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THE LINEAR RESPONSE OF A MAGNETIC FLUX TUBE TO BUFFETING BY EXTERNAL *p*-MODES. I.

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ABSTRACT

The linear response of a thin vertical magnetic flux tube to buffeting by *p*-modes in the ambient atmosphere is examined with the aim of understanding the interaction of acoustic modes with sausage tube waves. The idealized case of an isothermal atmosphere is considered, which has the mathematical advantage that the differential equation for the vertical component of the Lagrangian displacement in the tube can be solved analytically. A lower boundary condition is employed that permits the tube wave to leak out through this boundary. This has the important consequence that the *p*-mode interaction with flux tubes does not exhibit a resonant behavior. The detailed behavior of the vertical displacement in the tube and its dependence on various parameters are examined. An equation for the wave energy in a thin flux tube is derived along with analytic expressions for the wave energy density and vertical energy flux. The variation of the tube response Ξ (defined as the ratio of the total wave energy, integrated over the length of the tube, to the *p*-mode energy in the external atmosphere) is investigated for different values of the dimensionless external horizontal wavenumber K_x, mode order n, and β, where β is the ratio of the gas to magnetic pressure in the tube that, by assumption, is constant with depth. It is found that when n is small, the response of the tube increases gradually with K_x until reaching a maximum, and thereafter it drops very sharply. As n increases, the maximum shifts to lower values of K_x. For fixed values of K_x and β, Ξ increases with n and then falls off after reaching a maximum. A similar dependence of Ξ on β is found. Line widths of *p*-modes are also calculated, and their dependence on K_x and frequency is studied. Finally, an application of the results to the solar atmosphere is discussed, and the limitations of the model are pointed out.

Subject headings: MHD — Sun: magnetic fields — Sun: oscillations

1. INTRODUCTION

Small-scale magnetic flux tubes in the solar atmosphere occur preferentially at the boundaries of supergranulation cells, outlined by the chromospheric Ca network, and in plage regions. They are important features of the solar atmosphere and have a significant influence on the structure and dynamics of the chromosphere and corona, as well as the solar wind. Their field strength is known empirically to be in the kilogauss range, and typically their diameters in the photosphere are believed to be in the range of 100–300 km.

In the photosphere, these vertical flux tubes are surrounded by a field-free medium. It is well known that this medium contains acoustic waves or *p*-modes, with typical periods in the 5 minute range. The purpose of this investigation is to consider some consequences arising from the buffeting action of the *p*-modes on flux tubes. Several years ago, Thomas, Cram, & Nye (1982) recognized the importance of 5 minute oscillations for probing the sunspot atmosphere. In recent years, observations have demonstrated that sunspots can absorb a large fraction (up to 50%) of the power in *p*-modes (Braun, Duvall, & Labonte 1987, 1988a; Bogdan et al. 1993; Bogdan & Braun 1995). Moreover, observations show that active regions exhibit absorption of *p*-modes also (Braun, Labonte, & Duvall 1988b; Braun & Duvall 1990). Thus, observations suggest clearly that there is a significant interaction between acoustic waves and magnetic flux tubes.

The interaction of *p*-modes with flux tubes has been investigated very extensively (for a recent review see Bogdan & Braun 1995 and references therein). Earlier work treated this interaction as a problem in classical scattering theory in order to model the shift in *p*-mode frequencies as a result of scattering off flux tubes (e.g., Bogdan & Zweibel 1985; Zweibel & Bogdan 1986; see the reviews by Bogdan 1992, 1994 and references therein). The details of the interaction are contained in the T matrix (for a definition of the T matrix see Morse & Feshbach 1953, chap. 11). For an unstratified flux tube embedded in a homogeneous medium, the poles of the T matrix describe scattering resonances, which occur when the frequency of the incident acoustic wave equals one of the natural tube frequencies. Resonant interactions of compact tubes (i.e., when the tube diameter is typically much less than the wavelength of the incident wave) have been studied with varying degrees of complexity by several authors (e.g., Ryutov & Ryutova 1976; Defouw 1976; Wilson 1980; Spruit 1982; Cally 1985; Bogdan 1989). This interaction results in a transfer of acoustic energy into flux-tube oscillations by a process that is akin to inverse Landau damping (Ryutova & Priest 1993). For thick flux tubes such as sunspots, *p*-modes can be absorbed in a resonant critical layer when the incident frequency equals the local cusp speed or Alfvén speed on a field line (Hollweg & Yang 1988). This process has been treated in considerable detail by numerous authors (e.g., Lou 1990; Sakurai, Goossens, & Hollweg 1991; Goossens & Poedts 1992; Keppens, Bogdan, & Goossens 1994). It is of some interest to ask whether the resonant behavior persists when the effects of gravitational stratification are taken into account. Recent calculations by Bogdan et al. (1996) and Hasan & Bogdan (1996) suggest that the resonances are eliminated effectively for thin flux tubes, when the tube is characterized by a continuous mode spectrum. This point will be discussed in further detail in § 7.

The object of this investigation is to analyze the response of a stratified thin flux tube when it is buffeted by p-modes from the ambient medium. The primary motivation is to understand the conditions under which a significant transfer of energy occurs from p-modes into flux-tube oscillations. In the most general case, this is best formulated as an initial-value problem, so that one can examine the buildup of energy in tube oscillations and also find out whether these oscillations last sufficiently long to play a significant role in heating the upper atmosphere. Clearly, the problem is rather complex and is best approached by making simplifying assumptions, which reveal the basic nature of the process. As a first step, let us work within the framework of linear theory and examine the asymptotic (in time) response of a flux tube when a p-mode is incident on it. For mathematical tractability, we assume that the atmosphere in the flux tube as well as in the ambient medium is isothermal. This approximation has the advantage that the differential equation for the vertical displacement in the tube can be solved analytically, thereby enabling us to understand the details of the interaction in a fairly straightforward way. The model that we examine is very similar to the one studied by Bogdan et al. (1996), who considered a polytropic stratification, which is a reasonable approximation for the atmosphere in the convection zone. However, their analysis is fairly involved from a mathematical point of view, although it may be more realistic. The present work sacrifices the latter advantage in favor of a more tractable mathematical treatment. Preliminary results using an isothermal model can be found in Hasan & Bogdan (1996). In this paper, a more comprehensive investigation is undertaken of the interaction of an isothermal flux tube with external p-modes, and it is demonstrated that the response of the tube is not uniform but increases with the horizontal wavenumber (for a fixed order) of the incident p-mode up to a maximum, followed by a sharp drop. This is related to the fact that for a large response, the eigenfunctions of the p-mode and of the excited tube wave need to remain in phase over a sufficient length of the tube.

The model that we adopt for p-mode absorption of energy is based on the mechanism proposed by Spruit (1991) (see also Spruit & Bogdan 1992 and the review by Spruit 1996), in which incident p-modes couple to internal tube waves. The latter can flow down unimpeded along the vertical magnetic field lines and thereby constitute a process by which p-mode energy is lost. This mechanism draws on the earlier work by Biront et al. (1982) and Roberts & Soward (1983) on oscillations of magnetic stars. Recently, it has been applied to sunspot fields by Cally, Bogdan, & Zweibel (1994) and to thin flux tubes by Bogdan et al. (1996).

The plan of the paper is as follows: the mathematical analysis begins in § 2, with the well-known linear equation for the interaction of a p-mode with a sausage wave in a thin flux tube, followed by § 3 on the p-mode solutions in the external atmosphere. The analytic solution for sausage waves in terms of the vertical displacement in the tube and its dependence on various parameters is given in § 4. In § 5, an equation for the wave energy in a thin flux tube is determined, along with expressions for the time-averaged energy density, flux, response, and p-mode line width. Results showing the dependence of the response, wave energy, and line width on different parameters are presented in § 6 and discussed in § 7. The main conclusions of the investigation are summarized in § 8.

2. LINEAR EQUATIONS

Let us consider a vertical flux tube surrounded by a field-free atmosphere. For simplicity, let us assume a uniform temperature everywhere. Furthermore, we adopt the thin flux-tube approximation (Defouw 1976; Roberts & Webb 1978). Stated briefly, this approximation consists of expanding all quantities about the axis. Assuming that the radial variation is small compared with, say, the pressure scale height, the MHD equations for an inviscid and infinitely conducting medium can be derived easily in the thin tube approximation. On linearizing these equations and assuming a time dependence of the form $e^{-i\omega t}$, the interaction of a thin flux tube with the external medium for the sausage mode is described by the following equation (Roberts 1983):

$$\left\{\frac{d^2}{dz^2} - \frac{1}{2H_0}\frac{d}{dz} + \frac{1}{c_{S,0}^2}\left[\omega^2\left(1 + \frac{\gamma\beta}{2}\right) - \frac{1}{2}\gamma\omega_{\rm BV}^2(1+\beta)\right]\right\}\xi_z = \frac{1+\beta}{2p_{e,0}}\left(\frac{d}{dz} + \frac{g}{c_S^2}\right)\Pi_e , \tag{1}$$

where z is the height in the atmosphere (positive upward), γ is the ratio of specific heats, g is the acceleration due to gravity, $\beta = 8\pi p_0/B_0^2$, ξ_z is the vertical component of the Lagrangian displacement in the tube, H_0 is the pressure scale height, $c_{S,0}$ is the sound speed, ω is the angular frequency of the perturbation, $\omega_{\rm BV} = g(\gamma - 1)^{1/2}/c_{S,0}$ is the Brunt-Väisälä frequency, p_0 is the internal gas pressure on the tube axis, B_0 is the magnitude, on the tube axis, of the vertical component of the magnetic field (assumed constant in the horizontal direction), $p_{e,0}$ is the external pressure, and Π_e is the (Eulerian) perturbation in the external pressure. The subscript 0 (which will hereafter be dropped) refers to quantities in the unperturbed atmosphere. We have assumed that β is independent of z, which follows from the assumption that the internal and external temperatures are the same.

Equation (1) can be put in canonical form by making the transformation $Q = \xi_z e^{-z/4H}$, which yields (Roberts 1983)

$$\frac{d^2Q}{dz^2} + \left(\frac{\omega^2 - \omega_V^2}{c_T^2}\right)Q = e^{-z/4H}\,\frac{1+\beta}{2p_e}\left(\frac{d}{dz} + \frac{g}{c_S^2}\right)\Pi_e , \tag{2}$$

where

$$\omega_V^2 = \omega_{\rm BV}^2 + \frac{c_T^2}{H^2}\left(\frac{3}{4} - \frac{1}{\gamma}\right)^2 , \tag{3}$$

and c_T is the tube speed (sometimes also referred to as the cusp speed), defined as

$$c_T^2 = \frac{c_S^2}{(1 + \gamma\beta/2)} . \tag{4}$$

We have assumed implicitly that the flux tube and external atmospheres are isothermal with the same temperature. The equilibrium pressure and magnetic field for an isothermal atmosphere are given, respectively, by

$$p(z) = p_0 e^{-z/H} , \tag{5}$$

$$B(z) = B_0 e^{-z/2H} , \tag{6}$$

where $p_0 \equiv p(0)$ and $B_0 \equiv B(0)$. Assuming that magnetic flux is conserved, then the cross section area of the tube A is given by

$$A(z) = \frac{B_0 A_0}{B(z)} = A_0 e^{z/2H} , \tag{7}$$

where $A_0 \equiv A(0)$. The above equation implies that the area of the flux tube increases with z with a scale height of $2H$. This means that the upper boundary must be chosen judiciously, so that the radius of the flux tube does not become larger than H, which would lead to a breakdown of the thin tube approximation.

The internal and external gas pressures are related to each other from the equation of horizontal pressure balance, which for a thin tube is

$$p + \frac{B^2}{8\pi} = p_e . \tag{8}$$

Dividing both sides in equation (8) by p and using the definition of β, we find

$$p = \frac{\beta}{\beta + 1} p_e . \tag{9}$$

Since β is a constant, the internal pressure at each depth is lowered with respect to the ambient pressure at the same depth by a fixed factor.

When $\Pi_e = 0$, equation (2) yields the dispersion relation

$$\omega^2 = k_z^2 c_T^2 + \omega_V^2 , \tag{10}$$

where k_z denotes the vertical wavenumber inside the tube. Equation (10) is the dispersion relation for a sausage wave in an isothermal flux tube. It may be noted that the wave is evanescent for $\omega < \omega_V$; thus ω_V represents the cutoff frequency inside the tube.

3. WAVES IN THE EXTERNAL MEDIUM

Let us now consider the solution of the linear wave equation in the external medium. We assume that we are dealing with a single p-mode with frequency ω, horizontal wavenumber $k_{x,e}$, and confined to a vertical cavity between $z = 0$ and $z = -D$, where D is the depth of the lower boundary. Clearly, for an isothermal atmosphere, there is no cavity. We assume this in order to mimic a situation where the upper reflection occurs because of a sudden increase in the acoustic cutoff frequency. The lower boundary is chosen below the Lamb depth, i.e., at a level where the p-mode is reflected by the increase in sound speed. This is idealized as a sudden increase in c_S at $z = -D$. Assuming that the vertical component of the displacement vanishes at the boundaries, it can be shown that $\xi_{z,e}$ has the form

$$\xi_{z,e} = Ce^{z/2H} \sin (k_{z,e} z) e^{i(k_{x,e}x - \omega t)} , \tag{11}$$

where x denotes the distance along the horizontal direction, C is a constant, and $k_{z,e}$ denotes the vertical wavenumber in the external atmosphere, which is related to $k_{x,e}$ by the dispersion relation

$$k_{z,e}^2 = \left(\frac{\omega^2 - \omega_{\rm ac}^2}{c_S^2}\right) - k_{x,e}^2\left(1 - \frac{\omega_{\rm BV}^2}{\omega^2}\right) , \tag{12}$$

where $\omega_{\rm ac} = \gamma g/2c_S$ is the acoustic cutoff frequency in the external atmosphere. Using equation (11), it is straightforward to show that the external pressure perturbation is given by

$$\frac{\Pi_e}{p_e} = -C \frac{e^{z/2H}}{H} \frac{\gamma\omega^2}{\omega^2 - c_S^2 k_{x,e}^2} \left[k_{z,e} H \cos (k_{z,e} z) + \left(\frac{1}{2} - \frac{1}{\gamma}\right) \sin (k_{z,e} z)\right] e^{i(k_{x,e}x - \omega t)} . \tag{13}$$

The horizontal component of the external displacement $\xi_{x,e}$ is related to Π_e as follows:

$$\xi_{x,e} = \frac{1}{\omega^2} \frac{\partial}{\partial x}\left(\frac{\Pi_e}{\rho_e}\right) = \frac{ik_{x,e}}{\omega^2} \frac{\Pi_e}{\rho_e} . \tag{14}$$

4. SAUSAGE WAVES IN THE TUBE

Once Π_e is known, it is fairly straightforward to solve equation (1) along with appropriate boundary conditions. For the lower boundary, we follow Cally & Bogdan (1993) and Bogdan et al. (1996) and demand that the solutions at large depths match onto downward propagating waves, so that $\xi_z \sim e^{-i(\omega t + k_z z)}$ as $z \to -\infty$. For the upper boundary condition, it is convenient to choose that ξ_z vanishes at $z = 0$. This boundary condition is chosen mainly for mathematical convenience since it ensures that the analysis remains tractable. Choosing a different boundary condition, such as the vanishing of the Lagrangian perturbation in pressure, could also be considered, although the analysis becomes somewhat more involved. However, the essential point is that the main findings of the investigation remain the same.

Our main aim is to develop the particular solution of equation (1), subject to the above boundary conditions. We start with the canonical form of this equation, namely, equation (2). The solution of this equation can be obtained conveniently in terms of the solutions of the homogeneous equation (2), which are $e^{\pm i k_z z}$. These homogenous solutions correspond to sausage waves in a thin tube obeying the dispersion relation given by equation (10). From the homogeneous solutions, the particular solution of equation (1) can be calculated using standard techniques and is given by (omitting the time dependence of the form $e^{-i\omega t}$, which is hereafter implied)

$$\xi_z = -C \frac{\gamma}{2c_S^2}(1+\beta)(\omega^2 - \omega_{\rm BV}^2) \frac{e^{2\alpha z}}{[(k_e^2 - k^2 + \alpha^2)^2 + 4k^2\alpha^2]} [(k_e^2 - k^2 - \alpha^2)\sin(k_e z) + 2k_e \alpha \cos(k_e z) - e^{-(ik+\alpha)z} 2k_e \alpha] , \tag{15}$$

where $k_e \equiv k_{z,e}$, $k \equiv k_z$, and $\alpha \equiv 1/4H$.

In the limit $z \to -\infty$, we find that

$$\xi_z = \frac{C}{4} \eta e^{(-iK + 1/4)z/H} , \tag{16}$$

where

$$\eta = \gamma(1+\beta)(\Omega^2 - \Omega_{\rm BV}^2) \frac{K_e}{[(K_e^2 - K^2 + 1/16)^2 + K^2/4]} , \tag{17}$$

$$K^2 = (\Omega^2 - \Omega_V^2)(1 + \gamma\beta/2) , \tag{18}$$

$$\Omega_V^2 = \Omega_{\rm BV}^2 + \frac{(3/4 - 1/\gamma)^2}{(1 + \gamma\beta/2)} , \tag{19}$$

$$\Omega = \frac{\omega H}{c_s} , \qquad \Omega_{\rm BV} = \frac{(\gamma - 1)}{\gamma^2} , \qquad K_e = k_e H, \qquad \text{and} \qquad K = kH . \tag{20}$$

For a p-mode, Ω is given by

$$\Omega^2 = \frac{1}{2}\left(K_x^2 + K_e^2 + \frac{1}{4}\right)\left(1 + \sqrt{1 - \frac{4K_x^2 \Omega_{\rm BV}^2}{K_x^2 + K_z^2 + 1/4}}\right) , \tag{21}$$

where $K_x = k_{x,e} H$.

Thus, at large depths, η is a measure of the amplitude of ξ_z. We shall now examine in some detail the behavior of η. The general form of η with Ω given by equation (21) is rather complicated and not easily comprehensible. However, the expression for η becomes more amenable to analysis in the limit

$$\Omega \gg \Omega_{\rm BV} \quad \text{or} \quad K_x \ll K_e , \tag{22}$$

when equation (21) simplifies to

$$\Omega^2 \approx K_x^2 + K_e^2 + \tfrac{1}{4} . \tag{23}$$

Substituting equation (23) into equation (17), we obtain, after some algebra,

$$\eta = \frac{240(1+\beta)K_e[1 + 100(K_x^2 + K_e^2)]}{(1 + 100K_e^2)[(6+\beta)^2 + 100\beta^2 K_e^2)] + 40(6 + 5\beta)(6 + \beta + 100\beta K_e^2)K_x^2 + 400(6 + 5\beta)^2 K_x^4} , \tag{24}$$

where we have assumed that $\gamma = 5/3$.

The approximate relation for η given by equation (24) turns out to be very useful in understanding the response of the flux tube to external p-modes.

4.1. *Variation of η with K_x*

It is convenient first to consider the asymptotic behavior of η in the limits $K_x \to 0$ and $K_x \to \infty$. Let us first examine the former limit, when equation (24) reduces to

$$\eta = \frac{240(1+\beta)K_e}{(6+\beta)^2 + 100\beta^2 K_e^2} + O(K_x^2) . \tag{25}$$

Thus, to leading order, η is constant with K_x as $K_x \to 0$. The lowest order correction to η varies as K_x^2.

In the opposite limit, corresponding to $K_x \to \infty$, equation (24) yields

$$\eta = \frac{60(1+\beta)K_e}{(6+5\beta)^2 K_x^2} . \tag{26}$$

Thus, as $K_x \to \infty$, η varies as $1/K_x^2$, which shows that the response becomes vanishingly small at large values of K_x. From equations (25) and (26), it would thus appear that η should have a maximum for some value of K_x, assuming that the other parameters remain fixed. It should be pointed out that the limit $K_x \to \infty$ is not incompatible with equation (22) and hence equation (24). This is because only one of the two inequalities in equation (22) needs to be satisfied, which for large K_x corresponds to $\Omega \gg \Omega_{BV}$.

It turns out that to calculate, with reasonable accuracy, the value of K_x at which η is maximum, we need to use equations (17) and (21). This is because the maximum of η occurs at nonzero values of K_x only for low mode orders, for which the approximation $K_x \ll K_e$ is not strictly appropriate. We omit the algebra, which is messy, and instead present the results of the calculation graphically. Figure 1 depicts the variation of $K_{x,\max}$ (the value of K_x for which η is maximum) with K_e for different values of β (which label the curves). The upper horizontal axis denotes the mode order (assuming $D = 40H$) corresponding to K_e. For a fixed value of β, we find that as K_e increases, $K_{x,\max}$ decreases, finally becoming zero when K_e is large enough. Thus, when K_e is sufficiently large, the maximum of η occurs at $K_x = 0$. The effect of reducing β is to increase the range of K_e or mode orders for which $K_{x,\max}$ is finite.

Since the maximum of η occurs typically for small values of K_x, it is convenient to examine analytically the dependence of η on K_e and β in the limit $K_x \to 0$.

4.2. *Variation of η with K_e*

We now investigate the variation of η with K_e when $K_x \to 0$ and β is fixed. From equation (25), we find that for small values of K_e, η varies linearly with K_e, whereas for large values of K_e, $\eta \sim 1/K_e^2$. The maximum value of η is given by

$$\eta_{\max} = \frac{12}{\beta}\frac{1+\beta}{6+\beta} , \tag{27}$$

and occurs when

$$K_{e,\max} = \frac{6+\beta}{10\beta} . \tag{28}$$

The above expressions are clearly not valid when $\beta = 0$. In the latter limit, equation (25) yields $\eta = 20/3K_e$, which shows that η increases monotonically with K_e. However, when $\beta > 0$, we find that $K_{e,\max}$ decreases as β increases. The minimum value of $K_{e,\max}$ is 0.1 and occurs for $\beta \to \infty$.

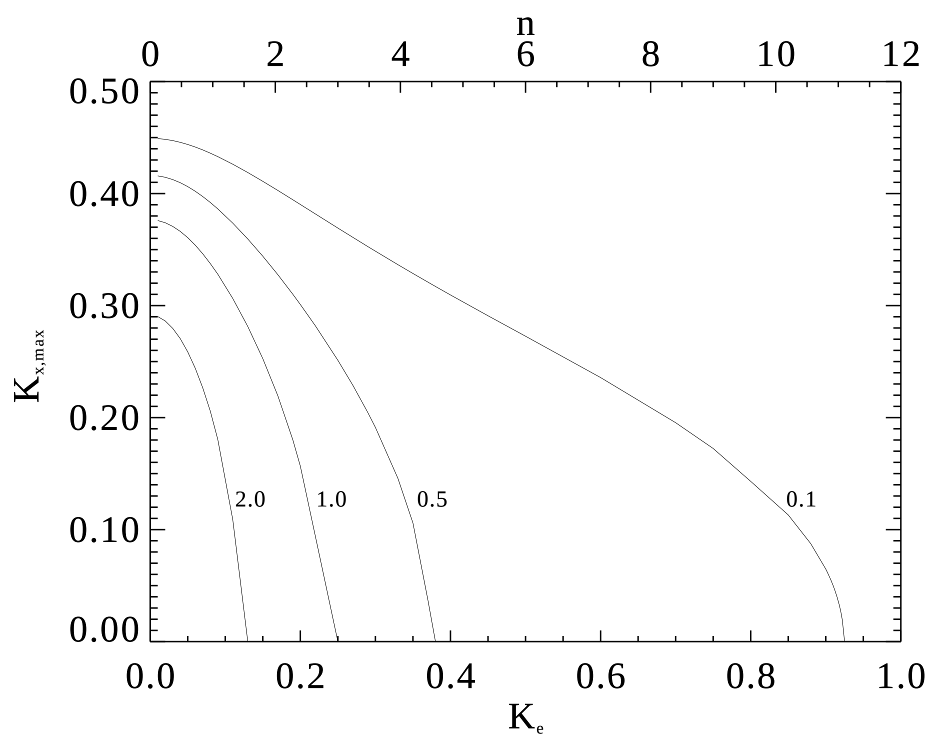

FIG. 1.—Variation of $K_{x,\max}$ with K_e (where $K_{x,\max}$ and K_e are in dimensionless units) for different values of β, which are used to annotate the curves. The top axis gives the order n of the p-mode corresponding to the vertical wavenumber K_e, assuming $D = 40H$.

4.3. *Variation of η with β*

Finally, let us consider the variation of η with β when $K_x \to 0$ and K_e is fixed. Using equation (25) once again, we find that as $\beta \to 0$, η is constant to leading order. The first-order correction scales linearly with β. On the other hand, for very large values of β, $\eta \sim 1/\beta$.

The maximum value of η occurs for

$$\beta_{\max} = -1 + \sqrt{1 + \frac{24}{1 + 100K_e^2}} . \tag{29}$$

Thus, as K_e increases, $\beta_{\max}$ decreases. In the limit $K_e \to 0$, we find from equation (29) that $\beta_{\max} \to 4$. As K_e increases, $\beta_{\max}$ decreases. For $K_e \to \infty$, $\beta_{\max} \to 0$; thus, $\beta_{\max}$ lies in the range 0–4.

Having obtained the linear displacement in the tube and its asymptotic form at large depths in terms of η, let us now go on to discuss the form of the energy density and energy flux associated with the tube oscillations. However, before doing that, we require an energy equation for waves in a thin flux tube.

5. WAVE ENERGY IN A THIN FLUX TUBE

The equation for the wave energy in a stratified atmosphere with a uniform magnetic field is well known and can be found, for instance, in the monograph by Bray & Loughhead (1974, chap. 6). The generalization of this equation for a thin flux tube, in which the magnetic field varies in the vertical direction, does not appear to have been discussed earlier. Let us now consider such an equation for a sausage wave in a thin flux tube when the latter is buffeted by a p-mode from the external medium. Starting from the thin flux-tube equations given by Roberts & Webb (1978), and following the procedure adopted in Bray & Loughhead (1974), it can be shown that the wave energy density in a thin flux tube satisfies the following equation:

$$\frac{\partial}{\partial t}(AE) + \frac{\partial}{\partial z}(AF_z) = -\Pi_e \frac{\partial\, \delta A}{\partial t} , \tag{30}$$

where E is the total wave energy density, F_z is the vertical wave energy flux, A is the unperturbed area of the tube, and δA is the area perturbation. The right-hand side term is a source term driving the tube oscillations and represents the work done by the external p-mode in compressing the tube. The wave energy density E and vertical energy flux F_z for a sausage wave are, respectively,

$$E = \frac{1}{2}\rho(\delta v_z)^2 + \frac{1}{2}\frac{(\delta p)^2}{\rho c_S^2} + \frac{1}{8\pi}(\delta B_z)^2 + \frac{1}{2}\rho\omega_{\rm BV}^2 \xi_z^2 , \tag{31}$$

$$F_z = \delta p\, \delta v_z , \tag{32}$$

where ρ is the internal gas density, δp is the pressure perturbation, δB_z is the perturbation in the vertical component of the magnetic field, and δv_z is the vertical velocity perturbation. In the above equations, the horizontal components of the velocity and magnetic field do not enter since they are assumed to be much smaller than their vertical counterparts, within the framework of the thin flux-tube approximation. In equation (31), the first term corresponds to the kinetic energy, the second to the internal energy, the third to the magnetic energy, and the last to the gravitational energy. It should be pointed out that equation (30) expresses the conservation of wave energy and should be distinguished from the nonlinear equation for the total energy density in a thin flux tube, such as the one derived by Spruit (1979).

5.1. *Time-averaged Wave Energy Density and Response*

Let us now consider the time-averaged total energy density $\langle E\rangle$ of the oscillations excited in the tube through the buffeting action of the external p-modes. Noting that the perturbed quantities are complex, it can be easily shown that $\langle E\rangle$ can be expressed as

$$\langle E\rangle = \frac{1}{4}\rho\omega^2 \xi_z \xi_z^* + \frac{1}{4}\frac{\delta p\, \delta p^*}{\rho c_S^2} + \frac{1}{16\pi}\delta B_z\, \delta B_z^* + \frac{1}{4}\frac{g^2}{c_S^2}\rho(\gamma - 1)\xi_z \xi_z^* , \tag{33}$$

where we have made use of the fact that $\delta v_z = -i\omega\xi_z$.

Let us now express all perturbed quantities in terms of ξ_z. It can be shown that the perturbations in pressure and vertical magnetic field are given, respectively, by

$$\frac{\delta p}{p} = \frac{1}{2 + \gamma\beta}\left[-2\gamma\frac{d\xi_z}{dz} + \frac{\gamma g}{c_S^2}(2 - \gamma)\xi_z + \gamma(\beta + 1)\frac{\Pi_e}{p_e}\right], \tag{34}$$

and

$$\frac{\delta B_z}{B} = \frac{1}{2 + \gamma\beta}\left[(\beta + 1)\frac{\Pi_e}{p_e} + \gamma\beta\frac{d\xi_z}{dz} - \frac{g}{c_S^2}\gamma\beta(2 - \gamma)\xi_z\right]. \tag{35}$$

It is convenient to recast equation (33) in the following form:

$$\langle E\rangle = \frac{\beta}{\beta + 1}\frac{p_e}{4}\left[\gamma(\Omega^2 + \Omega_{\rm BV}^2)\frac{\xi_z}{H}\frac{\xi_z^*}{H} + \frac{1}{\gamma}\frac{\delta p}{p}\frac{\delta p^*}{p} + \frac{2}{\beta}\frac{\delta B_z}{B}\frac{\delta B_z^*}{B}\right]. \tag{36}$$

In equation (36), the kinetic and gravitational energies have been put together, since they have the same functional dependence and differ only by a constant factor (the Brunt-Väisälä frequency). It is instructive to consider the asymptotic form of the tube energy at large depths, for which ξ_z is given by equation (16). The contribution due to the external pressure perturbation in equations (34) and (35) can be neglected as $\Pi/p_e \sim e^{z/2H}$, whereas $\xi \sim e^{z/4H}$. Thus, as $z \to -\infty$, Π/p_e is very much smaller than ξ_z. In this limit, we find that equations (34) and (35) become, respectively,

$$\frac{\delta p}{p} \to \frac{\eta}{2+\gamma\beta}\frac{C}{8H}[(4-3\gamma)+4i\gamma K]e^{-(iK+1/4)z/H}, \tag{37}$$

and

$$\frac{\delta B_z}{B} \to \frac{\eta\beta}{2+\gamma\beta}\frac{C}{16H}[(5\gamma-8)-4i\gamma K]e^{-(iK+1/4)z/H}. \tag{38}$$

Substituting equations (37) and (38) into equation (36), we find that the asymptotic form for the total energy density is given by

$$\langle E\rangle \to \frac{p_e}{512}e^{z/2H}\frac{\beta}{\beta+1}\frac{C^2}{H^2}\frac{\eta^2}{\gamma}\left\{8\gamma^2(\Omega^2+\Omega_{\rm BV}^2)+\frac{2}{(2+\gamma\beta)^2}[(4-3\gamma)^2+16\gamma^2K^2]+\frac{\gamma\beta}{(2+\gamma\beta)^2}[(5\gamma-8)^2+16\gamma^2K^2]\right\}. \tag{39}$$

The first term in equation (39) is the sum of the kinetic and gravitational energies, the second term is the internal energy, and the last term is the magnetic energy associated with the wave in the flux tube. Substituting the value of K from equation (18) and taking $\gamma = 5/3$, equation (39) simplifies to

$$\langle E\rangle \to \frac{p_e}{192}e^{z/2H}\frac{\beta}{\beta+1}\frac{C^2}{H^2}\eta^2\left[10\Omega^2-\frac{1}{12}\frac{\beta}{(1+5\beta/6)}\right]. \tag{40}$$

In equation (40), the second term in brackets involving β is very much smaller than the first term for p-mode frequencies and can be neglected for all practical purposes. If ϵ denotes the time-averaged total energy per unit length of the tube, defined as $\epsilon = \langle E\rangle A$, then ϵ tends to a constant value at large depths since $A \sim e^{z/2H}$ and $p_e \sim e^{-z/H}$. Furthermore, for large depths, it is easy to see from equation (39) that the magnetic and internal energies are comparable (for $\beta \sim 1$). The ratio of the kinetic energy to the internal energy is

$$\left(1+\frac{5}{6}\beta\right)\frac{\Omega^2}{\Omega^2-\Omega_{\rm BV}^2} \to \left(1+\frac{5}{6}\beta\right) \quad \text{for} \quad \Omega \gg \Omega_{\rm BV}. \tag{41}$$

We now consider the response of the tube when it is buffeted by a p-mode in the external atmosphere. Let us define the response Ξ as follows:

$$\Xi = \frac{\int dz\,\epsilon}{\int dz(\langle E_e\rangle A_e)}, \tag{42}$$

where A_e denotes the area in the horizontal plane in the external medium, and $\langle E_e\rangle$ is the time-averaged energy density associated with the p-modes. For simplicity, we assume that A_e does not vary in the vertical direction. The time-averaged external energy density is given by

$$\langle E_e\rangle = \frac{p_e}{4}\left\{\frac{\gamma}{c_S^2}[\omega^2(\xi_{z,e}\xi_{z,e}^*+\xi_{x,e}\xi_{x,e}^*)+\omega_{\rm BV}^2\xi_{z,e}\xi_{z,e}^*]+\frac{1}{\gamma}\frac{\Pi_e}{p_e}\frac{\Pi_e^*}{p_e}\right\}. \tag{43}$$

Substituting the value of $\xi_{x,e}$ from equation (14) into the above equation, we find

$$\langle E_e\rangle = \frac{p_e}{4}\left[\gamma(\Omega^2+\Omega_{\rm BV}^2)|\xi_{z,e}|^2+\frac{1}{\gamma}\left(1+\frac{K_x^2}{\Omega^2}\right)\left|\frac{\Pi_e}{p_e}\right|^2\right]. \tag{44}$$

We see from equations (11) and (13) that $\langle E_e\rangle$ is independent of z.

5.2. *Energy Flux and p-Mode Line Width*

Using equation (32), it can be shown that the time-averaged vertical energy flux in a thin flux tube is given by

$$\langle F_z\rangle = \tfrac{1}{4}(\delta p\,\delta v_z^* + \delta p^*\,\delta v_z). \tag{45}$$

Noting that $\delta v_z = -i\omega\xi_z$ and using equation (34), we find that

$$\langle F_z\rangle = \frac{\beta}{(\beta+1)(2+\gamma\beta)}\frac{i\omega p_e}{4}\left[\gamma(\beta+1)\frac{\Pi_e}{p_e}-2\gamma\frac{d\xi_z}{dz}+\frac{\gamma g}{c_S^2}(2-\gamma)\xi_z\right]\xi_z^*+cc, \tag{46}$$

where cc stands for the complex conjugate of the first expression.

In equation (46), the first term in brackets, which represents the contribution due to the external pressure perturbation, can be neglected compared with the other terms at large depths, since $\Pi_e/p_e \sim e^{z/2H}$, whereas $\xi_z \sim e^{z/4H}$. Thus, as $z \to -\infty$, Π_e/p_e

is very much smaller than ξ_z. In this limit, we find that

$$\langle F_z \rangle \to -\frac{1}{32}\frac{C^2}{H^2}\frac{\gamma\beta}{(\beta+1)}\eta^2\Omega\sqrt{\frac{(\Omega^2-\Omega_V^2)}{(1+\gamma\beta/2)}}\,p_e e^{z/2H}\,. \tag{47}$$

From equation (47), it is evident that the average power going down the tube $\langle F_z \rangle A$ tends to a constant value, independent of depth.

Following Bogdan et al. (1996), the average power going down the tube is related to the p-mode line width Γ through the relation

$$\Gamma = -\frac{1}{2\pi}\frac{\langle F_z \rangle A}{\langle E_e \rangle A_e}\,. \tag{48}$$

6. RESULTS

In the previous sections, expressions have been derived for various quantities, such as the time-averaged wave energy density and flux, in terms of the vertical displacement in the tube. Let us now consider the results of calculations that illustrate in greater detail the dependence on various parameters of the total energy density and vertical energy flux associated with sausage tube waves. The solid curves in Figure 2 shows Ξ/f, where $f = A_0/A_e$ denotes the magnetic filling factor, as a function of the dimensionless horizontal wavenumber K_x ($K_x \equiv k_{x,e}H$) in the external atmosphere for p-modes of different order, assuming $\beta = 0.5$ and $D = 40H$, where D is the depth of the lower boundary. Without loss of generality, we choose $C = H$. Hereafter, unless specified otherwise, these will be the default parameters. The dotted curves denote the frequencies (with reference to the right axis) of the p-modes in the external medium. These modes have a discrete spectrum because of the assumption that the vertical displacement vanishes at the top and bottom boundaries. Clearly, the p-modes have frequencies above the acoustic cutoff frequency, which, in our dimensionless units, is 0.5.

For a fixed order, corresponding to a specific value of the dimensionless vertical wavenumber K_e ($K_e \equiv k_{z,e}$), we find that the response is almost flat for $K_x \ll 1$. For low orders, Ξ/f shows a gradual increase with K_x until it reaches a maximum, and thereafter it decreases. This maximum value increases with n, whereas the value of K_x at which the maximum occurs decreases as n increases. For large enough n, the maximum of Ξ/f is at $K_x = 0$. Furthermore, whereas the response Ξ/f for small K_x increases with mode order, the asymptotic value of Ξ/f (other than for $n = 1$) appears to be independent of n. We shall now attempt to understand these features based upon the analysis in the preceding sections.

Let us first examine the response for small values of K_x. In order to understand its behavior, it is instructive to examine the depth variation in the tube of the total energy density and its various constituents, in order to see which one dominates the energy budget at various depths for small values of K_x. Figure 3 shows the variation with z/H of $\epsilon/A_0 p_0$ (*solid curve*) for $K_x = 0.1$ and $\beta = 0.5$, when the tube is buffeted by a p_3 mode from the external atmosphere. The curve has been normalized with respect to the maximum value of $\epsilon/A_0 p_0$ over the interval. The dashed curves denote the depth variation of the different components, which we use to label the curves. Let us first consider the solid curve corresponding to the total energy density.

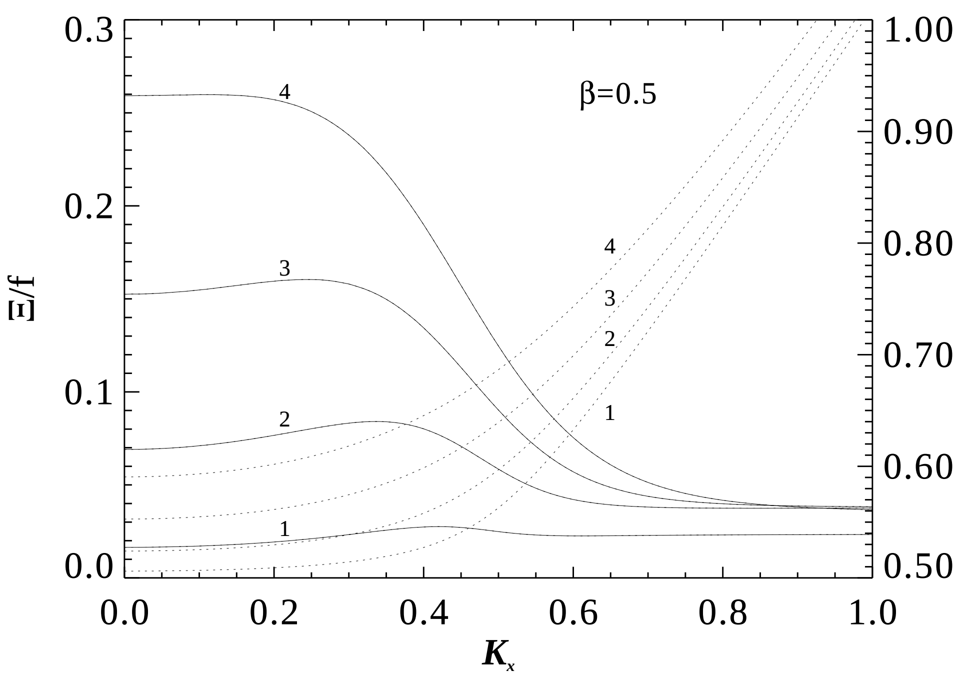

FIG. 2.—Dependence of the response Ξ/f (*solid curves*) on the p-mode dimensionless horizontal wavenumber K_x for different mode orders n, which label the curves, assuming $D = 40H$ and $\beta = 0.5$ (hereafter the default parameters). The dotted curves denote the variation of the p-mode dimensionless frequency Ω with K_x for different values of n.

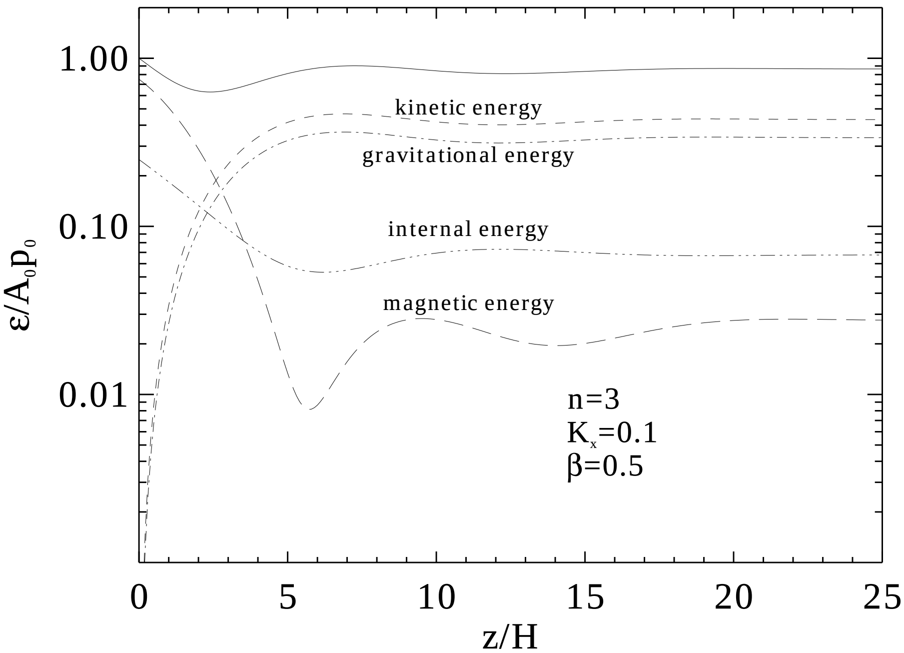

FIG. 3.—Variation with z/H of the dimensionless total energy density (*solid curve*) $\epsilon/A_0 p_0$ in sausage waves at each depth on the tube axis, for a p_3 mode with $K_x = 0.1$. The other curves denote the various constituents of the wave energy density. All curves have been normalized with respect to the maximum of $\epsilon/A_0 p_0$ in the interval.

We find that in the upper layers of the tube, the amplitude variation essentially reflects the driving action of the p_3 mode in the ambient medium. As z increases, the amplitude decreases while the wavelength increases. At very large depths, $\epsilon/A_0 p_0$ approaches a constant value, and the amplitude modulation is barely noticeable. Let us now consider the form of the kinetic and gravitational energy per unit length. As mentioned in § 5.1, the two differ by a constant factor $(\Omega^2 - \Omega_{\rm BV}^2)$ and have the same z-dependence. We find that other than in the layers close to the upper boundary, the contribution of the internal and magnetic energy to the total energy is very small. Since the main contribution to the integrated energy density in the tube comes from the deep layers of the tube, we can use the expression for $\langle E \rangle$ at large depths to examine the response when K_x is small. Using equation (40), we find that

$$\epsilon \sim \eta^2 \Omega^2 \; , \tag{49}$$

and from equation (44) (assuming $\Omega \gg \Omega_{\rm BV}$),

$$\langle E_e \rangle \sim \Omega^2 + O(K_x)^2 \qquad \text{as} \qquad K_x \to 0 \; . \tag{50}$$

Substituting equations (49) and (50) in equation (42), and recalling from equation (25) that $\eta \sim$ constant $+ O(K_x^2)$, we find that

$$\Xi/f \sim \text{constant} \qquad \text{as} \qquad K_x \to 0 \; . \tag{51}$$

The constant in equation (51) is proportional to K_e^2 or n^2. Actually, this is only true as long as K_e is sufficiently small. We defer a further discussion of this aspect until later. The decrease in the value of K_x at which Ξ has a maximum with increasing n is due to reasons similar to those discussed in § 4.1.

Let us now examine the behavior of Ξ/f as K_x increases. The dependence of ϵ on K_x at large depths can be expressed roughly using equations (26) and (39) as $\epsilon \sim \eta^2\Omega^2 \sim K_x^{-2}$ when K_x is sufficiently large. On the other hand, it is not difficult to see from equations (13), (23), and (44) that for large K_x, $\langle E_e \rangle \sim \Omega^4 \sim K_x^4$, and consequently from equation (42), we find that $\Xi \sim K_x^{-6}$. This dependence is valid as long as $\Omega \gg \Omega_{\rm BV}$ or $K_x \gg K_e$. Clearly, for small values of K_x and K_e, the above assumptions are not satisfied, which is the reason why, for instance, the curve corresponding to $n = 1$ looks different from the others.

In order to understand why the asymptotic value of Ξ/f is practically independent of K_e or n, it is instructive to look at the variation of the energy density with depth. Figure 4 shows the variation of $\epsilon/A_0 p_0$ as a function of z/H for $n = 3$ and $n = 4$, assuming $K_x = 1.0$ and $\beta = 0.5$. The dashed and dot-dashed curves denote, respectively, the contributions due to the internal and magnetic energy densities. The kinetic energy density (as well as the gravitational energy density) have not been plotted since they are negligible for this case. We find that the main contribution to the integrated energy density comes from the top layers of the flux tube. In these layers, the magnetic and internal energies dominate the energy budget. Furthermore, it is not difficult to see that for $z \approx 0$, $\delta p/p \sim \Pi_e/p_e$ and $\delta B/p \sim \Pi_e/p_e$. Similarly, for K_x sufficiently large, the external energy density varies roughly as $|\Pi_e/p_e|^2$, so that from equations (40) and (42), we find that Ξ/f is practically independent of n.

Figure 5 shows the response as a function of β when the flux tube is buffeted by p-modes of different order n (used to label the different curves), for a fixed value of the horizontal wavenumber $K_x = 0.1$. The results do not depend upon the precise value of K_x, as long as it is sufficiently small, for reasons that have already been pointed out in the previous paragraphs. The

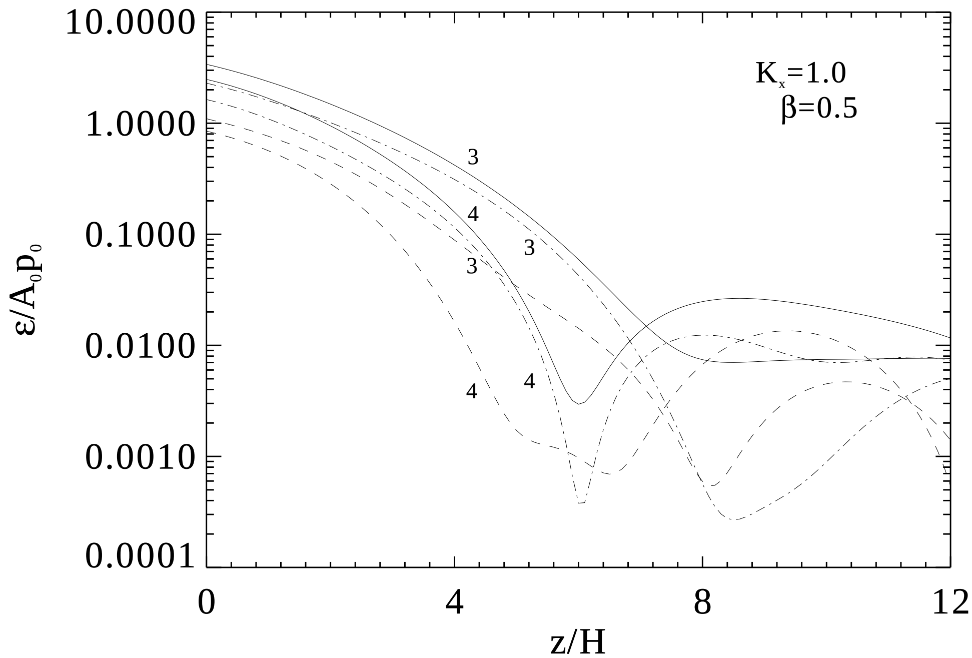

FIG. 4.—Variation with z/H of the dimensionless total energy density (*solid curve*) $\epsilon/A_0 p_0$ in sausage waves at each depth on the tube axis, for p_3 and p_4 modes with $K_x = 1.0$. The dashed and dot-dashed curves denote the contributions due to the internal and magnetic energy, respectively.

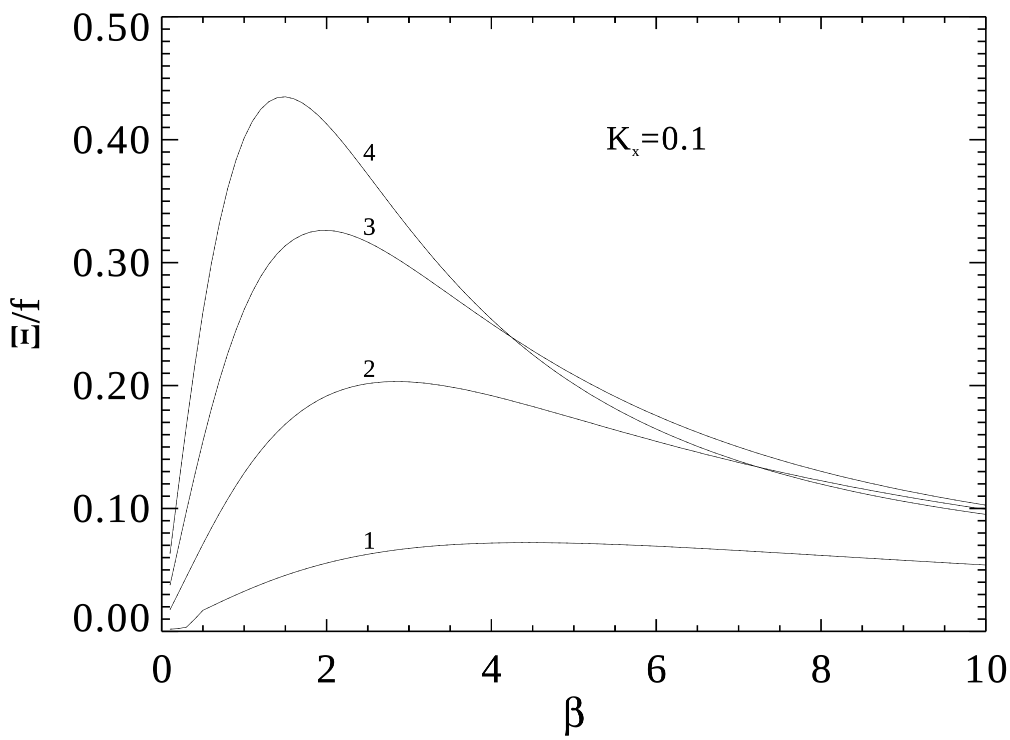

FIG. 5.—Dependence of the response Ξ/f on β for different mode orders n, which label the curves, assuming $K_x = 0.1$

most evident feature from the figure is that the response does not increase monotonically with β but has a maximum, as was noted in § 4.3, which shifts to lower values of β as n increases. When $K_x \to 0$, equation (49) yields

$$\frac{\Xi}{f} \sim \frac{\beta}{\beta + 1} \eta^2 \qquad \text{as} \qquad K_x \to 0 , \tag{52}$$

where η is given by equation (25). It can be shown that the value of β for which Ξ has a maximum is given by the solution of the following cubic equation:

$$2(1 + 100K_e^2)\beta^3 + 3(1 + 100K_e^2)\beta^2 - 60\beta - 36 = 0 . \tag{53}$$

It turns out that the value of β_{max} obtained from equation (53) is somewhat larger than the one given by equation (29). Equation (53) predicts that the largest value of β_{max} is 5.09, which occurs when $K_e = 0$, in contrast to $\beta_{max} = 4$ found from equation (29). From equation (53), we can easily verify that β_{max} decreases as K_e increases and that the values of β_{max} agree with those found from Figure 5.

We now consider the variation of Ξ/f with K_e, for different values of β when K_x is small. This is shown in Figure 6 by the solid curves, where the values of β are used to label the curves. The dotted curve shows the variation of the p-mode frequency. We have assumed the default parameters and have chosen $K_x = 0.1$, similar to the previous case. We find once again that the response does not increase monotonically but goes through a maximum, and thereafter it decreases with K_e. Also, the maximum value of the response maximum increases as β decreases. In order to understand this, let us recall that $\Xi \sim \eta^2$ for $K_x \to 0$ (eq. [52]), with η given by equation (25). Thus, the maximum of Ξ and η occurs at the same value of K_e, which is given by equation (28).The values of $K_{e,\max}$ decrease as β increases and are in excellent agreement with those found from Figure 6. Substituting equation (27) into equation (40), we find that the response has the following β-dependence:

$$\frac{\Xi_{max}}{f} \sim \frac{1+\beta}{\beta(6+\beta)^2} \qquad \text{as} \qquad K_x \to 0 \,, \tag{54}$$

where Ξ_{max} is the maximum value of the response. In view of the above, Ξ_{max} decreases as β increases. This situation is in contrast to the one discussed in the preceding paragraph, where we found that for sufficiently small values of K_e, the response increases at first with β and subsequently decreases. We find, therefore, that the value of K_e, or the mode order, influences the β-dependence of the response.

Finally, let us consider the dependence of the line width on K_x and Ω. Let us recall from § 5.2 that Γ is the ratio of the time-averaged power going down the tube (which approaches a constant at large depths) to the time-averaged p-mode energy density in the external medium. Figure 7 shows $\Gamma/\Omega f$ as a function of K_x for the $n = 2$ mode for two values of β, which are used to annotate the curves. The values of Ω^2 corresponding to K_x are given on the top horizontal axis. We find that the general behavior of the curves is very similar: a gradual increase of $\Gamma/\Omega f$ with K_x at low values of the horizontal wavenumber to a maximum and then a sharp falloff with K_x. For small K_x, the line width increases with β. Let us first understand the variation of the line width with K_x by examining the time-averaged energy flux at large depths, given by equation (47). For $K_x \to 0$, $\Omega \to K_e^2 + \frac{1}{4}$ (see eq. [23]), and η is given by equation (25). Thus, in this limit, $\langle F_z \rangle$ is independent of K_x. Using equation (50), it is easily verified that for small K_x, $\Gamma/\Omega f \sim$ constant $+ O(K_x^2)$. For large K_x, it can also be shown that $\Gamma/\Omega f \sim K_x^{-7}$.

Let us now consider the effect of changing β on the line width. From equation (47), we find that the β-dependence of $\langle F_z \rangle$ can be expressed as

$$\langle F_z \rangle \sim \eta^2 \frac{\beta}{(\beta+1)\sqrt{1+\gamma\beta/2}} \sim (1+\beta) \frac{\beta}{\sqrt{1+\gamma\beta/2}} \,, \tag{55}$$

where we have considered the limit $K_x \to 0$ and used equation (25) for η, assuming that $\beta \sim 1$. In this limit, $\langle F_z \rangle$ and hence Γ

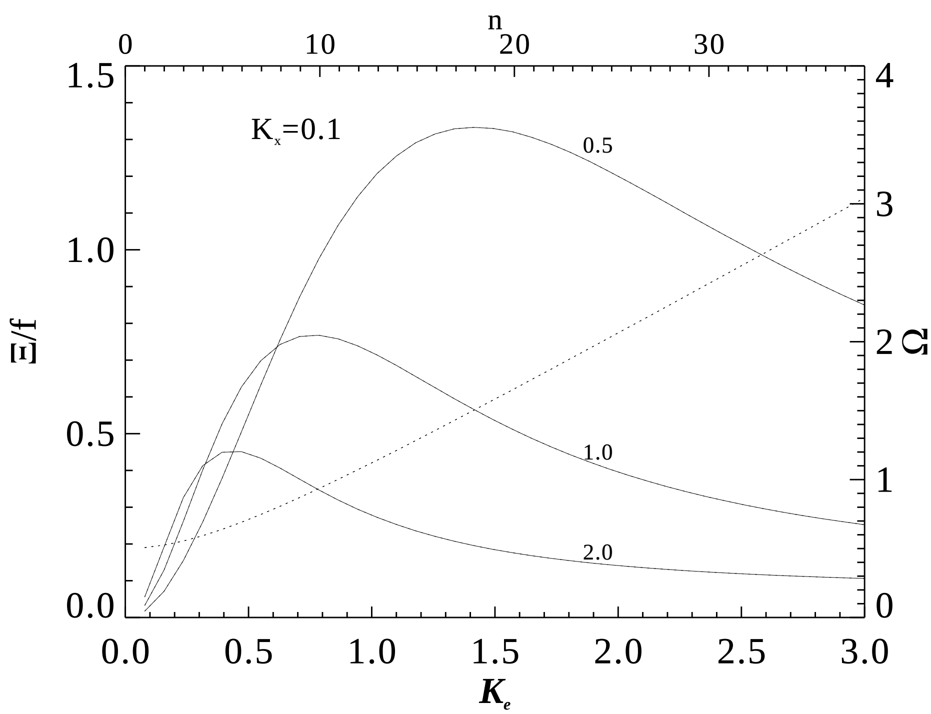

FIG. 6.—Variation of the response Ξ/f (*solid curves*) with K_e for different values of β (which label the curves), assuming $K_x = 0.1$. The upper horizontal axis denotes the values of n corresponding to K_e. The dotted curve denotes the frequency variation of the p-mode when K_x is fixed.

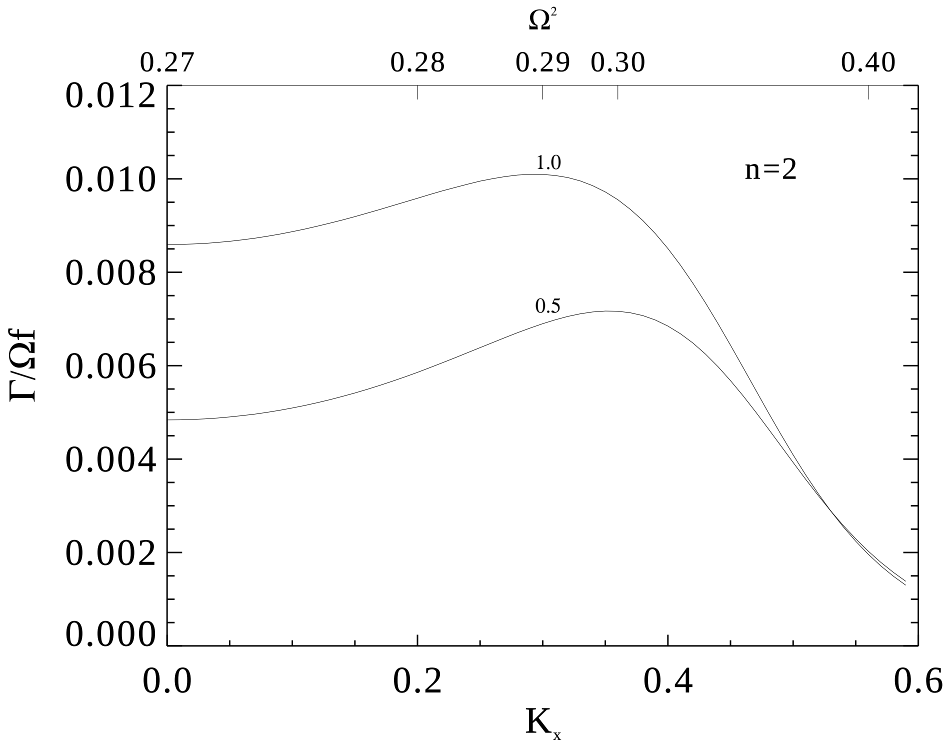

FIG. 7.—Dependence of the line width $\Gamma/\Omega f$ on the horizontal wavenumber K_x and frequency Ω^2 (*upper horizontal axis*) for different values of β, which label the curves, assuming $n = 2$.

increase with β. However, when β becomes sufficiently large, then $\eta \sim 1/\beta$, and it follows that $\langle F_z \rangle \sim \beta^{-5/2}$.

7. DISCUSSION

We now discuss some of the main features of our calculations, keeping in mind the limitations of the equilibrium isothermal atmosphere, which is at best reasonable for a few scale heights in the photosphere, but not in the convection zone. An important finding of the present investigation is that no resonance condition appears to exist that would lead to the tube having an infinite or sharply peaked response for a certain choice of parameters. For the unstratified case, a resonance can occur when $k_e = k$, i.e., when the vertical wavenumbers in the tube and external medium are equal. This follows from equation (15) in the limit $\alpha \to 0$, so that

$$\xi \sim (k_e^2 - k^2)^{-1} . \tag{56}$$

However, for a stratified atmosphere, no such condition exists, as has been shown through a detailed analysis of the behavior of ξ_z. The absence of a resonance condition was also noted by Bogdan et al. (1996) and Hasan & Bogdan (1996).

Furthermore, it is found that for *p*-modes of fixed radial order, the response of a tube typically increases with the horizontal wavenumber or mode degree up to a maximum, followed by a sharp decrease. As the radial order increases, the peak in the response shifts to a lower degree. Physically, this behavior is a consequence of the fact that a large response requires that the eigenfunctions of the forcing *p*-mode and the excited tube wave remain in phase over a significant extension of the tube. This effect is buried mathematically in the functional form of ξ_z given by equation (15). As K_x increases, the depth dependence of the external and internal wave eigenfunctions shows a decrease in phase correlation, so that the integrated effect over the extension of the tube is negligibly small.

For modes of low degree, but different orders, the response increases with K_e or the radial order n up to a maximum and then decreases. The maximum value shifts to lower values of n as β increases. Also, for low degree modes of a fixed order, the response exhibits a peak at a value of β that lies typically in the range 0–5.5.

Let us now consider the application of our results to intense flux tubes on the Sun. It is convenient to assume that the $z = 0$ level is at the temperature minimum in the external solar atmosphere, so that the base of the photosphere is located approximately at $z = -500$ km. We choose a pressure scale height such that the acoustic cutoff frequency corresponds to the frequency associated with 5 minute oscillations. This yields a scale height $H = 256$ km and a sound speed of 10.7 km s^{-1}. Our lower boundary is at $z = -40H$, which translates to a depth of about 10^4 km. This is the location of the lower turning point of the acoustic cavity, where ξ_z vanishes. We justify this assumption on the grounds that the temperature increases in the convection zone, which in our model is idealized as a sudden jump in temperature at the lower boundary. According to the model atmosphere of Christensen-Dalsgaard, Profitt, & Thompson (1993), the sound speed at our lower boundary is roughly 37 km s^{-1} (for $\gamma = 5/3$). For *p*-modes in the 5 minute range, only modes with $k_x \geq 6.310^{-9}$ cm $^{-1}$ ($K_x \geq 0.16$) or with degree $l \geq 439$ will be reflected at the lower boundary. This limits the applicability of our analysis to *p*-modes with degrees greater than the above value.

Regarding an appropriate choice of β, we appeal to semiempirical models and theoretical calculations that suggest a value

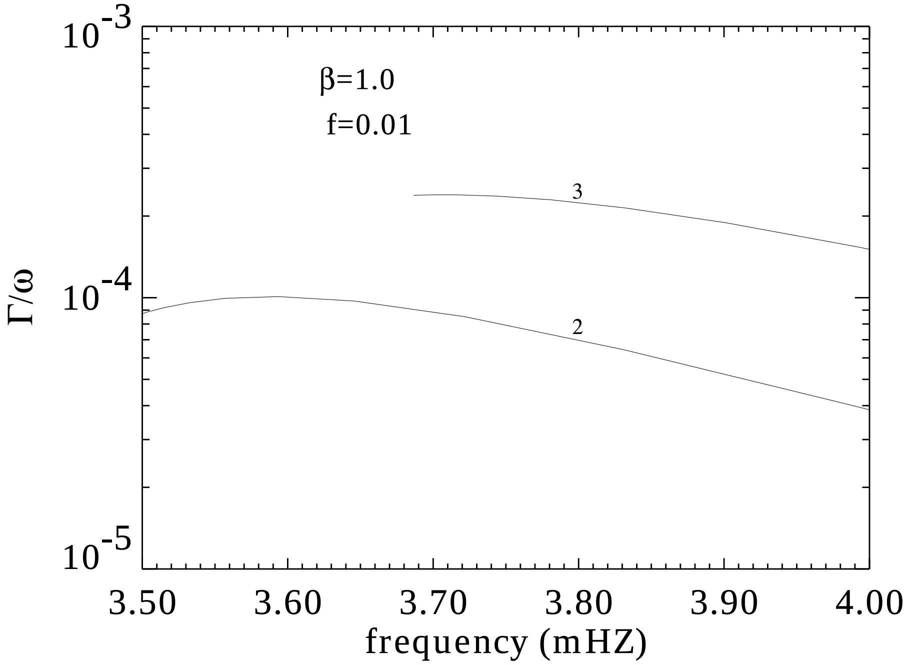

FIG. 8.—Variation of the line width Γ/ω with p-mode frequency (in mHz) for two values of n, which label the curves, assuming $\beta = 1.0$ and $f = 0.01$

of β in the range 0–1. The flux-tube models of Hasan & Kalkofen (1994) and the analysis of Kneer, Hasan, & Kalkofen (1995) based on these models favor values of β between 0.5 and 1.0, since highly evacuated tubes with, say, $\beta = 0.1$ are too hot. From Figure 5, we find that the response has a maximum at $K_e \approx 1.5$ for $\beta = 0.5$, which corresponds to a radial order of $n = 19$. On the other hand, when $\beta = 1.0$, we find that the maximum shifts down to $n = 10$. It thus appears that for p-modes with frequencies in the 5 minute range and with degrees of a few hundred or more, the response is appreciable for modes of moderate order (say, $n < 20$). The response of the tube to modes with high order or l larger than several hundred drops off very sharply. The values that we have given are rather crude, given the limitations of the model.

We now turn to a comparison of the line widths that we have calculated with observations. Figure 8 shows the variation of Γ/ω as a function of frequency, assuming $\beta = 1.0$ and $H = 256$ km, or an acoustic cutoff frequency of 3.3 mHz. The numbers above each curve denote the mode order. Following Bogdan et al. (1996), we choose a filling factor $f = 0.01$ that is appropriate when $\beta \sim 1$. The curve for $n = 3$ begins at a higher frequency, as is to be expected from equation (21), or from the dotted curves in Figure 2. Let us compare our results with Figure 9 of Bogdan et al. (1996), which shows theoretical (for a polytrope with an index of 1.5 and $\beta = 0.1$) and observed (using the data of Korzenik 1990) line widths. We find that our calculations yield a line width that is typically between 10% and 20% of the observed values for the $n = 2$ and $n = 3$ modes, respectively. However, as the frequency increases, we find that the line widths decrease, whereas the observations and also the results of Bogdan et al. (1996) suggest the opposite trend. We should point out that our calculations do not include the effect of the kink mode, which according to Bogdan et al. (1996) becomes more important as β decreases. However, as pointed out by them, p-modes tend to excite mainly sausage modes in flux tubes with $\beta \approx 1$. Nevertheless, it would be interesting to see the trend of the line widths when the kink mode is considered. This is presently under investigation.

8. CONCLUSIONS

The response of a flux tube to forcing by external p-modes has been examined. At the outset, it is important to emphasize that the primary motivation of the present study is not to present results that can be applied directly to observations. Rather, the aim of this investigation is to attempt an insight into the nature of the p-mode interaction with flux tubes. For mathematical tractability, the idealized case of an isothermal atmosphere has been considered. Although this assumption is not realistic for the atmosphere below the photosphere, it has the advantage that it enables us to solve the equations analytically and thereby understand in detail the factors affecting the response of a tube to buffeting by p-modes. This calculation in some ways complements the work of Bogdan et al. (1996), where a more realistic stratification was used, which enabled them to make a comparison with p-mode data. However, their analysis is less tractable mathematically and involves complicated transcendental functions. The present analysis has the advantage that it puts the mathematical treatment in a simple and transparent manner since the response can be examined using fairly simple analytic expressions.

Despite the limitations of assuming an isothermal stratification, it is tempting to attempt a comparison with observations of p-mode line widths. For p-modes in the 5 minute range, the line widths cannot be explained solely by the loss of energy through conversion into tube waves—this result appears to hold even when the influence of the f-mode on tube kink waves is included (Bogdan et al. 1996). As pointed out by the latter authors, additional processes such as collective effects, energy leakage at the upper boundary, and mode-mixing could lead to an increase of the line widths. However, although the effect on line widths of p-mode absorption is probably at most 20%–30%, it may be an important mechanism for the excitation of flux-tube oscillations. In future studies, we hope to consider the question of whether sufficient energy can be accumulated in these oscillations to be relevant for chromospheric heating. As a first step, one can work within the framework of the thin flux-tube approximation, despite its limitations at chromospheric heights. Eventually, a two-dimensional calculation would be required in order to properly treat the flaring geometry of the magnetic field and also to take into account the back-reaction of the tube oscillations on the *p*-modes.

Another question for future work that will be considered in detail is a solution of the time-dependent equations, which involve formulating the interaction as an initial-value problem. Some work has already been done in this respect (Hasan 1995), but clearly more exhaustive calculations are needed, which we hope to take up in subsequent papers. Also, it is of some interest to assess the importance of nonlinear effects and compare the linear and nonlinear treatments. This is presently under investigation.

Finally, we have neglected nonadiabatic effects involving radiative transport, which are known to be important in the photospheric layers and above. Preliminary calculations, in which the nonlinear time-dependent MHD equations have been solved with radiative and convective transport (Hasan 1995), show that although these processes reduce the amplitude of the oscillations compared with the adiabatic case, they do not alter in a qualitative way the essential results stated above. This problem is also currently under examination.

In conclusion, the present study is the beginning of a comprehensive investigation into the dynamical interaction of flux tubes with *p*-modes. It was important to make simplifying assumptions in order to delineate the essential nature of this process. At best, the analysis presented in this paper suggests a broad scenario for examining this interaction. It is hoped that subsequent work will be able to refine the treatment and raise the sophistication of the model in order to enable a closer contact with observations.

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