigma^q\sigma$, $V_\omega=g_\omega^q\omega$ and $V_\rho=g_\rho^q R_{03}$ with σ, ω, and R_{03} are the mean meson fields. g_σ^q, g_ω^q, and g_ρ^q are the quark couplings with the σ, ω, and ρ mesons, respectively. m_q^0 is the current quark mass and τ_{3q} is the third component of the Pauli matrices. Here $q=u$ or d quarks only.

The normalized ground state for a quark (in an s state) in the bag is given as

$$\psi_q(\vec{r},t)=N\exp\left(-i\frac{\epsilon_q t}{R}\right)\begin{pmatrix} j_0(xr/R) \\ i\beta_q\vec{\sigma}\cdot\hat{r}j_1(xr/R)\end{pmatrix}\frac{\chi_q}{\sqrt{4\pi}}. \quad (2)$$

*Electronic address: prafulla@iiap.ernet.in

At finite temperatures, the quarks inside the bag can be thermally excited to higher angular momentum states. However, for simplicity, we shall still assume the bag describing the nucleon to be spherical with radius R which is now temperature dependent.

The single-particle quark and antiquark energies in units of R^{-1} are given as

$$\epsilon_q^{n\kappa}=\Omega_q^{n\kappa}+R\left(V_\omega \pm \frac{1}{2}V_\rho\right), \quad \text{for} \begin{pmatrix} u \\ d \end{pmatrix} \text{ quarks}, \tag{3a}$$

$$\epsilon_q^{n\kappa}=\Omega_q^{n\kappa}-R\left(V_\omega \pm \frac{1}{2}V_\rho\right), \quad \text{for} \begin{pmatrix} u \\ d \end{pmatrix} \text{ quarks}, \tag{3b}$$

where

$$\Omega_q^{n\kappa}=(x_{n\kappa}^2+R^2 m_q^{*2})^{1/2}, \tag{4}$$

$m_q^*=m_q^0-g_\sigma^q\sigma$ is the effective quark mass. The boundary condition at the bag surface determines the quark momentum $x_{n\kappa}$ in the state characterized by specific values of n and κ.

The total energy density at finite temperature T and at finite baryon density ρ_B is

$$\epsilon=\frac{\gamma}{(2\pi)^3}\int d^3k\sqrt{\mathbf{k}^2+M_N^{*2}}(f_B+\bar{f}_B)+\frac{g_\omega^2}{2m_\omega^2}\rho_B^2+\frac{1}{2}m_\sigma^2\sigma^2 + \frac{g_\rho^2}{8m_\rho^2}\rho_3^2, \tag{5}$$

where γ is the degeneracy factor (2 for neutron matter and 4 for symmetric nuclear matter), f_B and $\bar{f}_B$ are the thermal distribution functions for the baryons and antibaryons,

$$f_B=\frac{1}{e^{[\epsilon^*(\mathbf{k})-\mu_B^*]/T}+1} \quad \text{and} \quad \bar{f}_B=\frac{1}{e^{[\epsilon^*(\mathbf{k})+\mu_B^*]/T}+1} \tag{6}$$

with $\epsilon^*(\mathbf{k})=(\mathbf{k}^2+M_N^{*2})^{1/2}$ the effective nucleon energy and $\mu_B^*=\mu_B-g_\omega\omega-\frac{1}{2}g_\rho R_{03}$ the effective baryon chemical potential. M_N^* is the effective nucleon mass. We have substracted the spurious zero-point energy as described in Ref. [13]. The proton or neutron density

$$\rho_{p/n}=\frac{2}{(2\pi)^3}\int d^3k(f_B-\bar{f}_B). \tag{7}$$

Here ρ_3 is the difference between the proton and neutron densities, i.e., $(\rho_p-\rho_n)$ and the baryon density is $\rho_p+\rho_n$. The pressure reduces to the familiar expression [14]

$$P=\frac{\gamma}{3}\frac{1}{(2\pi)^3}\int d^3k\frac{\mathbf{k}^2}{\epsilon^*(k)}(f_B+\bar{f}_B)+\frac{g_\omega^2}{2m_\omega^2}\rho_B^2+\frac{g_\rho^2}{8m_\rho^2}\rho_3^2 - \frac{1}{2}m_\sigma^2\sigma^2. \tag{8}$$

At finite temperatures and for a given μ_B, the effective nucleon mass is known for given values of the meson fields once the bag radius R and the quark chemical potential μ_q are calculated [13].

The calculation of the neutron star [15] is then performed using the tabulated zero-temperature equation of state obtained above within our QMC model. The star is assumed to be rapidly rotating, relativistic, and compact. The details of the model are given by Kamatsu *et al.* [16] [Komatsu-Eriguchi-Hachisu (KEH) method] and Cook *et al.* [17]. For completeness, a few important steps are outlined below.

The stars are assumed to be stationarily rotating and hence have axially, equatorially symmetric structures. The metric can be written in spherical coordinates (t, r, θ, ϕ)

$$ds^2=-e^{2\nu}dt^2+e^{2\alpha}(dr^2+r^2d\theta^2) + e^{2\beta}r^2\sin^2\theta(d\phi-\omega dt)^2, \tag{9}$$

where α, β, ν and ω are the potentials which depend only on r and θ. The geometrized units c and G have been set to unity. The stellar matter is assumed to be a perfect fluid so that the energy momentum tensor T^{ab} is given by

$$T^{ab}=(\epsilon+P)U^aU^b+Pg^{ab}, \tag{10}$$

where ϵ, P, U^a, and g^{ab} are the energy density, pressure, four velocity, and metric tensor, respectively. It is further assumed that the four velocity U^a is simply a linear combination of the time and angular Killing vectors. The Einstein equations for ν, β, and ω are written as

$$\Delta[\rho e^{\gamma/2}]=S_\rho(r,\mu), \tag{11}$$

$$\left(\Delta+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\gamma e^{\gamma/2}=S_\gamma(r,\mu), \tag{12}$$

$$\left(\Delta+\frac{2}{r}\frac{\partial}{\partial r}-\frac{2}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega e^{(\gamma-2\rho)/2}=S_\omega(r,\mu), \tag{13}$$

where

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{1}{r^2}\cot\theta\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}, \tag{14}$$

$$\gamma=\beta+\nu, \tag{15}$$

$$\rho=\nu-\beta. \tag{16}$$

The detailed expressions for the source terms S_ρ, S_γ, and S_ω are given in [16]. The above differential equations can be transformed into an integral representation so as to enable us to handle boundary conditions in a much easier manner. Using a three-dimensional Green's function and introducing cylindrical coordinates $\bar{\omega}=r\sin\theta$ and $z=r\cos\theta$, the integral equations are given as

$$\rho = -\sum_{n=0}^{\infty} e^{-\gamma/2}\int_0^{\infty} dr' \int_0^1 d\mu' r'^2 f_{2n}^2 \times (r,r') P_{2n}(\mu) P_{2n}(\mu') S_\rho(r',\mu'), \tag{17}$$

$$r \sin\theta\gamma = -\frac{2}{\pi} e^{-\gamma/2} \sum_{n=1}^{\infty} \int_0^{\infty} dr' \times \int_0^1 d\mu' r'^2 f_{2n-1}^1(r,r') \frac{1}{2n-1} \times \sin(2n-1)\theta \sin(2n-1)\theta' S_\gamma(r',\mu'), \tag{18}$$

$$r \sin\theta\omega = -\sum_{n=1}^{\infty} e^{(2\rho-\gamma)/2} \int_0^{\infty} dr' \int_0^1 d\mu' r'^3 \sin\theta' \times f_{2n-1}^2(r,r') \frac{1}{2n(2n-1)} P_{2n-1}^1(\mu) \times P_{2n-1}^1(\mu') S_\omega(r',\mu'). \tag{19}$$

Here P_n is the Legendre polynomial, P_n^m is the associated Legendre function. The expressions for $f_n^1(r,r')$ and $f_n^2(r,r')$ are given in [16]. The expression for the potential α is also given in Ref. [16]. Thus the calculation involves solving the four field equations for ρ, γ, ω, and α.

We assume the bag constant to be density dependent and to be of the form

$$B = B_0 \exp\left(-\frac{4 g_\sigma^B \sigma}{M_N}\right) \tag{20}$$

with g_σ^B as a parameter. For $g_\sigma^q = 1$, the values of the vector meson coupling and the parameter g_σ^B are obtained by fitting the saturation properties of nuclear matter [7] and asymmetry energy and are given as $g_\omega^2/4\pi = 5.24$, $g_\rho^2/4\pi = 4.91$, and $(g_\sigma^B)^2/4\pi = 3.69$. The calculation at finite temperature proceeds in the following steps. For a given value of temperature and neutron chemical potential μ_n, the thermodynamic potential is obtained in terms of the effective nucleon mass which depends on the bag radius R, the quark chemical potential μ_q, and the mean fields σ,ω. For given values of σ and ω, the bag radius and the quark chemical potential are determined [13], respectively. The value of the σ mean field is determined by minimizing the thermodynamic potential Ω with ω determined from the self-consistency calculation.

In our calculation for neutron matter without ρ-meson coupling, we get a shallow bound state of neutron matter. If we introduce the ρ-meson coupling, the neutron matter becomes unbound. Similar results were obtained by Saito and Thomas [6] in their calculation of neutron matter within their quark meson coupling model. Thus the ρ-meson plays an important role in the neutron equation of state.

In Fig. 1, we plot pressure against density at different temperatures for pure neutron matter. We have included the contribution of the ρ meson. As the temperature increases, the equation of state becomes stiffer. The pressure has a nonzero value for zero baryon density at and above $T = 200$ MeV. This implies that the pressure has a contribution arising from the thermal distribution functions for baryons and antibaryons as well as from the nonzero value for the σ field. Similar results were also obtained by Panda *et al.* [13] for symmetric nuclear matter. The Walecka model [18] has also been observed to give a nonzero value for the scalar σ field. The entropy density for neutron matter is similar to the symmetric nuclear matter case [13].

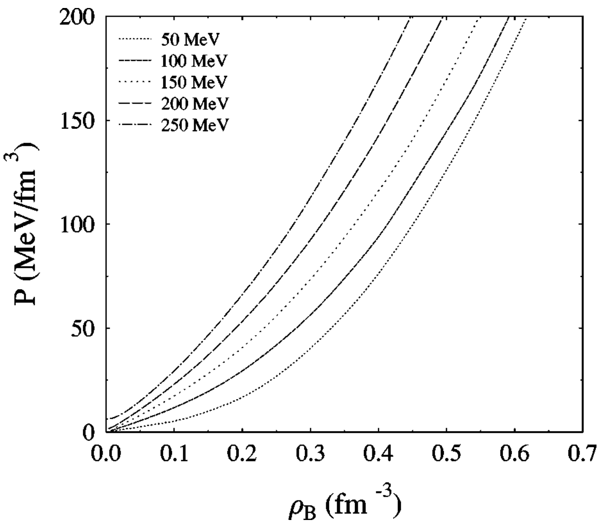

FIG. 1. The pressure for neutron matter as a function of the baryon density ρ_B for various values of temperature showing an increase with temperature (including the ρ-meson contribution).

We next proceed to calculate the properties of a rapidly rotating neutron star using the QMC equation of state at zero temperature obtained for neutron matter. In Fig. 2, we plot the radius of the neutron star as a function of its mass for the static case. We have also plotted the result obtained from the Walecka model. We find that the QMC model gives a maximum stable mass at 2.1 $M_\odot$. However, the Walecka model gives the stable mass at 2.6 $M_\odot$. The radius of the neutron star corresponding to the above maximum stable mass is 11.5 and 13.3 Km, respectively. In Fig. 3, the angular veloc-

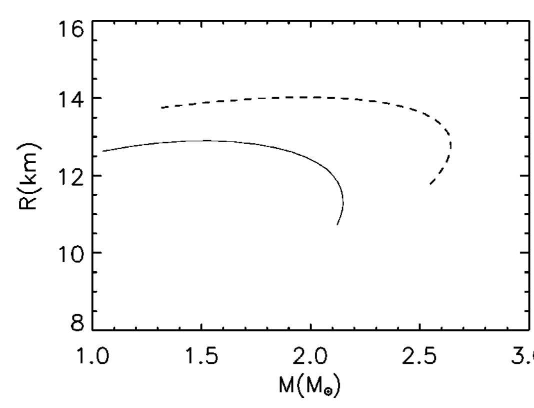

FIG. 2. The radius versus mass (static case) of the neutron star for the Walecka model (dashed line) and the QMC model (solid line).

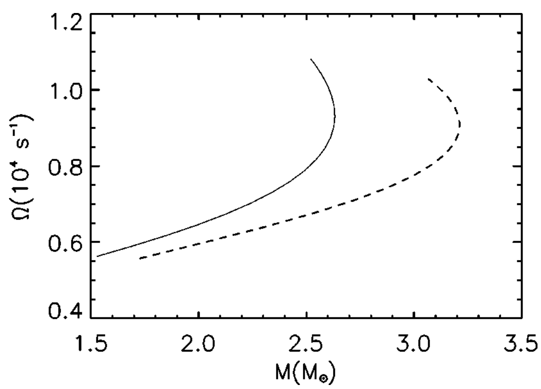

FIG. 3. The angular velocity versus mass of the neutron star for the Walecka model (dashed line) and the QMC model (solid line).

ity of the neutron star is plotted as a function of its mass. Walecka model predictions are also given for comparision.

We have tried to describe the properties of the neutron matter and neutron star within the quark meson coupling model. We assume the quarks to be bound inside nonoverlapping MIT bags and interacting through the exchange of scalar (σ), vector (ω), and isovector ρ mesons within the mean-field approximation. We assume the bag constant to be density dependent. We have performed the calculation of neutron matter at zero temperature and at finite temperature $T=50$, 100, 150, 200, and 250 MeV. We find that the neutron matter at $T=0$ MeV shows a shallow bound state if we do not include the effect of the ρ meson. The neutron matter becomes unbound after introducing the ρ-meson contribution. Thus we find the ρ-meson plays a very important role. Similar results were also obtained by Saito and Thomas [6]. Hence we have included the ρ-meson interaction in our subsequent calculations. We find that at $T \geqslant 200$ MeV, pressure has a nonzero value at vanishing density, similar to the result obtained by Panda *et al.* [13]. The Walecka model also gives a similar result [18].

We then apply the equation of state obtained for neutron matter at zero temperature to calculate the mass, radius, and angular velocity of the neutron star in the KEH method. We obtain results similar to the other relativistic and nonrelativistic calculations. For example, we obtain the maximum stable mass at 2.1 $M_\odot$ corresponding to the radius 11.5 km. We also compare the results obtained from the Walecka equation of state.

Thus starting from quark and meson degrees of freedom, the calculated neutron matter equation of state gives a good description of neutron star properties.

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