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Optical colours and polarization of a model reflection nebula IV. Mixture of grains in the case of the star within the nebula

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Abstract. The *UBV* colour differences and polarization that can be caused by mixtures of interstellar grains in a model reflection nebula have been presented. The geometrical configuration of the nebula is adapted in the form of a homogeneous plane-parallel slab with the star within the nebula. The materials of the grain species include ice, graphite, enstatite silicate and silicon carbide. We have considered separately one-, two-, and three-component mixtures of the grains in various proportions of their number densities. Each of the grain species follows an exp $(-a^3)$-type size distribution function. The wavelength dependent indices of refraction have been used throughout.

Key words : reflection nebula—interstellar grains—*UBV* colours—polarization—mixture of grains

1. Introduction

In continuation of earlier work (Shah 1974, 1977a, b; Papers I, II, III respectively) on optical colours and polarization of light from a model reflection nebula with the star in the rear (hereafter referred to as SBN) it is proposed to study here similar results in the case of star within the nebula (SWN) containing mixture of grains composed of ice, graphite, enstatite silicate and silicon carbide taken one, two or three materials at a time in various proportions. As in Paper III (Shah 1977b), the wavelength dependent indices of refraction have been incorporated throughout the present work. It may be noted that the mixture of grains in the case of SWN has not been treated so far in the literature.

In a recent attempt Voshchinnikov (1980) has constructed models of reflection nebula by considering gains in the shape of infinite circular cylinders. The model consists of a central star surrounded by a luminous zone bounded by a spherical surface. The size distribution of cylindrical grains and space-random orientation or perfect (picket fence) alignment have been incorporated in the theoretical formulation. The grains are supposed to be stationary, *i.e.* spin is ignored, for simplicity. The model calculations demonstrate that the observed colours and polarization of a reflection nebula may depend both on the shape of the grains and on their orientation. The results do not seem to reproduce the observations of any particular nebula. However, they certainly indicate that no matter what the structure and the shape of the grains are, the colour indices and especially the polarization depend strongly on the geometrical configuration of the nebula. If the nebula is located between the star and the observer (the SBN case), it will be considerably bluer than the star. In general, its polarization while small will increase monotonically as one goes from the centre to the periphery of the nebula. However, if the illuminating star happens to be in front of the nebula, the colour of the scattered light from the nebula, will be appreciably redder compared to that of the star. The polarization of the emergent radiation will be quite high and will depend weakly on the offset angle. It has been suggested that definite conclusions regarding the shape, structure and chemical composition of the grains will have to await accurate new observations of reflection nebulae with a simple structure.

Some useful references related to the models of reflection nebulae can be traced through the works of Greenberg (1968, 1978), Shah (1974, 1977a, b), Shah & Krishna Swamy (1978, 1979, 1981), Voshchinnikov (1977, 1978, 1980) and Andriesse *et al.* (1977).

2. Outline of the model reflection nebula with the star within

The geometrical model of the reflection nebula, illustrated schematically in Figure 1, consists of a homogeneous plane-parallel slab with the illuminating star within the nebula (the case referred to as SWN). The relevant analytical expressions have been given by Shah (1974) and Shah & Krishna Swamy (1978, 1981). The colours and polarizations are defined according to Greenberg & Roark (1967) and Shah (1974). The grains are assumed to be perfectly homogeneous smooth spheres. We shall consider only single and independent scattering. The intensity phase functions for scattering of electromagnetic waves by grains follows from the theory of Mie (see *e.g.* van de Hulst 1957). The wavelength dependent indices of refraction for the grain materials have been chosen from Greenberg (1968) and Shah (1967) for ice, Taft & Philipp (1965) for graphite, Huffman & Stapp (1971) for enstatite silicate and Greenberg (1975) for silicon carbide. Since the imaginary part of the index of refraction of ice in the visual wavelengths is not known accurately, we have assumed it to be $m'' = 0.02$ in the range $\lambda = 0.3$ μm to 0.6 μm.

The symbols, notation and relevant geometrical parameters with their values have been adopted as follows (*cf.* Figure 1).

SWN = the case of the star within the nebula,
S = the location of the illuminating star,
O = the location of the observer,
T = the thickness of the nebula = 1.0 pc,
D = the distance of the star from the observer = 126 pc,
H = the perpendicular distance between the star and the front surface of the nebula = 0.25 pc,

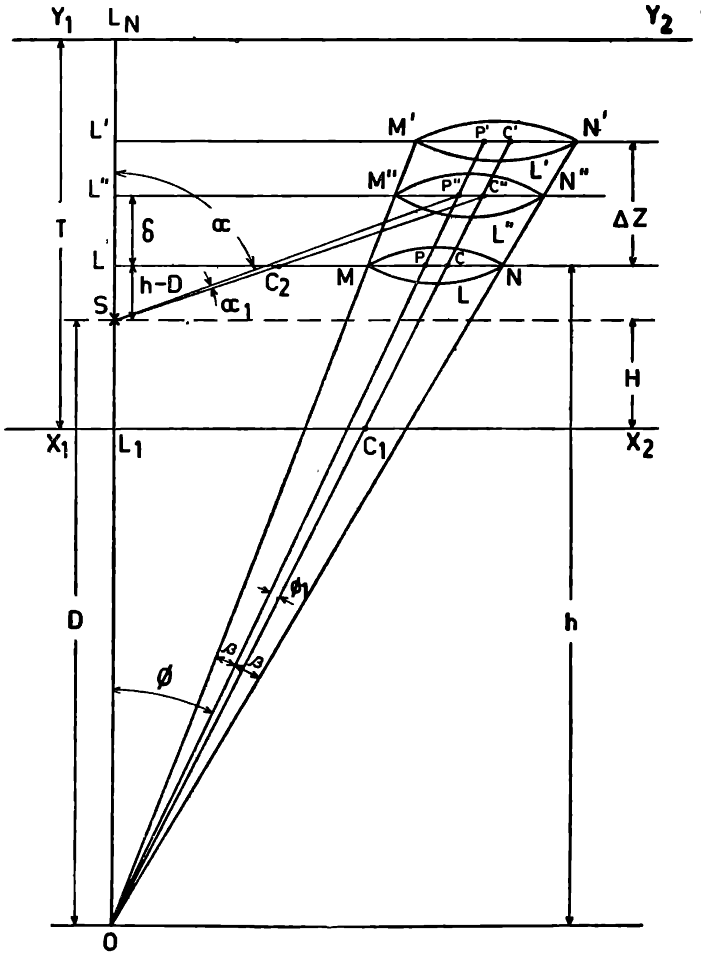

Figure 1. Schematic representation of plane-parallel slab model of reflection nebula with the star within (SWN).

ΔZ = the width of the elementary slab LNN′L′,

$\phi = \,<$ SOP, $\phi_1 = \,<$ POC

$\phi + \phi_1$ = the corrected offset angle,

β = semi-vertex angle of the telescope = 0.0015 rad,

h = the distance OL of the bottom of a typical elementary slab L′LNN′ from the observer,

$\Delta\Omega$ = the solid angle of the telescope = $\pi \tan^2 \beta$

λ = the wavelength of light,

a = the size of an interstellar grain,

a_{0i} = the characteristic parameter in the size distribution function for the *i*th component of the mixture,

$n_i(0)$ = the number density of the grains per cm^3 for the *i*th species of the mixture,

$m(\lambda) = m'(\lambda)\ im(\lambda)$ = the complex index of refraction of the grain material,

$l_1 = OC''$ = the distance between the observer and the scattering centre C″ within a typical volume element MNN′M′

$l_{1N} = CC''$ = the portion of l_1 lying within the nebula to account for extinction within the nebula,

$l_2 = SC''$ = the distance between the star and the scattering centre C″,

$l_{2N} = l_2$, defined similar to l_{1N}.

One is referred to Shah & Krishna Swamy (1978, 1981) for details of the analytical expressions. We note here that the exact volume corresponding to the element MNN′M′ is given by

$$\Delta V = h^2 \Delta Z \Delta\Omega \frac{\{1 + (\Delta Z/h) + \frac{1}{3}(\Delta Z/h)^2\}}{\{\cos^2\phi - \sin^2\phi \tan^2\beta)^{3/2}}\ . \qquad \ldots(1)$$

If $\beta = 0°$ and $(\Delta Z/h) \ll 1$, this volume element approximates to the expression given by Greenberg & Roark (1967).

The size distribution function for each component of the mixture of grains in properly normalized form (Shah 1974) is given by

$$dn_i(a) = \{Cn_i(0)/a_{0i}\} \exp\{-5(a/a_{0i})^3\}\ da, \qquad \ldots(2)$$

where $i = 1, 2, 4$ or 4 corresponds to ice, graphite, enstatite silicate and silicon carbide, respectively; a is the running value of the grain size; C the normalization constant equal to $5^{1/3}/\Gamma(4/3) \simeq 1.914$.

The factor a_{0i} in the denominator of the first bracket in equation (2) is necessary while considering a mixture of grains to account for variation in the dispersion of grain sizes among different species of the grains. The size parameter a_{0i} in the present work has been held constant throughout for a given grain species. In particular, we have chosen $a_{0i} = 0.5\ \mu$m for ice and $a_{0i} = 0.25\ \mu$m for $i = 1, 2$ and 3 corresponding to graphite, silicate and silicon carbide, respectively. The proportion of the grains for ice has been varied in the range $n_1(0) = 0, 1, 2(2)\ 10$, and for the rest of the materials, we have adopted $n_i(0) = 0(2)\ 10$, $i = 2, 3, 4$. The contribution to the observed intensity comes from all the nebular volume elements along the line of sight. We consider intensities I_1 and I_2 for two orthogonal states of polarization coming from all the grains belonging to one species along the line of sight. These intensities are then weighted by appropriate $n_i(0)$ and summed up for all the mixture to derive sort of mean values $\langle I_1 \rangle$ and $\langle I_2 \rangle$. The mean intensities have been further used to calculate the nebular colours and polarization according to the scheme described by Shah (1974). The extinction within the nebula has been taken into account on the basis of the observations by Boggess & Borgman (1964). The filter response functions for the U, B and V bands have been adopted from Allen (1973). For the sake of brevity, we shall denote a_{0i} and $n_i(0)$ by a_i and n_i respectively. For colours and polarization the following notation has been used :

$(J - K)_{*-\mathrm{N}}$ = the colour difference between the star and the nebula for off-set angle ϕ.

$P_X(\phi)$ = the degree of polarization as function of offset angle ϕ and for a given wavelength band $X(=U, B, V)$.

Note that the band intensity J can be for B (or U) corresponding to K for V (or B). Thus the definition of the colour difference implies that the nebular colour difference between the bands J and K has to be subtracted from that of the star. This means that a minus value for the quantity $(B - V)_{*-\mathrm{N}}$ or $(U - B)_{*-\mathrm{N}}$ indicates reddening, *i.e.* the nebula is redder relative to the star. Analytical expressions for colours and polarization in terms of intensities can be found in Paper I.

3. Computed colours and polarization of nebular light

3.1. *Single species of grains*

The computational results on colours and polarization caused by single species of the grains in the case SWN are plotted in Figure 2. Here the grains are composed of one of the materials, *viz.*, ice, graphite, enstatite silicate or silicon carbide. The smoothed out observations based on data by Elvius & Hall (1966, 1967) and Greenberg & Roark (1967) for Merope nebula have also been included as dots in

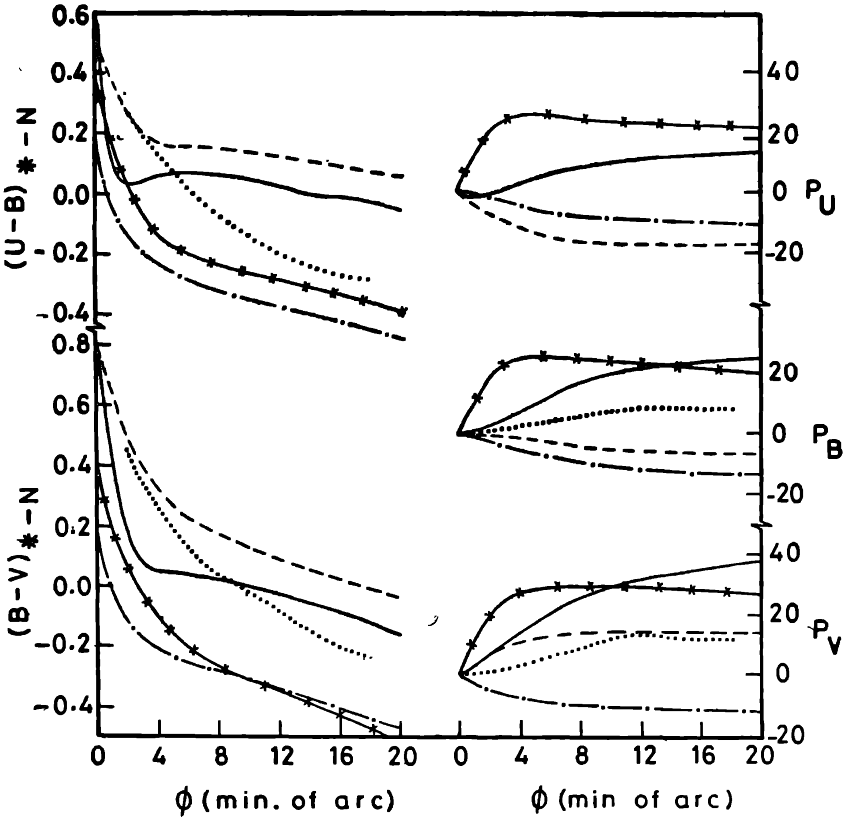

Figure 2. The optical colours and polarization of the model reflection nebula containing one-component grains. —— for ice grains ($a_0 = 0.5$ μm), – x – x – for graphite ($a_0 = 0.25$ μm), – – – – for enstatite silicate ($a_0 = 0.25$ μm) and – · – · – · for silicon carbide ($a_0 = 0.25$ μm). The dotted curve follows from the observations by Greenberg & Roark (1967) and Elvius & Hall (1967).

Figure 2. They should be regarded as a rough guide for comparison with the calculated models. Note that we have chosen fixed values of the size distribution parameter a_{0i}, for each species of the grains. The observed colour differences $(B - V)_{*-N}$ and $(U - B)_{*-N}$ in the range $\phi = 0$ to $4'$ approximately follow the corresponding results for one component grains composed of enstatite silicate. For $\phi > 4'$ some other species need to be included in the mixture. The polarization in visual P_V, for $\phi \gtrsim 10'$, produced by enstatite silicate grains, lies close to the observational points. The observed polarization in the blue band, P_B, does not follow any particular curve but it lies, almost symmetrically between the results for ice (or graphite) and silicate (silicon carbide). There are no observations available for polarization in the U-band. Individually none of the species of the grains considered here satisfy the observations.

4. Two-component mixture of the grains

The present model of the reflection nebula with the star within has been examined by considering two-component mixture of grains consisting of (i) ice and graphite, (ii) ice and enstatite silicate, (iii) ice and silicon carbide and (iv) graphite and enstatite silicate. The number densities of the grains in each mixture are n_1 and n_2 respectively. The representative proportions $n_1 : n_2$ have been chosen to be 1 : 2, 2 : 1, 1 : 3, 3 : 1, 1 : 4 and 1 : 5 for each mixture. The calculated colours and polarization of nebular light have been set out in Figures 3, 4, 5 and 6 corresponding to the above mentioned four mixtures. In Figure 3, the colour difference

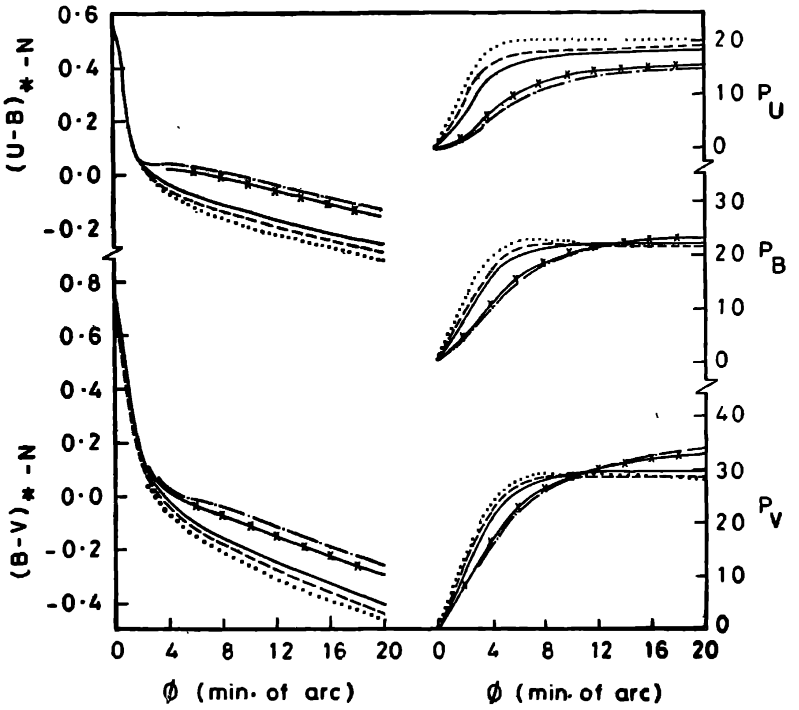

Figure 3. Colours and polarization produced by two-component mixture of grains composed of ice and graphite in proportion $n_1 : n_2 = 1 : 2$ for ——, 2 : 1 for – x – x –, 1 : 3 for - - - -, 3 : 1 for – · – · –, 1 : 4 for –··–··– and 1 : 5 for ......

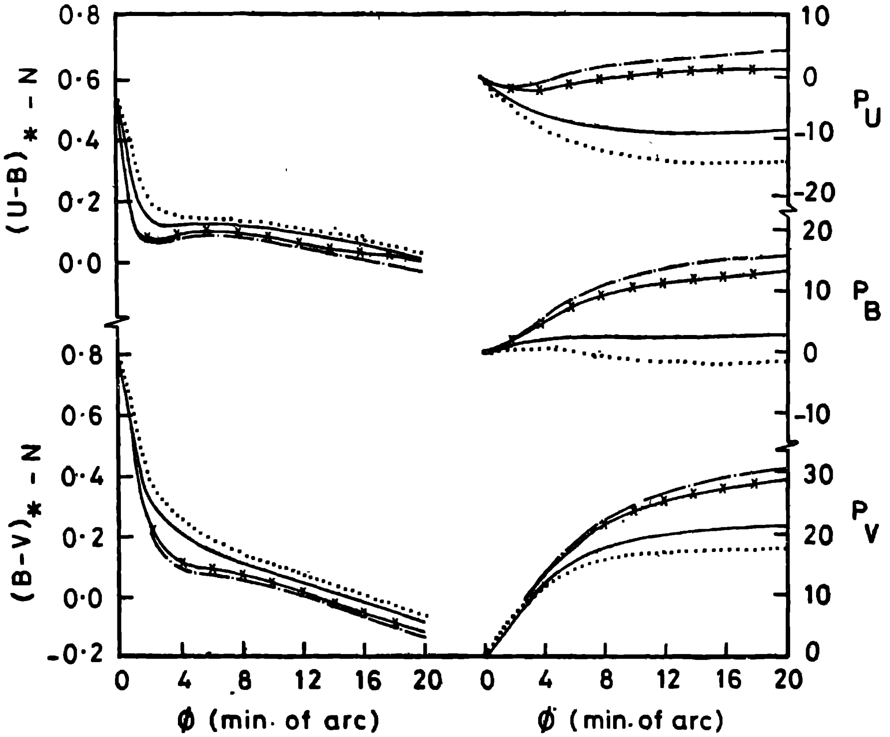

Figure 4. The same as in Figure 3 but with the mixture of ice and enstatite silicate.

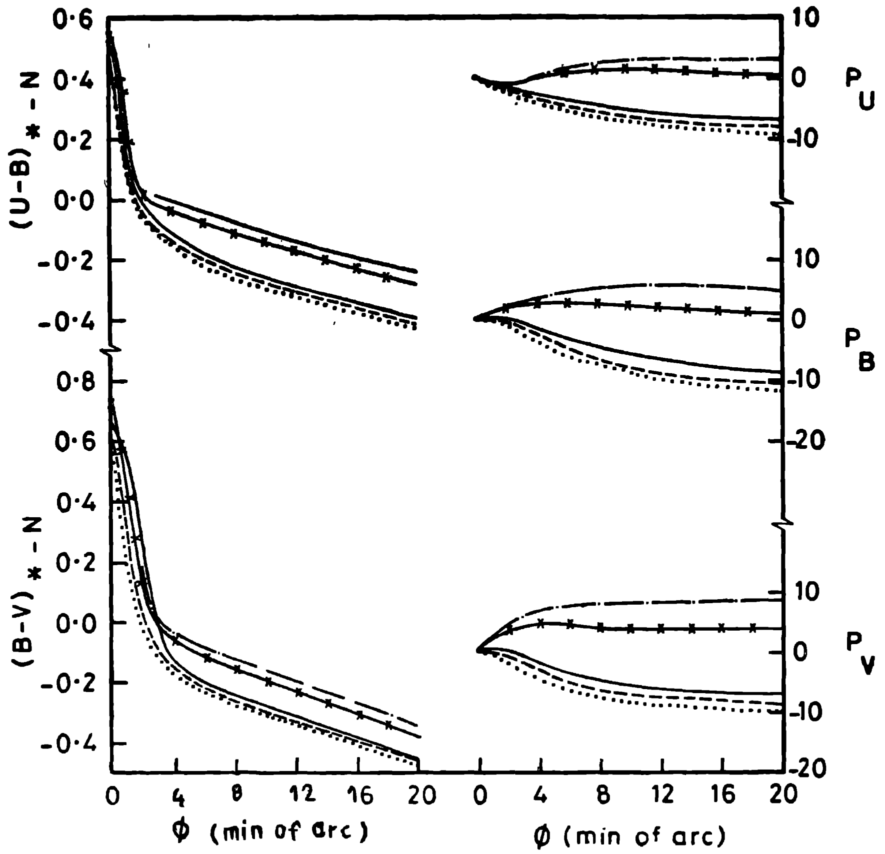

Figure 5. The same as in Figure 3 but with the mixture of ice and SiC.

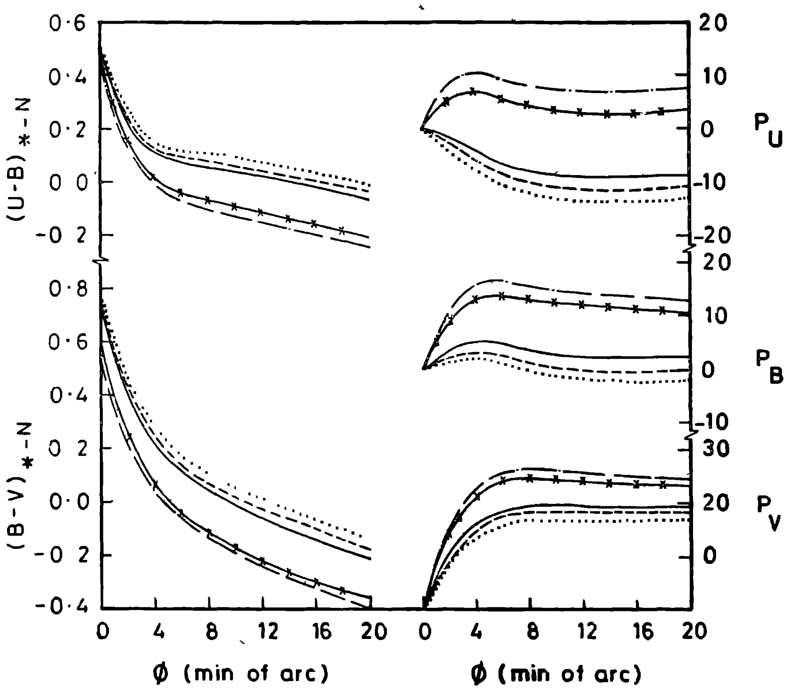

Figure 6. The same as in Figure 3 but the mixture components are graphite and enstatite silicate.

$(B - V)_{*-N}$ becomes negative at too early a value of offset angle ϕ as compared to the observational data. The polarization in visual or blue band, though positive for ϕ up to 20′, is rather too high. In general, more proportion of ice is favoured vis-a-vis graphite.

The mixture of ice and silicate grains in Figure 4 shows that reddening of the nebula starts at much later value of ϕ compared to observations. This is indicated by the change of sign in the colour differences as one goes from $\phi = 0$ to 20′. The polarization in the visual for $n_1 : n_2 = 1 : 5$ seem to favour the observational trend. However, the corresponding polarization in the blue band is negative contrary to the observations.

In Figure 5 for mixture of ice and silicon carbide, the reddening begins too early. The positive polarization in the case of the *V*- or the *B*-band can occur only if we choose higher proportion of ice grains relative to silicon carbide grains.

In Figure 6 for mixture of graphite and enstatite silicate grains, the trends of variation in polarization with ϕ in the *B*-band rule out $n_1 : n_2 > 1$. On the other hand, if we choose $n_1 : n_2 < 1$, the polarization in *V*-band becomes too large. Thus there must be other components like ice in the mixture in order to soften the polarization level.

5. Three-component mixture of grains

It is natural to extend the consideration of mixture of grains to more than two components out of the four materials adopted in the present work. Here we have studied four types of three-component mixtures:

(i) ice, graphite and enstatite silicate,

(ii) ice, graphite and silicon carbide,
(iii) ice, enstatite silicate and silicon carbide, and
(iv) graphite, enstatite silicate and silicon carbide.

The number densities of the grains of different species are represented by n_1, n_2 and n_3 in the same order as the components mentioned in each of the mixtures. The proportions $n_1 : n_2 : n_3$ are selected as 1 : 1 : 2, 1 : 2 : 1, 2 : 1 : 2, 2 : 2 : 1, 1 : 2 : 4 and 1 : 4 : 2. Note that in Figure 7, the proportions $n_1 : n_2 : n_3 = 1 : 1 : 2$ and 1 : 2 : 4 give almost the same results for $(B - V)_{*-\mathrm{N}}$ and so only one curve is plotted. This is true for the case of $(U - B)_{*-\mathrm{N}}$ as well. In Figure 8, the similar situation occurs for $n_1 : n_2 : n_3 = 1 : 4 : 2$ and 1 : 1 : 2.

Figures 7 to 10 show certain general trends in the observations. For example, the nebula is bluer in the vicinity of the star and redder for sufficiently large value of the offset angle ϕ. The mixture of ice, graphite and enstatite silicate is the only one which shows nearly the correct location for $(B - V)_{*-\mathrm{N}} = 0.0$ for $n_1 : n_2 : n_3 = 1 : 2 : 4$ or 2 : 1 : 2. However, corresponding ϕ for $(U - B)_{*-\mathrm{N}} = 0.0$ is rather too large. The negative polarization in the blue band for large ϕ goes against the mixture (iii) and (iv).

It appears from Figures 8, 9 and 10 as well as the earlier study on two-component mixture that whenever SiC is included there is a possibility of unwanted negative polarization for cases of interest as judged from the colour differences. Thus, as in

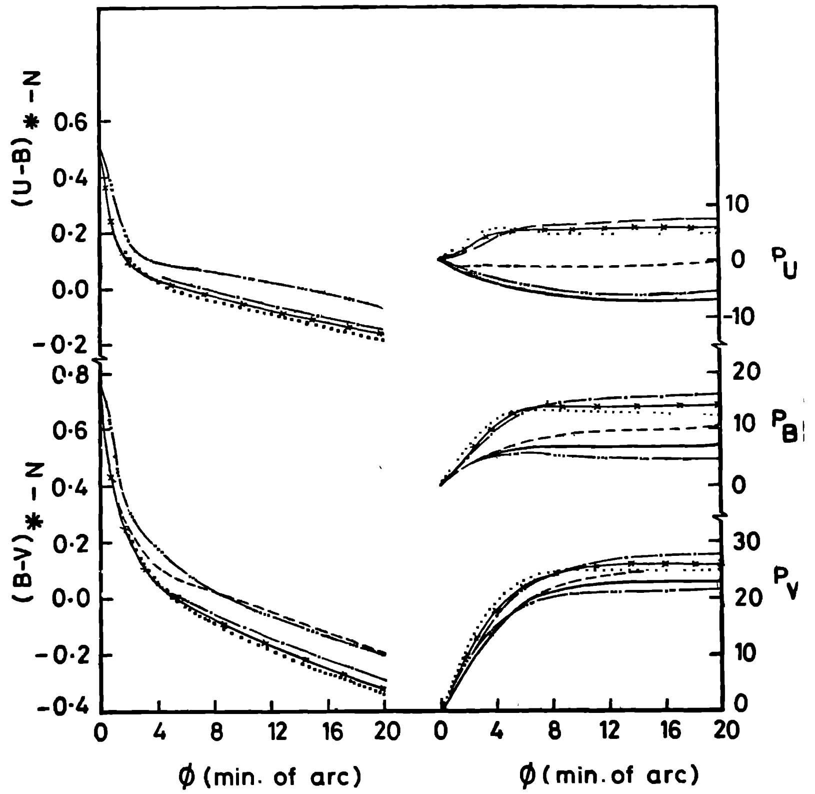

Figure 7. Colours and polarization for the case SWN with the three-component mixture of ice (n_1), graphite (n_2) and enstatite silicate (n_3). The mixture proportions are $n_1 : n_2 : n_3 = 1 : 1 : 2$ for ——, 1 : 2 : 1 for - x - x -, 2 : 1 : 2 for - - -, 2 : 2 : 1 for - • - • - • -, 1 : 2 : 4 for -··-··-·· and 1 : 4 : 2 for ......

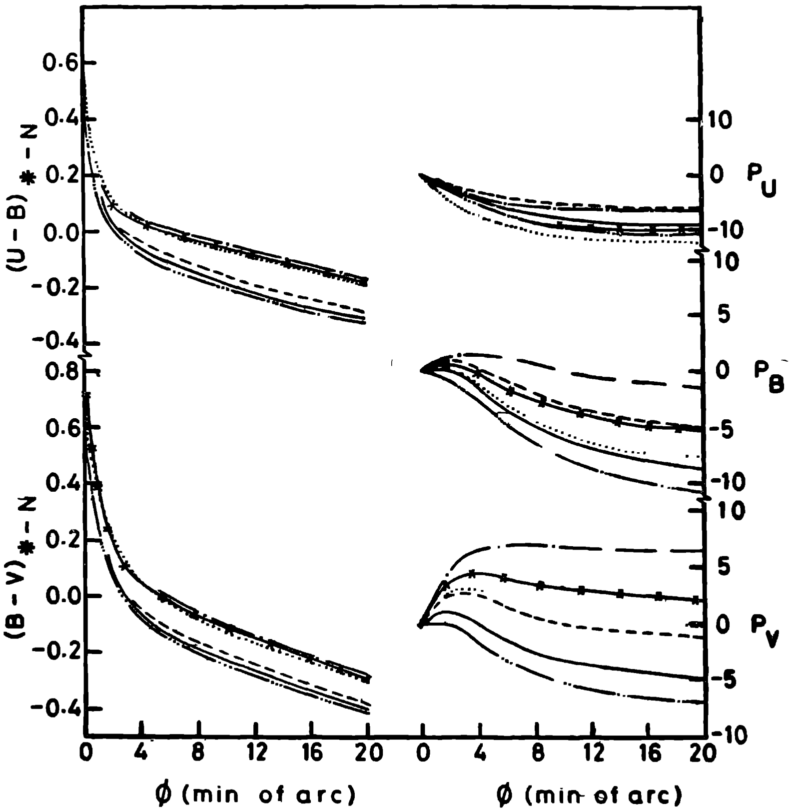

Figure 8. The same as in Figure 7 but the mixture consists of ice, enstatite silicate and SiC grains.

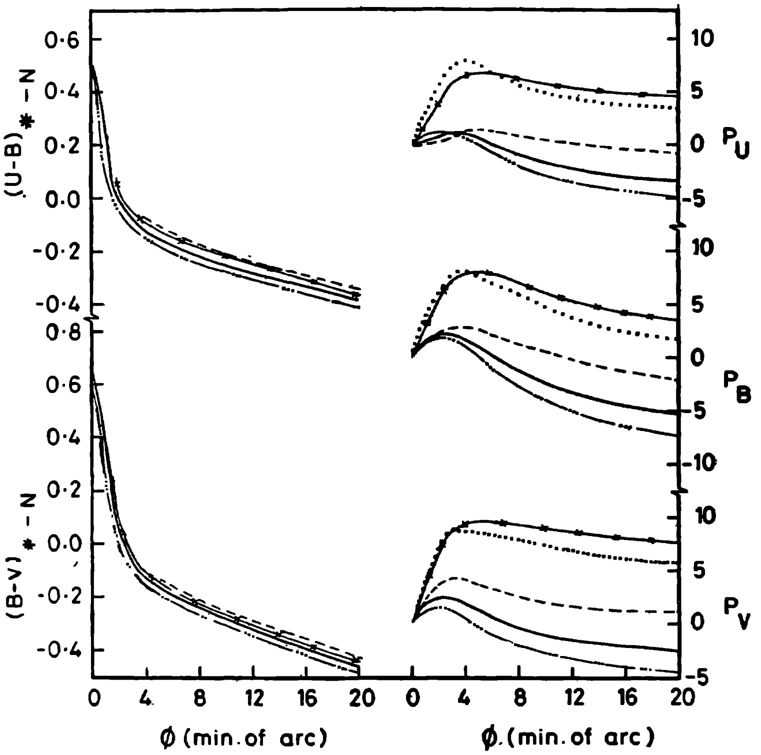

Figure 9. The same as in Figure 7 but the mixture consists of ice, graphite and SiC grains.

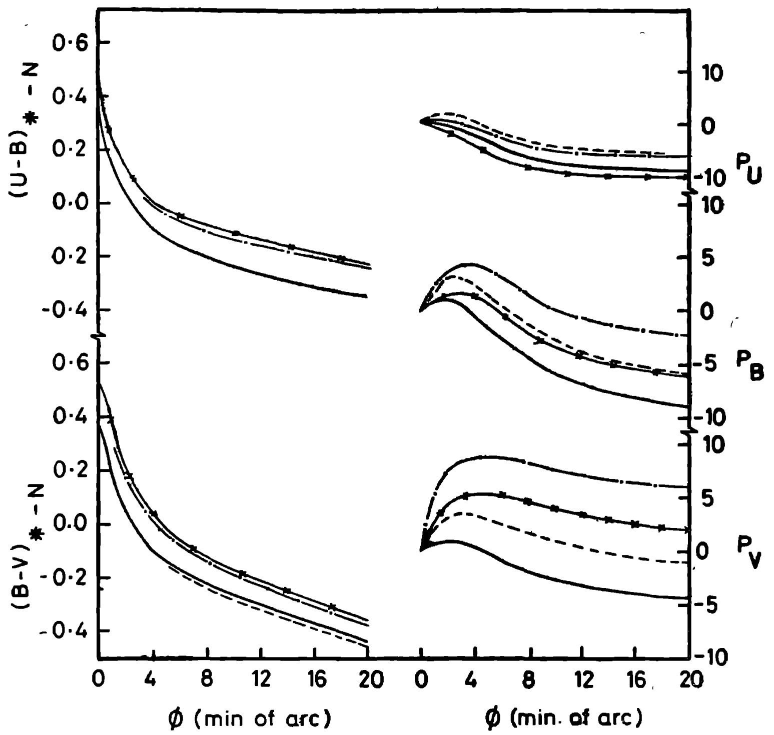

Figure 10. The same as in Figure 7 but the mixture consists of graphite, enstatite silicate and SiC.

the case of SBN in Paper III higher proportion of ice, graphite or silicate in comparison with SiC is preferable.

6. Conclusions

The colours and polarization caused by mixtures of grains within a model reflection nebula with the star within show some general observational trends provided ice, graphite and enstatite silicate materials are considered. The inclusion of SiC grains contributes to negative polarization for most cases of interest. Therefore, it seems that SiC is not an important component of the grain mixture at least in the case of Merope reflection nebula. We have not exhausted all the possibilities for the case SWN. We need to experiment further on this case. Also, we propose to study the third geometrical case of the reflection nebula with the star in the front along similar lines in order to be able to judge the nature of the grains and the location of the star relative to the nebula. More accurate observations on reflection nebulae with simple structure are desirable to justify detailed and elaborate calculations of the models.

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