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Estimation of the depths of initial anchoring and the rising-rates of sunspot magnetic structures from rotation frequencies of sunspot groups

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Abstract. The mean variation of the 'initial' rotation frequency of sunspot groups, with the life span of spot groups in the range 12 to 2 days, has its trend similar to the radial variation of plasma rotation frequency, $\Omega(r)$, across the convective envelope at 15^o latitude as given by helioseismology.

The mean variation, with age, of the rotation frequency of sunspot groups which live for 10-12 days in latitudes $10^o - 20^o$ also has its trend similar to $\Omega(r)$.

These results suggest: (i) In all latitudes, the magnetic structures (*e.g*, bunch of flux tubes) of spot groups which live for 2 days are initially anchored near the surface (radial distance $r \sim 650$ Mm), and those of spot groups which live successively longer are initially anchored in successively deeper layers, by ~ 21000 km per day, (ii) the structures which yield spot groups living 10-12 days are initially anchored near the base of the convective envelope.

For a spot group which lives 10-12 days in latitudes $10^o - 20^o$, the 'anchoring layer' of its magnetic structure rises at a rate ~ 21000 km day^{-1}, as the spot group ages.

Key words: Sun: sunspots – Sun: rotation

1. Introduction

Studies of the rotation of plasma and magnetic structures beneath the Sun's surface are vital for understanding the magnetohydrodynamic (MHD) interactions beneath the surface. For studying the rotation of sub-surface magnetic structures, it is important to determine the variations in the rotation frequencies of sunspots and sunspot groups with respect to their ages, areas and lifetimes. Several such studies have been carried out by a number of authors using the Greenwich photoheliographic results (GPR) for sunspot groups (see Schröter 1985). It is believed that the rotation rate of spot groups decreases with their age (e.g. Ward 1966; Godoli & Mazzucconi 1979; Gokhale and Hiremath 1984; Balthasar et al. 1986; Tuominen & Virtanen 1987; Zappalà & Zuccarello 1991; Zuccarello 1993). This could be interpreted as an indication that the Sun's deeper layers rotate faster than the shallower layers assuming that the magnetic structures of the spot groups rise to shallower depths as the groups grow older. This interpretation supports the traditional dynamo theory (e.g. Stix 1981), but contradicts the radial gradient in the Sun's angular velocity inferred from helioseismology (e.g. Brown et al. 1989; Libbrecht 1989; Dziembowski et al. 1989; Goode et al. 1991; Tomezyk et al. 1995; Antia & Chitre 1996). It is also known that 'large' or 'long lived' spot groups rotate *slower* than the 'small' or 'short lived' groups (e.g. Howard et al. 1984). Hence it is not ruled out that one obtains the larger mean angular velocity for the younger spot groups because of the presence of a larger number of 'small' and 'short-lived' groups among the sample of the 'young' groups. To resolve the aforementioned discrepancy it is necessary to study the rotation rates of the samples of spot groups of different life spans (lengths of life) separately. Such studies have been carried out recently by Zappalà and Zuccarello (1991) and Zuccarello (1993).

In this paper we investigate in more detail how the rotation frequency of a spot group varies with its age (t) and what is the relation between the 'initial' rotation frequency of a spot group and its life span (τ).

Using the compilation of Greenwich sunspot data during 1874-1939, provided to us by Balthasar, we find the following results.

For spot groups, which live for 2-12 days, the mean variation of the "*initial*" rotation frequency, $\omega_{ini}(\tau)$, of a spot groups, with respect to its life span τ, is found to have a trend similar to that of the radial variation, $\Omega(r)$, of the rotation frequency of the solar plasma across the convective envelope.

For sunspot groups occurring in latitude interval $10^o - 20^o$ and living 10-12 days, we find that the mean variation, $\omega(t)$, also has a trend similar to $\Omega(r)$.

We determine the relation between the radial location (r_o), of the initial anchoring of the 'magnetic structure' of the spot group, and its life span (τ), by maximizing the correlation between $\omega_{ini}(\tau)$ and $\Omega(r)$. Similarly, we also obtain the relation

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Table 1. Columns 2,3,4 contain the number of sunspot groups of different specified life spans (τ, in days) in different 10^o latitude intervals.

τ	latitude intervals		
	$0-10^o$	10^o-20^o	20^o-30^o
1.5	183	277	109
2.5	138	217	95
3.5	107	235	62
4.5	122	201	89
5.5	123	222	81
6.5	100	292	70
7.5	92	183	65
8.5	85	159	53
9.5	84	154	52
10.5	64	142	58
11.5	16	49	26

between the radial position 'r' of the magnetic structure of the spot group, and its age t. From these relations we estimate the depths at which the magnetic structures of spot groups of given life spans are anchored when they are first seen, and the rate of rise of the magnetic structures of the spot groups which live for 10-12 days in latitude 10^o-20^o.

2. Data analysis

The magnetic tape of the GPR data on the sunspot groups during the years 1874-1976 which was kindly provided by H. Balthasar, include the observation time, (the date and the fraction of the day), heliographic latitude, and longitude for each spot group on each day of its observation, besides other parameters.

In the present study we have to ensure that we know the first and the last days of the life spans of the sunspot groups without any ambiguity. For spot groups living longer than 9 days the identification of the first and the last days is possible for much fewer groups during 1940-1976. For these years the magnetic tape contains data on spot groups only with central meridian longitudes (CML) having absolute values $\leq 58^o$. Hence we first confined our study to years 1874-1939. (Subsequently we have verified that the inclusion of the few spot groups with identifiable first and last days during 1940-1976 do not yield any significant changes in the results.) Even from the years 1874-1939 we have eliminated the entire data on those spot groups which have $|CML| > 75^o$ on any day of their life. (This leads to elimination of the second and the subsequent disc passages of the recurrent spot groups.) Thus, all spot groups with identifiable first and last days, have life spans $\leq$ 12 days.

These spot groups are further binned into latitude intervals of width 10^o each. In order to have better statistics, groups in the same latitude intervals in the northern and southern hemispheres were taken in combination. In Table 1 we give the total number of spot groups of different specified life spans in three latitude intervals of 10^o each.

The sidereal rotation frequency (ω) of a spot group of a specified life span in a specified latitude interval, at its age t = $(n+1/2)$ day is computed using the epochs of its observations on the (n)th and the (n+1)th day, and the heliographic longitudes of the spot group at these epochs. Rotation frequencies between non-consecutive days were not determined. Thus the uncertainty in t is $\leq$ 0.5 day.

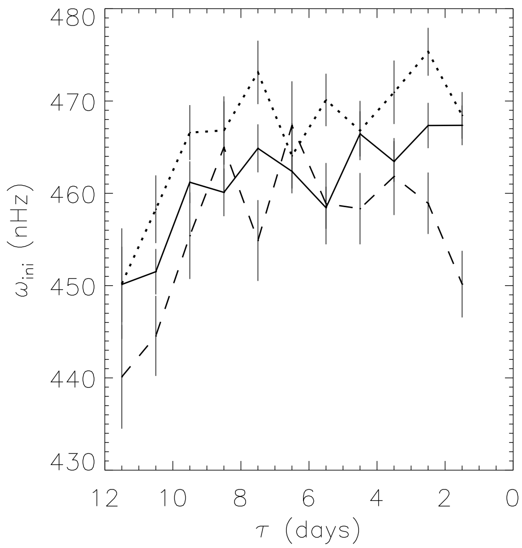

Fig. 1. The initial rotation frequency ω_{ini} as a function of the lifespan τ, for spot groups occurring in latitude intervals 0^o-10^o (dotted curve), 10^o-20^o (continuous curve) and 20^o-30^o (dashed curve).

For each specified value of the age 't' of a spot group, the mean rotation frequency ($\omega(t)$), and the standard deviation (σ), were computed for spot groups of given life spans in given latitude bins.

For spot groups of given life span τ in a given latitude bin, the value of ω at t = $3/2$ is defined as the mean "initial" rotation frequency $\omega_{ini}(\tau)$.

While determining the values of $\omega(t)$ and $\omega_{ini}(\tau)$ we excluded the data corresponding to the 'abnormal' motions, e.g. displacements exceeding 3^o day^{-1} in the longitude or $2^o day^{-1}$ in latitude. This reduces the data sample by about 3% but guards against errors in recording and in identification of small spot groups from one day to the next (Ward 1966). This precaution substantially reduces the uncertainties in the mean rotation frequencies (cf. Javaraiah & Gokhale 1995).

3. Comparison of $\omega_{ini}(\tau)$ and $\omega(t)$ with $\Omega(r)$

3.1. Comparison of $\omega_{ini}(\tau)$ with $\Omega(r)$

In Fig. 1 we show the dependence of the "initial" rotation frequency ($\omega_{ini}(\tau)$) on the life span 'τ' of a spot group, for spot groups of life spans 2 to 12 days, in different latitude intervals, obtained from the data during 1874-1939. The 'error-bars' correspond to the uncertainties at the respective σ-levels.

In Fig. 2 we show the radial variation of $\Omega(r)$, of the Sun's plasma rotation frequency (for fractional radius $\geq$ 0.65) at latitudes 5^o, 15^o and 25^o, as provided to us by Dr. H. M. Antia, who determined it from the Big Bear Solar Observatory (BBSO)

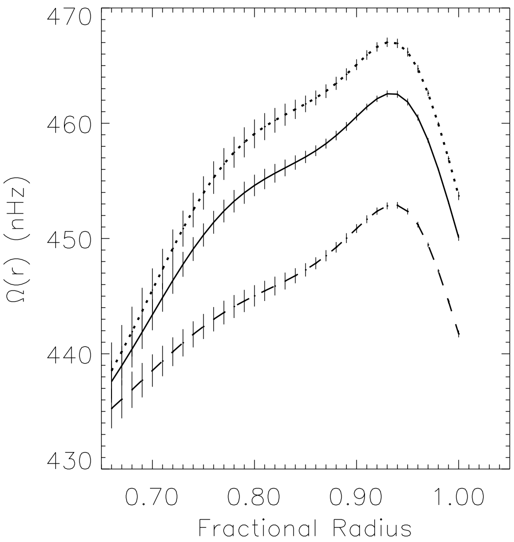

Fig. 2. Plasma rotation frequency $\Omega(r)$ as a function of r at latitudes 5^o (dotted curve), 15^o (continuous curve) and 25^o (dashed curve) provided to us by Dr. H. M. Antia, as determined from the BBSO helioseismoic data (Woodard & Libbercht 1993) using the inversion method of Antia and Chitre (1996).

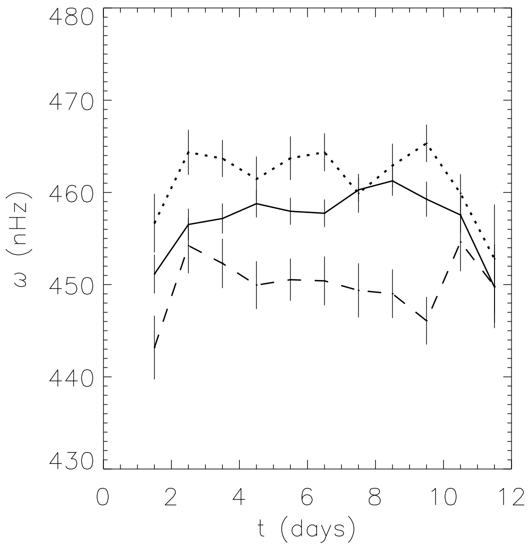

Fig. 3. The mean rotation frequency ω as a function of the age t for spot groups with life spans of 10-12 days, occurring in latitude intervals $0^o - 10^o$ (dotted curve), $10^o - 20^o$ (continuous curve) and $20^o - 30^o$ (dashed curve).

helioseismic data (Woodard & Libbrecht 1993) by using the inversion method of Antia & Chitre (1996).

From Figs. 1 and 2, it can be seen that in latitude intervals $0^o - 10^o$ and $10^o - 20^o$, $\omega_{ini}(\tau)$ has trends similar to the trend of $\Omega(r)$, across the convective envelope. In the interval $20^o - 30^o$, there is relatively large difference between the corresponding trends of $\omega_{ini}(\tau)$ and $\Omega(r)$ which could be due to the poor statistics. However, the similarity between $\omega_{ini}(\tau)$ and $\Omega(r)$ at 15^o latitude is even better for spot groups in the entire sunspot latitude belt (see Fig. 4). In each interval, the values of $\omega_{ini}(\tau)$ are 5-15 nHz higher than the values of $\Omega(r)$. This might be due to the 'differential buoyancy' which makes the flux loops rotate faster than the local plasma (D'Silva & Howard 1994). Inspite of the differential buoyancy effects, the similarity of trends of $\omega_{ini}(\tau)$ and $\Omega(r)$ suggests that the magnetic structures of spot groups with successively longer life spans (2-12 days), are initially anchored in successively deeper layers of the Sun.

3.2. Comparison of $\omega(t)$ with $\Omega(r)$

In Fig. 3 we show $\omega(t)$ as a function of age t for spot groups of life spans 10-12 days in latitude bins ($0^o - 10^o$), ($10^o - 20^o$) and ($20^o - 30^o$), obtained from data during 1874-1939. We notice the following :

(i) In each latitude interval, the rotation is accelerated during the *first* day, and in first two intervals the rotation is decelerated $after$ the 9th day, and

(ii) between the 2nd and the 9th days we see the following trends : (a) in latitudes ($0^o - 10^o$): neither acceleration nor deceleration, (b) in ($10^o - 20^o$): slow acceleration, (c) in ($20^o - 30^o$): slow deceleration.

We believe that the 'decreasing' trend of rotation rate with age of spot groups, reported by earlier authors was due to mixing of shorter and longer lived groups in the data sample for each given age. Such a mixture in data sample used, affects largely the initial phase of $\omega(t)$. This follows from the variation of $\omega_{ini}(\tau)$ shown in Fig 1.

In latitudes $0^o - 10^o$ the trend in $\omega(t)$ is generally similar to that found by Zuccarello (1993) from the GPR sunspot groups with life spans of 11 days during 1874-1976.

From Figs. 2 and 3, it can be seen that in each of the latitude intervals ($0^o - 10^o$), ($10^o - 20^o$) and ($20^o - 30^o$), the range of values of $\omega(t)$ is approximately same as that of $\Omega(r)$ across the convective envelope. In the interval $10^0 - 20^o$, even the trend of $\omega(t)$ is similar to that of $\Omega(r)$ across the convective envelope at 15^o. This may be interpreted as due to the rise of the magnetic structures of spot groups across the convective envelope as the spot groups grow older. (This is consistent with the inference that spot groups are 'anchored' in deeper layers when they are younger and in shallower layers when they are older: e.g., Schüssler 1987).

[The similarity between $\omega(t)$ and $\Omega(r)$ depends on the statistical distribution of the magnetic structure of the sunspot groups with respect to various factors, e.g., initial field strength, total magnetic flux, drag force, magnetic tension, etc. (e.g., Fan et al. 1993, and references therein). The decrease of the similarity of trends in latitude intervals $0^o - 10^o$ and $10^o - 20^o$ must be due to the effects of such factors. Which of these factors lead to maximum loss of similarity between $\omega(t)$ and $\Omega(r)$ is not clear.]

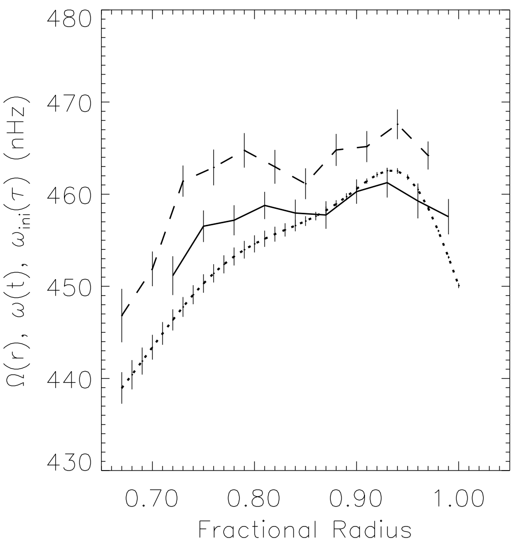

Fig. 4. Curves $\omega_{ini}(\tau)$ (dashed curve) in entire sunspot latitude belt and $\omega(t)$ (continuous curve) in latitude interval $10^o - 20^o$, plotted using values of r_1, k_1, r_2 and k_2 which yield *maximum correlations* with $\Omega(r)$ at latitude 15^o (dotted curve). From left to right the values of t are 1.5, 2.5,...,10.5 days, and those of τ are 11.5, 10.5,...,1.5 days respectively.

4. Estimation of the location of depths of initial anchoring and the rate of emergence of the magnetic structures

To estimate the initial anchoring depths and the rate of rise of the magnetic structures of the spot groups, we adopt working hypothesis that the relations between the initial radial location r_o and life sapn τ, and between the subsequent radial position r and age t are linear with :

$$r_o = r_1 + k_1\tau \ ,$$

$$r = r_2 + k_2 t \ ,$$

and maximize the correlations, between $\omega_{ini}(\tau)$ and $\Omega(r)$, and between $\omega(t)$ and $\Omega(r)$ as follwows. We choose various combinations of trial values for the coefficients r_1 and k_1 in wide ranges around the values visually estimated from Figs. 1 and 2. For each pair of the trial values of r_1 and k_1, we computed the correlation between $\omega_{ini}(\tau)$ and $\Omega(r)$. The pair which yields *maximum correlation* is taken as our best estimates of r_1 and k_1. The best estimates of r_2 and k_2 are also obtained by *maximizing* in a similar way the correlation between $\omega(t)$ and $\Omega(r)$.

In Fig. 4 we show the plots of $\omega_{ini}(\tau)$ for spot groups in the entire sunspot latitde belt, $\omega(t)$ in latitude interval $10^o - 20^o$, and $\Omega(r)$ at latitude 15^o, with the maximized correlations by a dashed curve, a continuous curve and a dotted curve, respectively.

The coefficients of the maximized correlations of $\omega_{ini}(\tau)$ in the latitude intervals $0^o - 10^o$, $10^o - 20^o$ and $20^o - 30^o$ to $\Omega(r)$ at their respective midpoints, are 0.868, 0.883 and 0.725 respectively. The coefficient of the maximized correlation in the entire latitude belt, is 0.908 and the corresponding 'r_o'-'τ' relation is as follows:

$$r_o(\tau) = (696.5 \pm 0.6) - (20.9 \pm 0.1)\tau. \qquad (1)$$

The coefficient of the maximized correlation between $\omega(t)$ of spot groups living 10-12 days in latitude interval $10^o - 20^o$ and $\Omega(r)$ at latitude 15^o is 0.863 for all 11 points and 0.911 for first 10 points. The 'r'-'t' relation corresponding the later value is as follows :

$$r(t) = (480.6 \pm 0.7) + (20.9 \pm 0.1)t. \qquad (2)$$

Here, $r_o(\tau)$ and $r(t)$ are in megameters (i.e., 10^3 km). The uncertainties in the intercepts and the slopes in Eqs. (1) and (2) are nominal uncertainties corresponding for uncertainties of 0.5 day in t and τ.

(We believe that the 87% and 88% correlations between $\omega_{ini}(\tau)$ and $\Omega(r)$ in the individual latitude intervals $0^o - 10^o$ and $10^o - 20^0$ indicate that the similarity of $\omega_{ini}(\tau)$ and $\Omega(r)$ is real. We have given here the values of r_1 and k_1 derived from the data in the entire sunspot latitude belt, because we had found that their values in individual latitude intervals do not differ by more than a few percent, and we do not want to claim that the differences are significant. We have considered the first 10 points of $\omega(t)$ because all of them represent the spot groups which live for 10-12 days, but the last point is for spot groups of life spans 11-12 days which are few in number. In the intervals $0^o - 10^o$ and $20^o - 30^o$, the coefficients of the maximized correlations are mere 0.49 and 0.21, respectively, and for the entire sunspot latitude belt it is 0.72.)

From Eq. (1) we infer that the initially the magnetic structures of sunspot groups of life spans $\leq$ 2 day are anchored at $r \sim 650$ Mm, i.e., near the Sun's surface and those of spot groups with successively longer life spans between 3 and 9 day are anchored at successively increasing depths at the rate $\sim$ 21 Mm/day (i.e., $\sim$ 240 m/s). The magnetic structures of the spot groups with $\tau > 9$ days are initially anchored at $r \leq 500000$ km.

For spot groups of life spans 10-12 days in latitudes $10^o - 20^o$, we infer from Eq. (2) that the magnetic structures are initially anchored at $r \sim 500000$ km (i.e. near the base of the convective envelope) and these structures rise across the envelope at the rate of $\sim$ 21 Mm/day (i.e., $\sim$ 240 m/s).

5. Discussion

The last conclusion (in Sect. 4) is consistent with the conventional belief that the 'main' mechanism of solar activity, (e.g. the one that generates spot groups living longer than 10 days), operates near the base of the convective envelope (see Rosner & Weiss 1992). The rate of rise of magnetic structures of spot groups having life span 10-12 days in latitudes $10^o - 20^o$, derived by us is consistent with the conclusion of Howard & LaBonte (1981) that the magnetic flux of an active region rises on a time scale of 10 days. Incidentally, in the model of Choudhuri & D'Silva (1990), the 10 day rise flux tube would correspond to initial strength $\sim 10^5$ G.

Whether the depths of the initial anchoring of the magnetic structures and the depths of their generation are same or different, is not clear. In either case, the 5-15 nHz difference between their rotation and the plasma rotation at that depth (cf. Sect. 3.1) may be due to 'differential buoyancy' (D'Silva & Howard 1994).

It must be noted that the results obtained here may represent only the average behavior of spot groups. The rotation rate of any single spot group will be affected by several processes such as splittings, expansions, proper motions, etc.. The data used by us in determining the rotation frequencies of spot groups extends over several decades whereas the helioseismic data extends over only about 4 yr, 1985 and 1988-1990, though the internal rotation of the Sun might vary with time besides depending upon depth and latitude (Gough et al. 1993; Hiremath 1994). Recurrent spot groups during their second and later appearances are not included in this study. It should be of interest to see how the rotation frequencies of spot groups living longer than 12 days vary in time.

[Note: preliminary results of this paper obtained using values of $\Omega(r)$ read from Dziembowski et al. (1989), with large uncertainties showed correlations only in the latitude interval $10^o - 15^o$ were presented in PRL Golden Jubilee Workshop (Javaraiah & Gokhale 1996).]

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