Kinetic theory of Jeans instability of a dusty plasma

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A kinetic theory of the Jeans instability of a dusty plasma has been developed in the present work. The effect of grain charge fluctuations due to the attachment of electrons and ions to the grain surface has been considered in the framework of Krook's collisional model. We demonstrate that the grain charge fluctuations alter the growth rate of the gravitational collapse of the dusty plasma. The Jeans length has been derived under limiting cases, and its dependence on the attachment frequency is shown. In the absence of gravity, we see that the damping rate of the dust acoustic mode is proportional to the electron-dust collision frequency.
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I. INTRODUCTION

Several observed features of galaxies can be explained assuming that the galactic dark halo is composed of dust grains of varying sizes [1]. Most of the invisible mass in the Universe may consist of the dust of ordinary matter. It is believed that most of the large-scale structure formation in the universe takes place due to the gravitational collapse of huge ($\sim 10^2-10^6$ pc, where 1 pc$\approx 3\times 10^{18}$ cm) cloud complexes consisting of dust and gas. The dust and gas particles are generally charged due to prevailing thermal and radiative conditions in such cloud complexes [2,3]. In the interplanetary medium, the charged grains are composed of graphite, silicate, and metallic compounds. The size distribution of grains has been investigated by comparing the observed interstellar extinction curve with the theoretical one. The observed extinction curves imply that the mass, size, and electric charge of the grains vary in a wide range. For example, the mass of a grain may vary typically from 10^{-5} g in the interplanetary medium [4,5] to 10^{-14} g in the interstellar medium [6]. The corresponding size may range from a few cm to a few submicrons. The grains may carry 10^3 (or more) down to zero electronic charges. As a result, there exists a parameter regime in which the electrostatic and gravitational effects become comparable.

The gravito-electromagnetic coupling of an atomic plasma in local thermal equilibrium was studied in the context of star formation by Eddington [7]. It was shown that self-gravity is unimportant for such a plasma as $R=\omega_J^2/\omega_{pd}^2\sim 10^{-36}$, where $\omega_J=(4\pi G n_d m_d)^{1/2}$ is the Jeans frequency and $\omega_{pd}=(4\pi n_d Q^2/m_d)^{1/2}$ is the plasma frequency of the grain with G as the gravitational constant and n_d, m_d, and Q as the number density, the mass, and the charge of a grain, respectively. Therefore, the scales at which the two forces operate are widely separated. The formation of large-scale structures has been entirely attributed to the gravitational condensation of matter, whereas radiation processes are attributed to electromagnetic interactions of the plasma particles. However, for grains of mass $m_d\sim 10^{-5}$ g, charge $Q\simeq 30$ electronic charge, $R\sim O(1)$, and thus electromagnetic (EM) and gravitational force (GF) effects compete. The broad spectrum of observed masses and charges [8,9] suggests that the dynamics of a dusty plasma can be studied in any of the following regimes: (a) EM force$\gg$GF, (b) EM force$\sim$GF, and (c) EM force$\ll$GF. The first case corresponds to the plasma processes like radiation, heating, etc., the second case has been shown to correspond to spoke formation in Saturn's rings, and the last case corresponds to the formation of stars and galaxies [10].

The Jeans collapse of electrically charged grains occurs essentially in the presence of an equilibrium electrostatic field. The resultant electrostatic repulsion among the grains will counteract the gravitational attraction, slowing or halting the collapse altogether.

Recently, the Jeans instability of a dusty plasma has been studied by several authors [11–18]. Time-dependent nonlinear solutions of a self-gravitating three-component dusty plasma showed [13] that even when self-gravity is canceled by the electric polarization of grains, i.e., $R\sim 1$, the system may become gravitationally unstable due to the properties of the background plasma in which the grains are embedded. Therefore, merely balancing the gravitational attraction of the grains with their electrostatic repulsion is not sufficient for determining the fate of the condensation process. Furthermore, a study of the Jeans-Buneman instability [12,18] suggested that streaming plasma ions can act as a precursor to the gravitational condensation of the grains.

In the present work, we investigate the Jeans instability of a dusty plasma in the framework of kinetic theory which has not been studied so far to the best of our knowledge. As is known, novel collective features of dusty plasma dynamics arise due to the fluctuations in the electric charge of the grains [18–22]. The charge fluctuations may affect the gravitational condensation of the grains by modifying the polarization field set up by the collapsing grains [18]. However, studies of the collapse of grains with fluctuating charges have been carried out in the framework of a multicomponent fluid model, where a model source and sink term in the elec-

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tron and ion continuity equation couples with the grain charge dynamics. The validity of such a model term is *a priori* unclear. Also, the grain charge fluctuation is described by two frequencies, namely, attachment β and charge decay η frequencies. The kinetic formulation of a dusty plasma describes the charge dynamics, making use of only one frequency or only one time scale.

Therefore, the purpose of the present work is twofold: (a) to formulate a proper self-consistent kinetic theory of the Jeans instability of a dusty plasma and (b) to check the validity of hitherto derived results on gravitational collapse using fluid models ([18] and references therein). The only recourse to treat grain charge dynamics self-consistently is to solve the full set of Boltzmann equations for electrons, ions, and grains and Poisson's equation. That such a task is beyond the analytical means is well known. However, a ''limited'' self-consistent analytical treatment of the problem can be given by approximating the electron and ion collision operators using Krook's model. This will allow us to examine the validity of the previous results obtained by using fluid models.

In a plasma consisting of dust grains, electrons, and ions, the grain is charged by the plasma currents at the grain surface, owing to the potential difference $\Phi_d-\Phi$ between the grain surface potential Φ_d and the local plasma potential Φ. The electron I_e and the ion I_i currents are [2,3,23]:

$$I_e=-\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_e \exp\left[\frac{e(\Phi_d-\Phi)}{T_e}\right], \quad (1)$$

$$I_i=-\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_i\left[1-\frac{e(\Phi_d-\Phi)}{T_i}\right]. \quad (2)$$

The grain acquires a steady-state electric charge Q_0 when

$$\frac{dQ}{dt}=I_e+I_i=0. \quad (3)$$

The electrostatic potential Φ_d due to a spherical charged grain of radius a is [24]

$$\Phi_d(r)=\frac{Q_0}{r}\left\{\left[\left(1-\frac{b}{\lambda_D}\right)e^{(b-r)/\lambda_D}-\left(1+\frac{b}{\lambda_D}\right)e^{(b-r)/\lambda_D}\right]\right.$$
$$\times\left[\left(1-\frac{b}{\lambda_D}\right)\left(1+\frac{a}{\lambda_D}\right)e^{(b-a)/\lambda_D}\right.$$
$$\left.\left.-\left(1+\frac{b}{\lambda_D}\right)\left(1-\frac{a}{\lambda_D}\right)e^{-(b-a)/\lambda_D}\right]\right\}, \quad (4)$$

where grain density n_d is related to b by $n_d^{-1}=(4\pi/3)b^3$ and $\lambda_D^{-2}=\Sigma_\alpha(n_\alpha q_\alpha^2/k_B T_\alpha)$, where α refers to electrons and ions and k_B is the Boltzmann constant. Thus, correspondingly, here is an equilibrium electrostatic field $\mathbf{E}$ defined as $\mathbf{E}=-\boldsymbol{\nabla}(\Phi_d-\Phi)$. The thickness of the non-neutral layer is of the order of the Debye length λ_D.

II. BASIC EQUATIONS

In the kinetic description of a three-component dusty plasma consisting of electrons, ions, and charged, spherical grains of radius a, the distribution function of electrons and ions satisfies a Boltzmann equation which assures the possibility of a variation of charges of the dust particles. There are many sources of the grain's charge fluctuations, e.g., associated with fluctuating electron and ion densities as well as due to electron-electron, electron-ion, and ion-ion collisions. The Boltzmann equation is

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{\mathbf{F}}{m_\alpha}\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}}=\left(\frac{\partial f_\alpha}{\partial t}\right)\bigg|_C, \quad (5)$$

where $f_\alpha=f_\alpha(\mathbf{r},\mathbf{v},t)$ is the distribution function for electrons and ions in the six-dimensional phase space $(\mathbf{r},\mathbf{v})$ and m_α is the mass of the particle. Using Krook's model for the collision term, Eq. (5) becomes

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{\mathbf{F}}{m_\alpha}\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}}=-\nu_{\alpha d}(f_\alpha-f_{\alpha 0}), \quad (6)$$

where $\nu_{\alpha d}$ is the attachment frequency of electrons and ions to the dust grains and $f_{\alpha 0}$ is the Maxwellian distribution to which the plasma particles relax over the collisional time scale [25]. As will be shown later in the text, collisional processes such as electron-electron, electron-ion, and ion-ion can be neglected when compared with charging processes which are governed by electron-grain and ion-grain collision processes. The force $\mathbf{F}$ in the above equation is the sum of the electrostatic and the gravitational forces, i.e.,

$$\frac{\mathbf{F}_\alpha}{m_\alpha}=\frac{e_\alpha}{m_\alpha}\mathbf{E}+\mathbf{g}, \quad (7)$$

where $\mathbf{E}$ and $\mathbf{g}$ are the electrostatic and the gravitational fields, respectively.

The dust distribution should be written in the enlarged seven-dimensional phase space $(\mathbf{r},\mathbf{v},Q)$ where the grain charge Q has been included as a new dynamical variable. Then the dust distribution function satisfies following continuity equation in $(\mathbf{r},\mathbf{v},Q)$ space [26,27]:

$$\frac{\partial f_d}{\partial t}+\mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}}+\frac{\mathbf{F}_d}{m_d}\cdot\frac{\partial f_d}{\partial \mathbf{v}}+\frac{\partial[I(Q)f_d]}{\partial Q}=0, \quad (8)$$

where $\mathbf{F}_d$ is given by

$$\frac{\mathbf{F}_d}{m_d}=\frac{Q}{m_d}\mathbf{E}+\mathbf{g}.$$

In Eq. (8), the effect of grain collisions with electrons and ions $(\partial f_d/\partial t)|_C$ has been ignored. This assumption is justified owing to large mass difference between the grains and the plasma particle (electrons and ions), e.g., $m_i/m_d \sim 10^{-9}-10^{-19}$. The current $I(Q)$ to the dust particle is given by

$$I(Q)=\Sigma_\alpha\int d\mathbf{v}\, e_\alpha\sigma_\alpha v f_\alpha(\mathbf{v}), \quad (9)$$

where σ_α is the grains cross section [2]:

$$\sigma_\alpha = \begin{cases} \pi a^2\left(1-\dfrac{2e_\alpha Q}{a m_\alpha v_\alpha^2}\right) & \text{if } \dfrac{2e_\alpha Q}{a m_\alpha v_\alpha^2}<1, \\ 0 & \text{if } \dfrac{2e_\alpha Q}{a m_\alpha v_\alpha^2}\geq 1. \end{cases} \quad (10)$$

The number density of the grain is defined as follows:

$$n_d(\mathbf{r}) = \int d\mathbf{v}\, dQ f_d. \quad (11)$$

The Poisson equation is written as

$$\nabla \cdot \mathbf{D} = -4\pi\left\{\Sigma_\alpha \int d\mathbf{v}\, e_\alpha f_\alpha + \int d\mathbf{v}\, dQ Q f_d\right\}, \quad (12)$$

where the polarization vector $\mathbf{D}=(\Sigma_\alpha \chi_\alpha + \chi_d)\mathbf{E}$. The Poisson equation for the gravitational field is

$$\nabla \cdot \mathbf{g} = -4\pi G m_d \int d\mathbf{v}\, dQ f_d. \quad (13)$$

The susceptibility χ and dielectric permitivity ϵ is related by $\epsilon = 1+4\pi\Sigma_\alpha \chi_\alpha + 4\pi\chi_d$.

In Eq. (13), contributions of electrons and ions have been neglected as $m_e n_e \ll m_i n_i \ll m_d n_d$ in most of the astrophysical plasmas. Equations (6), (8), (12), and (13) define the entire dynamics of the dusty plasma.

III. EQUILIBRIUM

In the equilibrium, the distribution functions of electrons and ions are functions of the total energy, a constant of motion, such that

$$f_\alpha = f_\alpha\left(\frac{1}{2} m_\alpha v_\alpha^2 + e_\alpha \Phi(r)\right), \quad (14)$$

where $\Phi(\mathbf{r})$ is the electrostatic potential in the plasma. Assuming that the grains carry identical charge Q_0, the equilibrium distribution function of the dust particles can be written as

$$f_{d0} = f_0(\mathbf{v}_d)\,\delta(Q-Q_0), \quad (15)$$

where $f_0(\mathbf{v}_d) = (m_d/2\pi k_B T_d)^{3/2} n_{d0} \exp[-m_d v_d^2/2k_B T_d]$ is the Maxwellian distribution function. Here, in the zeroth order, the electric force $Q_0\mathbf{E}_0$ has been balanced by the gravitational force $m_d\mathbf{g}$. This equilibrium defines a constraint on the zeroth-order grain density [13]. In order to simplify the kinetic treatment, we resort to the approximation that the electrostatic and gravitational energy everywhere is smaller than the thermal energy so that a Maxwellian distribution for electrons and ions can be assumed. In equilibrium, the net current on the grain is zero, i.e.,

$$I_0(Q_0) = \frac{1}{n_{d0}} \int I_0(Q) f_{d0} dQ\, d\mathbf{v} = 0, \quad (16)$$

where the subscript 0 corresponds to the equilibrium quantities. For Maxwellian electrons and ions, Eq. (16) becomes

$$\frac{\omega_{pe}^2}{v_{te}} \exp(-z) = \frac{\omega_{pi}^2}{v_{ti}} (\tau + z), \quad (17)$$

where $\omega_{p\alpha} = \sqrt{4\pi n_\alpha e_\alpha^2/m_\alpha}$ is the plasma frequency, $v_{t\alpha} = \sqrt{k_B T_\alpha/m_\alpha}$ is the thermal velocity with T_α as the temperature of the αth species, $\tau = T_i/T_e$, and $z = Q_0 e/(a k_B T_e)$.

In equilibrium, Eq. (8) becomes

$$\frac{\partial[I_0(Q) f_{d0}]}{\partial Q} = 0, \quad (18)$$

which, on making use of Eq. (15), can be written as

$$I_0(Q) = -\Omega_c (Q - Q_0), \quad (19)$$

where

$$\Omega_c = -\left(\frac{\partial I(Q)}{\partial Q}\right)\bigg|_{Q=Q_0}. \quad (20)$$

In order to calculate the charge fluctuation frequency Ω_c, we shall evaluate the integrals

$$\left(\frac{\partial I_{i,e}(Q)}{\partial Q}\right)\bigg|_{Q=Q_0} = \pm e \frac{\partial}{\partial Q} \int d\mathbf{v}\, \sigma_{i,e}(Q,v) v f_{i,e}(\mathbf{v}).$$

With $f_{i,e} = (m_{i,e}/2\pi k_B T_{i,e})^{3/2} n_{i,e} \exp[-m_{i,e} v^2/2k_B T_{i,e}]$, we get

$$\left(\frac{\partial I_i(Q)}{\partial Q}\right)\bigg|_{Q=Q_0} = \frac{a}{\sqrt{2\pi}} \frac{\omega_{pi}^2}{v_{ti}}, \quad (21)$$

and

$$\left(\frac{\partial I_e(Q)}{\partial Q}\right)\bigg|_{Q=Q_0} = \frac{a}{\sqrt{2\pi}} \frac{\omega_{pe}^2}{v_{te}} \exp\left(-\frac{Q_0 e}{a T_e}\right) \quad (22)$$

gives

$$\Omega_c = \frac{a}{\sqrt{2\pi}}\left[\frac{\omega_{pi}^2}{v_{ti}} + \frac{\omega_{pe}^2}{v_{te}} \exp\left(-\frac{Q_0 e}{a T_e}\right)\right]. \quad (23)$$

As the electrons diffuse to the grain surface faster than the ions, grains acquire negative charge over electron plasma time scale ω_{pe}^{-1} [second term in Eq. (23)]. This induces an ion response [at an ω_{pi}^{-1} time scale, first term in Eq. (23)] to prohibit the negative charge buildup. After the initial buildup, electrons and ions get attached to the grain surface at such a rate that their total flux is zero, i.e., $I_{e0}+I_{i0}=0$. This situation is very similar to plasma behavior near the wall of a plasma container. Making use of $I_{e0}+I_{i0}=0$, Eq. (23) can be written as

$$\Omega_c = \frac{1}{\sqrt{2\pi}} \frac{\omega_{pi}^2 a}{v_{ti}} (1+\tau+z). \quad (24)$$

The attachment frequency in Eq. (6) $\nu_{\alpha d} = n_d \langle \sigma_\alpha v_\alpha \rangle$, for the Maxwellian distribution can be written in the following form [28]:

$$\nu_{ed}=4\sqrt{\frac{\pi}{2}}a^2 v_{te} n_{d0}\exp[-z]=\Omega_c P\,\frac{\tau+z}{1+\tau+z}, \quad (25)$$

where $P=(a n_{d0} k_B T_e)/(n_{e0}e^2)$ is a dimensionless parameter which determines approximately the ratio of grain charge density to the electron charge density in equilibrium.

The ion-dust collision frequency is given by

$$\nu_{id}=\sqrt{\frac{m_e}{m_i}}\,\tau\nu_{ed}\exp[z]. \quad (26)$$

It is clear from the above expression that electron-dust and ion-dust collision time scales $\nu_{ed}^{-1},\nu_{id}^{-1}$ are of the order of grain charge fluctuation time scale Ω_c^{-1} and thus they will play a vital role in the attenuation or amplification of dusty plasma modes. Let us compare the ion-ion collision frequency ν_{ii} with Ω_c. Making use of the following expression for ν_{ii} [29],

$$\nu_{ii}=\frac{\ln\lambda}{3(2\pi)^{3/2}}\,\frac{\omega_{pi}}{n_i\lambda_{Di}^3}, \quad (27)$$

we get

$$\frac{\Omega_c}{\nu_{ii}}=\frac{3}{2\ln\lambda}\,\frac{t(1+t+z)}{z}\,\frac{Q_0}{e}, \quad (28)$$

where $\ln\lambda$ is the Coulomb logarithm and has a typical value between 10 and 20. Except for the factor Q_0/e (which varies from 10 to 100 in the HI region to 10^4 in the HII region and may be as large as 10^6 in the planetary rings), all other factors are of the order of unity and hence ν_{ii} can be ignored. Similarly, since $\nu_{ii}=(m_e/m_i)^{1/2}\nu_{ee}$, it is clear that for $\nu_{ee}<\Omega_c$, the (Q_0/e) ratio should be $\sim 10^2$ or large. For such a dusty plasma, electron-electron collisions are unimportant. Since $\nu_{ei}=\nu_{ee}/(2\sqrt{2})$, electron-ion collisions are as well unimportant. And last, as $\nu_{ie}\approx(m_i/m_e)^{1/2}\nu_{ii}$, ion-electron collisions can as well be ignored for a dusty plasma for $(Q_0/e)\simeq 10^2$.

It is apparent from Eqs. (25) and (26) that the attachment frequency of the electrons and ions is intimately related to the charging frequency of the grain. Therefore, the neglect of the attachment frequency in the ion and electrons equations [Eq. (6)] is valid under very restricted conditions. Assuming $1+\tau_i+z\approx\tau+z$, we see that for $P\sim O(1)$, attachment and charging frequencies are of the same order. Therefore, the neglect of the attachment frequency for a fluctuating grain charge in Eq. (6) is valid only for a very tenuous dust cloud $(P\ll 1)$. Furthermore, when $P\gg 1$, it is the attachment frequency which plays the dominant role in comparison with the charge fluctuation frequency. It is well known [30] that P can vary in a wide range (for example, for Saturn's E, G, and F rings, the value of P is 10^{-3}, 10, and 1, respectively) and, thus, the above conclusion needs to be borne in mind before ignoring the attachment frequency in comparison with the grain charge fluctuation frequency.

IV. LINEAR STABILITY ANALYSIS

We perturb the distribution function such that

$$f_\alpha=f_{\alpha 0}+f_{\alpha 1}, \quad f_d=f_{d0}+f_{d1}, \quad (29)$$

where $f_{\alpha 1}\ll f_{\alpha 0}$, $f_{d1}\ll f_{d0}$. Then we plug Eq. (29) into Eq. (6) and carry out a normal-mode analysis by choosing the spatial and temporal dependence of the fluctuations in the form $\exp[-i\omega t+\mathbf{k}\cdot\mathbf{x}]$. As our system is inhomogeneous, a normal-mode analysis is applicable only for $kx\gg 1$, i.e., for short-wavelength fluctuations. Then Eq. (6) can be written as

$$f_{\alpha 1}=\frac{\left(\frac{e_\alpha}{m_\alpha}E_1+g_1\right)\hat{\mathbf{k}}\cdot\frac{\partial f_{\alpha 0}}{\partial\mathbf{v}}}{i(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{\alpha d})}, \quad (30)$$

where $\hat{\mathbf{k}}=\mathbf{k}/k$. In the above expression it has been assumed that the linearized force $\mathbf{F}_1$ is parallel to the direction of wave propagation $\mathbf{k}$. The resultant charge density fluctuation is

$$\rho_{\alpha 1}=\int e_\alpha\frac{\left(\frac{e_\alpha}{m_\alpha}E_1+g_1\right)\hat{\mathbf{k}}\cdot\frac{\partial f_{\alpha 0}}{\partial\mathbf{v}}}{i(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{\alpha d})}d\mathbf{v}. \quad (31)$$

Linearizing Eq. (8) gives

$$\frac{\partial f_{d1}}{\partial t}+\mathbf{v}\cdot\frac{\partial f_{d1}}{\partial\mathbf{r}}+\frac{\mathbf{F}_{d1}}{m_d}\cdot\frac{\partial f_{d0}}{\partial\mathbf{v}}+\frac{\partial[I_0(Q)f_{d1}+I_1(Q)f_{d0}]}{\partial Q}=0. \quad (32)$$

Fourier analyzing the above equation in space and time leads to

$$(\omega-\mathbf{k}\cdot\mathbf{v})f_{d1}-i\frac{\partial}{\partial Q}[\Omega_c(Q-Q_0)f_{d1}]$$
$$=-i\left(\frac{Q}{m_d}E_1+g_1\right)\hat{\mathbf{k}}\cdot\frac{\partial f_0}{\partial\mathbf{v}}\,\delta(Q-Q_0)$$
$$-i\frac{\partial}{\partial Q}[I_1(Q)f_0\delta(Q-Q_0)]. \quad (33)$$

The current $I_1(Q)$ is given by

$$I_1(Q)=\Sigma_\alpha\int d\mathbf{v}\,e_\alpha\sigma_\alpha(Q)v f_{\alpha 1} \quad (34)$$

and use has been made of $I_0(Q)=-\Omega_c(Q-Q_0)$. Defining $\rho_{d1}=\int Q\,dQ\,d\mathbf{v}\,f_{d1}$, the grain charge density can be written as

$$\rho_{d1}=\int\frac{\left(\frac{Q_0^2}{m_d}E_1+Q_0 g_1\right)\hat{\mathbf{k}}\cdot\frac{\partial f_0}{\partial\mathbf{v}_\alpha}d\mathbf{v}}{i(\omega-\mathbf{k}\cdot\mathbf{v})}$$
$$+i\int\frac{\left(E_1+\frac{m_d}{e_\alpha}g_1\right)Sf_0 d\mathbf{v}}{(\omega-\mathbf{k}\cdot\mathbf{v})}, \quad (35)$$

where

$$S=\int \Sigma_\alpha \frac{e_\alpha^2 v\sigma_\alpha(Q)\hat{\mathbf{k}}\cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}}}{i m_\alpha(\omega-\mathbf{k}\cdot\mathbf{v}+i\Omega_c)}d\mathbf{v}. \tag{36}$$

Before calculating dielectric permitivity, we eliminate g_1 in terms of E_1 by making use of Poisson's equation

$$\nabla\cdot g_1=\frac{\omega_J^2}{n_{d0}}\int d\mathbf{v}\,dQ f_{d1}, \tag{37}$$

where $\omega_J^2=4\pi G m_d n_{d0}$ is the Jeans frequency. Substituting for f_{d1} from Eq. (33) and integrating Eq. (37) for $\omega\ll\mathbf{k}\cdot\mathbf{v}$, one gets

$$g_1=-\frac{Q_0\omega_J^2}{m_d(\omega^2+\omega_J^2)}E_1. \tag{38}$$

Writing the dielectric permittivity as

$$\epsilon(\omega,k)=1+4\pi\Sigma_\alpha\chi_\alpha(\omega,k)+4\pi\chi_d(\omega,k), \tag{39}$$

from Poisson's equation (12), one gets

$$ik[\Sigma_\alpha\chi_\alpha(\omega,k)+\chi_d(\omega,k)]E_1=-4\pi(\Sigma_\alpha\rho_{\alpha 1}+\rho_{d1}). \tag{40}$$

Substituting for $\rho_{\alpha 1}$ and ρ_{d1} from Eqs. (31) and (35) in Eq. (39), we get the dielectric permittivity. Its two components are

$$\chi_\alpha(\omega,k)=\frac{4\pi e_\alpha^2}{m_\alpha k}\left[1-\frac{Q_0 m_\alpha\omega_J^2}{e_\alpha m_d(\omega^2+\omega_J^2)}\right]\int\frac{\hat{\mathbf{k}}\cdot\frac{\partial f_{\alpha 0}}{\partial\mathbf{v}}}{\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_\alpha} \tag{41}$$

and

$$\chi_d(\omega,k)=\frac{4\pi Q_0^2}{m_d k}\left[1-\frac{\omega_J^2}{(\omega^2+\omega_J^2)}\right]\int\frac{\hat{\mathbf{k}}\cdot\frac{\partial\Phi}{\partial\mathbf{v}}}{\omega-\mathbf{k}\cdot\mathbf{v}}-\frac{4\pi}{k}\times\left[1-\frac{Q_0 m_\alpha\omega_J^2}{e_\alpha m_d(\omega^2+\omega_J^2)}\right]\int\frac{S\Phi}{\omega-\mathbf{k}\cdot\mathbf{v}+i\Omega_c}. \tag{42}$$

For a Maxwellian distribution, when $\omega\gg k v_{td}$ and $\omega\ll\nu_{\alpha d}<k v_{t\alpha}$, the integration of Eq. (36) gives

$$S=\frac{a^2}{\sqrt{2\pi}i}\left(\left\{\frac{\omega_{pe}^2}{k v_{te}}e^{-z}\left[1-\frac{\pi\nu_{ed}}{2\sqrt{2}k v_{te}}\left(\sqrt{z}+\frac{\sqrt{\pi}(1-2z)e^z}{2}\times[1-\mathrm{erf}(\sqrt{z})]\right)\right]\right\}+\frac{\omega_{pi}^2}{k v_{ti}}\left(1-\frac{\pi\nu_{id}}{4\sqrt{2}k v_{ti}}\right)\right). \tag{43}$$

Then the integration of Eqs. (41) and (42) combined with the Eqs. (25), (26), and (43) gives the following expression for the dielectric permittivity:

$$\epsilon(\omega,k)=1+\Sigma_\alpha\frac{1}{k^2\lambda_\alpha^2}\left(1+\sqrt{\frac{\pi}{2}}\frac{\nu_{\alpha d}}{k v_{t\alpha}}\right)^{-1}-\frac{\omega_{pd}^2}{\omega^2+\omega_J^2}+\frac{iP\Omega_c}{k^2\lambda_{D0}^2(\omega+i\Omega_c)}\left[1-\frac{\sqrt{\pi}}{4}\frac{\nu_{ed}}{k v_{te}}\frac{a\omega_{pe}^2}{v_{te}\Omega_c}\times\exp(-z)\left(\sqrt{z}+\frac{\sqrt{\pi}(1-2z)e^z}{2}[1-\mathrm{erf}(\sqrt{z})]\right)-\frac{\sqrt{\pi}}{8}\frac{\nu_{id}}{k v_{ti}}\frac{a\omega_{pi}}{\lambda_i\Omega_c}\right], \tag{44}$$

where $\lambda_\alpha=v_{t\alpha}/\omega_{p\alpha}$ is the Debye length of the plasma particles and $\lambda_{d0}^2=k_B T_e/(4\pi n_{d0}e^2)$. Finally, we write the dispersion relation, given by $\epsilon(\omega,k)=0$, in a more compact form

$$\omega^2\approx\frac{\omega_{pd}^2\Sigma_\alpha k^2\lambda_\alpha^2 C_\alpha}{1+\Sigma_\alpha k^2\lambda_\alpha^2 C_\alpha+\frac{iP\Omega_c\Sigma_\alpha\lambda_\alpha^2 C_\alpha L}{\lambda_{D0}^2(\omega+i\Omega_c)}}-\omega_J^2, \tag{45}$$

where

$$C_\alpha=\left(1+\sqrt{\frac{\pi}{2}}\frac{\nu_{\alpha d}}{k v_{t\alpha}}\right)$$

and

$$L=\left[1-\frac{\sqrt{\pi}}{4}\frac{\nu_{ed}}{k v_{te}}\frac{a\omega_{pe}^2}{v_{te}\Omega_c}\exp(-z)\left(\sqrt{z}+\frac{\sqrt{\pi}(1-2z)e^z}{2}\times[1-\mathrm{erf}(\sqrt{z})]\right)-\frac{\sqrt{\pi}}{8}\frac{\nu_{id}}{k v_{ti}}\frac{a\omega_{pi}}{\lambda_i\Omega_c}\right].$$

The dispersion relation equation (45) is similar to dispersion relation equation (21) of Pandey and Lakhina [18] in the absence of thermal effects, i.e., when $C_\alpha\approx 1$ and $L\approx 1$. However, there is a notable difference with the dispersion equation (21) of Ref. [18]. The description of the charge dynamics of a dusty plasma in a three-fluid model [18] requires the introduction of two frequencies, namely, attachment and charge decay frequency β and η, respectively. Their role in gravitational dynamics varies; whereas in the present kinetic description, only one frequency Ω_c is sufficient to describe the effect of grain charge fluctuations on the Jeans collapse. As a result, we shall see that charge fluctuations do not affect gravitational collapse in any significant way and the Jeans length is not dependent on the grain charge fluctuation, unlike in a fluid model [18].

In the absence of gravity, Eq. (45) reduces to the following dispersion relation:

$$\omega^2\approx\frac{\omega_{pd}^2\Sigma_\alpha k^2\lambda_\alpha^2 C_\alpha}{1+\Sigma_\alpha k^2\lambda_\alpha^2 C_\alpha+\frac{iP\Omega_c\Sigma_\alpha\lambda_\alpha^2 C_\alpha L}{\lambda_{D0}^2(\omega+i\Omega_c)}}. \tag{46}$$

This is the familiar dispersion relation of Melandso *et al.* [31] for the dust-acoustic mode. However, as they have described the grain charge fluctuation in terms of two frequencies similar to β and η, the damping of the dust-acoustic

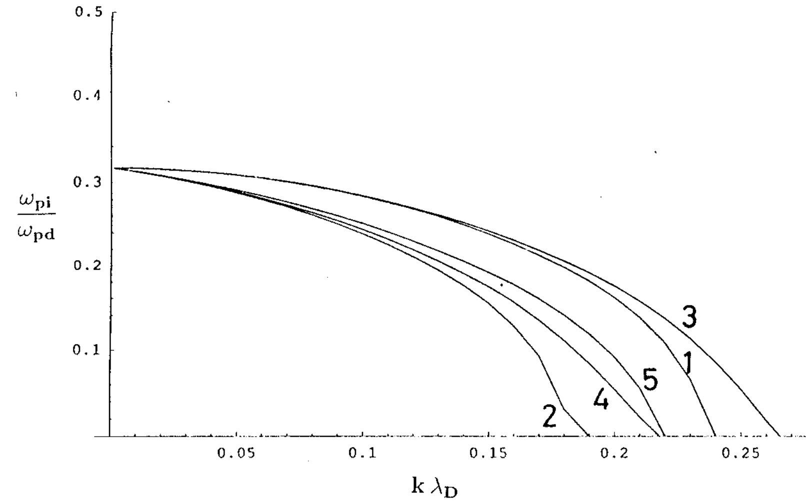

FIG. 1. The unstable root (ω_i/ω_{pd}) of Eq. (45) is plotted against $k\lambda_D$ for $P=1$, $R=0.1$, and $z=2.5$ with $C_e=C_i$, $L=1$, and $\lambda_{D0}=\lambda_e$. Curve 1 corresponds to a pure gravitational mode in the absence of collisions and grain charge fluctuation so that $a=\nu_{ed}/\omega_{pe}=0$, $b=\Omega_c/\omega_{pd}=0$. Curve 2 represents the case with a collision and without grain charge fluctuation $a=0.1$ and $b=0$. Curve 3 corresponds to the case with grain charge fluctuation and no collision, i.e., $b=0$ and $a=0.1$. Curve 4 corresponds to the case with a collision and grain charge fluctuation, i.e., $a=b=0.1$. Curve 5 corresponds to the case when $a=0.1$ and $b=0.8$.

wave is due to the $\beta\,[\Omega_{V_0}$ in the notation of Ref. [31], Eq. (20)] and the real part of dust-acoustic dispersion relation is proportional to $\eta\,[\Omega_{d0}$ in the notation of Melandso *et al.*, Eq. (19)]. As the present kinetic treatment requires only one frequency to describe the grain charge fluctuation, it will be illustrative to derive the approximate form of the real ω_r and imaginary ω_i parts of ω from Eq. (46) in the $\omega_i \ll \omega_i$ limit when $C_\alpha \approx 1$ and $L\approx 1$. This gives

$$\omega_r \approx \omega_{pd}\sqrt{\Sigma_\alpha \lambda_{D\alpha}^2}\left(\frac{1}{1+\Sigma_\alpha k^2\lambda_{D\alpha}^2}\right)^{1/2} k, \tag{47}$$

$$\omega_i \approx -\Omega_c \frac{1+\dfrac{P\Sigma_\alpha \lambda_{D\alpha}^2}{(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)\lambda_{D0}^2}-\dfrac{\omega_{pd}^2\Sigma_\alpha k^2\lambda_{D\alpha}^2}{\omega_r^2(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)}}{3-\dfrac{\omega_{pd}^2\Sigma_\alpha k^2\lambda_{D\alpha}^2}{\omega_r^2(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)}}. \tag{48}$$

As

$$\frac{\omega_{pd}^2\Sigma_\alpha k^2\lambda_{D\alpha}^2}{\omega_r^2(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)} \approx 1,$$

we see that the dust-acoustic mode is damped as

$$\omega_i \approx -\Omega_c \frac{P\Sigma_\alpha \lambda_{D\alpha}^2}{(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)\lambda_{D0}^2}. \tag{49}$$

Making use of Eq. (25), the damping of the dust-acoustic wave can be directly expressed in terms of the electron-dust attachment frequency

$$\omega_i \approx -\frac{\nu_{ed}\Sigma_\alpha \lambda_{D\alpha}^2}{(1+\Sigma_\alpha k^2\lambda_{D\alpha}^2)\lambda_{D0}^2}, \tag{50}$$

where $1+\tau+z\approx\tau+z$ has been assumed. The damping of the dust-acoustic mode is caused by the delay in the charging of the dust grains. As the electron sticks to the grain surface, a repulsive electrostatic field builds up, which subsequently delays the arrival of the electrons. As a result, the dust-acoustic mode dampened.

Next, we solve the dispersion relation (45) numerically and plot the unstable roots (normalized with respect to the dust plasma frequency ω_{pd}) for $R=0.1$, $\tau=1$, $z=2.5$, $n_{e0}\approx n_{i0}$, and $P=1$ [3–5,11,12] (see Fig. 1). Here, we have assumed that $C_e=C_i$, $L=1$, and $\lambda_{D0}=\lambda_e$. Curve 1 is the usual Jeans mode in the absence of charge fluctuations. When $\Omega_c/\omega_{pd}=0$ and $\nu_{ed}/\omega_{pe}=0.1$, the Jeans mode approaches zero faster (curve 2) and at somewhat shorter $k\lambda_D$. As the charge fluctuation is switched on, the growth rate is increased towards a large $k\lambda_D$ (curve 3). Therefore, charge fluctuations may facilitate the condensation of the grain to a somewhat shorter scale length than would have been possible otherwise. Clearly, then, the attachment of the plasma particles to the grain surface resets the repulsive electrostatic field in such a fashion that the collapsing grains are dispersed while approaching each other under the influence of their self-gravitational field. Curves 4 and 5 correspond to $\nu_{ed}/\omega_{pe}=0.1$ and $\Omega_c/\omega_{pd}=0.1$ and 0.8, respectively.

From the figure, it is clear that the growth rate remains unaffected by collisional processes and grain charge fluctuation below $k\lambda_D<0.05$. Note that we have carried out a local analysis, i.e., $kx \gg 1$. These two conditions put a constraint on the region of applicability of the result. If the scale size of the system x is larger than $20\lambda_D$, the gravitational collapse rate is unaffected by collisions between the plasma particles and dust grains. This is a manifestation of the short-range nature of collisional processes. Gravitational condensation operates at a long range, and when the distance between the charged grains becomes few Debye lengths, binary collisions and ensuing sticking or recombination of plasma particles to the grain surface alters the ratio of electrostatic to gravitational force R in favor of the former, causing a decrease in ω_i/ω_{pd}. Therefore, the gravitational collapse rate beyond $20\lambda_D$ is not affected by grain charge fluctuations.

Last, we calculate the Jeans length λ_J from Eq. (45) in various limiting cases. The Jeans length is the critical length at which instability vanishes. Before calculating the Jeans length numerically, we give an approximate analytical expression for some interesting astrophysical situations. It is known that in the planetary rings of Saturn, $P\ll 1$ [30]. Then, using marginal stability condition $(\omega=0)$, we get, from Eq. (45),

$$\lambda_J=\frac{2(\lambda_e^2+\lambda_i^2)}{-\sqrt{\pi/2}\left[\frac{\lambda_e^2\nu_{ed}}{v_{te}}+\frac{\lambda_i^2\nu_{ed}}{v_{ti}}\right]+\sqrt{\frac{\pi}{2}\left(\frac{\lambda_e^2\nu_{ed}}{v_{te}}+\frac{\lambda_i^2\nu_{ed}}{v_{ti}}\right)^2+4\frac{R(\lambda_e^2+\lambda_i^2)}{1-R}}}. \tag{51}$$

We see from the above expression that the plasma thermal pressure force along with the collisional force affects the gravitational collapse of a dusty plasma. When the gravitational attraction between the grains is much smaller than the corresponding electrostatic repulsion $(R\to 0)$, $\lambda_J\to\infty$, i.e., there is no collapse. When two forces are comparable, i.e., $R\to 1$, we have

$$\lambda_J=\lambda_e\sqrt{\frac{(1-R)}{2R}}. \tag{52}$$

As the gravitational attraction overcomes the electrostatic repulsion, the Jeans length shrinks and ultimately becomes zero. This is because we have treated the grains as cold. In reality, kinetic effects would prevent the grain from collapsing to a zero Jeans length.

Next, we derive the Jeans length for $P\geqslant 1$ when $C_e \approx \nu_{ed}/(k\lambda_{De}\omega_{pe})$, $C_i\approx C_e$ (this will cause a slight overestimation of the length, but allow for simplification), $\lambda_e=\lambda_i=\lambda_D$, and approximating $\Sigma_\alpha k^2\lambda_{D\alpha}^2C_\alpha\approx 2k^2\lambda_D^2C_e$. Then, for $R=1$, we have,

$$\left(\frac{\lambda_J}{\lambda_D}\right)\approx\left(\frac{\lambda_{D0}}{\lambda_D}\right)^2\frac{\omega_{pe}}{P\nu_{ed}}\left[-1+\left(1+\frac{\omega_{pe}\lambda_{D0}^2L_1}{P\nu_{ed}\lambda_D}\right)^{1/2}\right]^{-1}, \tag{53}$$

where

$$L_1=\frac{\nu_{ed}}{2\Omega_c}\frac{a\omega_{pe}}{v_{te}}\exp(-z)\left(\sqrt{z}+\frac{\sqrt{\pi}(1-2z)e^z}{2}\times[1-\mathrm{erf}(\sqrt{z})]\right)+\frac{\nu_{id}}{4\Omega_c}\frac{a\omega_{pi}}{v_{ti}}.$$

The above expression suggests that the Jeans length increases with the increase of v_{te}. Also, with the increase of the grain charge Q the Jeans length increases as $\exp[eQ/ak_BT]$. When $\nu_{ed}\to 0$, $\lambda_J\to\infty$, implying that there will be no collapse. As $R=1$, the collapse is halted by the self-repulsion of the grain, and unless there is a change in this ratio, condensation of the grain will not take place. In the other limit $\nu_{ed}\to\infty$, $\lambda_J\to 0$, a large recombination rate at the grain surface neutralizes the electrostatic field, setting $\lambda_J=0$, and collapse takes place without any critical length.

We note, however, that the above limiting cases are at best indicative of a trend as, in a strict sense, the limits $\nu_{ed}\to 0$ and $\nu_{ed}\to\infty$ are not valid since the dispersion relation, Eq. (45), has been derived in the $\omega\gg kv_{td}$ and $\omega\ll\nu_{ed}\ll kv_{td}$ limits.

V. CONCLUSIONS

We have studied the Jeans instability of a dusty plasma in the present work using a kinetic formulation which allows for the self-consistent treatment of charge dynamics. The self-consistent treatment of charge dynamics guarantees that the charge conservation law is not violated. The findings of the present investigation are the following.

(i) When the grain is carrying 100 or more electronic charges, the two most important collisional processes responsible for the grain charge fluctuation are electron-dust and ion-dust collisions.

(ii) The relation between electron-dust and ion-dust collision frequencies with grain charge fluctuation frequency [Eqs. (25) and (26)] suggest that a fluid description of a dusty plasma with grain charge dynamics without the electron and ion loss terms in the electron and ion continuity equations, respectively, is valid only for very tenuous plasma clouds $(P\ll 1)$.

(iii) In the absence of gravity, the effect of dust charge fluctuation on the dust-acoustic mode is manifested through electron-dust collisions. The damping of the dust-acoustic mode is directly proportional to the electron-dust collision frequency.

(iv) In the presence of gravity, the charge fluctuation on the dust grain leads to a reduction in the growth rate of gravitational collapse. Also, the growth rate is unaffected for distances larger than $20\lambda_D$ owing to the short-range nature of the collisional processes responsible for the charge fluctuation.

(v) In addition to the usual thermal pressure and electrostatic repulsion, in a self-gravitating dusty plasma, in the presence of grain charge fluctuations; the Jeans length is also dependent on collisional processes [see Eq. (51)]. The dependence of the Jeans length on collision frequencies may allow the gravitational attraction to operate beyond the Debye length.

These results may help us to understand the real physical system better. Implications of this result for spoke formation in Saturn's ring or on the thickness of the Jovian ring are interesting. For example, as the charge fluctuation modifies the Jeans mode, the lifetime of grain levitation over the ring plane may increase before grains succumb to the planet's gravity.

Observations of molecular clouds (Shu *et al.* [32]) suggest that their dense core has embedded infrared sources, implying that such clouds are the main sites of star formation in galaxies. However, the Jeans mass $M_J=(2\pi\lambda_J)^3\rho_0$ associated with the average condition of cloud clumps of density ρ_0 is much smaller than the mass of the clump, *M*. Then all such clumps should be unstable to the Jeans instability. If this were the case, all such molecular clouds would be collapsing on a gravitational time scale ω_J^{-1}; otherwise, the rate of star formation in the galaxy would far exceed the observed rate. The presence of a repulsive electrostatic field in such a dusty (plasma) cloud clump will oppose the gravitational collapse. As a result, the star formation rate will be affected. It is quite possible that such a repulsive electrostatic field set up during the gravitational collapse of the charged grain is opposing the gravitational attraction of the cloud complex and thus reducing the chance of an excessive rate of star formation.

Our results are important for understanding the dynamics of self-gravitating plasmas and, thus, to the formation of stars, clusters, etc. Specific applications will be studied in future work.

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