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On the existence of interfacial waves with inclined magnetic fields

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Abstract. The dispersive characteristics of interfacial waves in low β plasma is studied. The condition for the existence of these waves is derived. It is assumed that the magnetic field and the propagation vector are inclined at different angles to the density discontinuity which is horizontal.The dispersion relation for such a configuration is solved for the interfacial (surface) waves as a function of the propagation angle for a given inclination of the magnetic field. The normalized phase speed of these waves are studied for different values of $\alpha = \rho_{02}/\rho_{01}$, γ_1 and γ_2.

Key words: interfacial waves – low β plasma – inclined magnetic fields

1. Introduction

Gravity and magnetism play an important role in stratifying and structuring the solar atmosphere. The combined action of these forces complicates the description of waves that occur in the solar atmosphere. Surfaces of discontinuity across which the plasma and magnetic properties vary rapidly can support magnetohydrodynamic interfacial (surface) waves.

By interfacial waves we mean waves that propagate on a sharp (discontinuous) interface. These waves are anisotropic and guided by the interface. The phase speed of these waves generally lies between the bulk speeds of the media on either sides of the interface.

Several authors have investigated wave propagation in a magnetically structured atmosphere (see Roberts (1991) for a review). Wentzel (1978), Roberts(1981), Somasundaram and Uberoi (1982), Miles and Roberts (1989), Jain and Roberts (1991), Singh and Talwar (1993) have investigated the properties of waves arising on a single magnetic interface. Recently, Satya Narayanan (1995), studied surface waves in a two layered fluid model wherein the magnetic field was inclined at an angle to the interface in the upper region, while the lower region was field free. The dispersion relation was solved for the normalized phase speed of the waves as a function of the propagation angle θ for different values of the parameters $\alpha = \rho_{02}/\rho_{01}$ and $\delta = c_2/v_{A1}$. Here ρ_{02} and ρ_{01} are the mass densities on either side of the interface, while v_{A1} and c_2 are the Alfven velocity and sound velocities in the media 1 and 2, respectively.

In this paper, we will be concerned with a plane surface, the single magnetic interface. Although the single magnetic interface is the most elementary of field structures, its detailed study provides a valuable insight into the general nature of the surface wave propagation. We also assume the plasma β to be very small. The dispersion relation is presented in the next section. Discussion of the results and concluding remarks are made in the subsequent sections.

2. Dispersion relation

Let $x = 0$ be the interface between two compressible media where the region $x < 0$ is denoted by the suffix "1" and the region $x > 0$ by the region "2". The magnetic fields which are inclined at an angle to the interface can be chosen to be of the form

$$B_{01,2} = (0, B_{01,2}Cos\gamma_{1,2}, B_{01,2}Sin\gamma_{1,2}) \tag{1}$$

The wave vector is chosen to be

$$K = (0, kSin\theta, kCos\theta) \tag{2}$$

Here $\gamma_{1,2}$ is in general different from θ. Let ρ_{01} and ρ_{02} be the mass densities on either side of the interface and $c_{1,2}$ and $v_{A1,2}$ be the sound and Alfven speeds, respectively. Solving the linearized magnetohydrodynamic equations at the interface leads to the dispersion relation (Uberoi and Satya Narayanan (1986))

$$\tau_1\epsilon_2(\omega, k) + \tau_2\epsilon_1(\omega, k) = 0 \tag{3}$$

where

$$\epsilon_1(\omega, k) = \rho_{01}(-\omega^2 + k^2v_{A1}^2Sin^2(\theta + \gamma_1)) \tag{4}$$

$$\epsilon_2(\omega, k) = \rho_{02}(-\omega^2 + k^2v_{A2}^2Sin^2(\theta + \gamma_2)) \tag{5}$$

and

$$\tau_{1,2}^2 = (\omega^4 - A + B)/(C - D) \tag{6}$$

where

$$A = k^2\omega^2(v_{A1,2}^2 + c_{1,2}^2)$$

$$B = k^4 c_{1,2}^2 v_{A1,2}^2 Sin^2(\theta + \gamma_{1,2})$$

$$C = k^2 c_{1,2}^2 v_{A1,2}^2 Sin^2(\theta + \gamma_{1,2})$$

$$D = \omega^2 (c_{1,2}^2 + v_{A1,2}^2)$$

Eq. (3) is very similar to Eq. (21) of Jain and Roberts (1991). In the incompressible limit, $c_{1,2} \to \infty$ $\tau_1^2 = \tau_2^2 = k^2$ and Eq. (3) reduces to

$$\rho_{01}(-\omega^2 + k^2 v_{A1}^2 Sin^2(\theta + \gamma_1))$$

$$+\rho_{02}(-\omega^2 + k^2 v_{A2}^2 Sin^2(\theta + \gamma_2)) = 0 \quad (7)$$

which gives

$$(\omega^2/k^2) = (B_{01}^2 Sin^2(\theta + \gamma_1) + B_{02}^2 Sin^2(\theta + \gamma_2))/E \quad (8)$$

where $E = (\rho_{01} + \rho_{02})$. Eq. (8) is a generalization to Eq. (27) of Jain and Roberts (1991).

The dispersion relation (3) with the coefficients (4), (5) and (6) is quite complicated to study analytically and one has to resort to numerical methods. However, under some specific approximations which are realistic and applicable to the solar atmosphere the dispersion relation can be simplified. Assume that the sound speed is small compared to the Alfven speed. In this case

$$\tau_1^2 = k^2(1 - \frac{\omega^2}{k^2 v_{A1}^2}), \tau_2^2 = k^2(1 - \frac{\omega^2}{k^2 v_{A2}^2}) \quad (9)$$

Define $\alpha = \rho_{02}/\rho_{01}$, $x = \omega/k v_{A1}$, $\lambda_1 = Sin(\theta + \gamma_1)$ and $\lambda_2 = Sin(\theta + \gamma_2)$. It is important to note that for the low β case the pressure balance condition given by

$$p_1 + B_{01}^2/2\mu = p_2 + B_{02}^2/2\mu \quad (10)$$

at the interface would yield $\rho_1 v_{A1}^2 \approx \rho_2 v_{A2}^2$. In this case $\tau_1 = k(1 - x^2)^{1/2}$ and $\tau_2 = k(1 - \alpha x^2)^{1/2}$. The dispersion relation can be simplified to yield

$$(1 - \alpha x^2)^{1/2}({\lambda_1}^2 - x^2)+$$

$$(1 - x^2)^{1/2}({\lambda_2}^2 - \alpha x^2) = 0 \quad (11)$$

Before discussing the roots of the above dispersion relation, we shall discuss some special cases.

The dispersion relation (11) for the case when $\gamma_1 = \gamma_2 = \pi/2$ reduces to

$$(1 - \alpha x^2)^{1/2}(Cos^2\theta - x^2)+$$

$$(1 - x^2)^{1/2}(Cos^2\theta - \alpha x^2) = 0 \quad (12)$$

which can be simplified to yield

$$\alpha x^4 - (1 + \alpha)x^2 + Cos^2\theta(1 + Sin^2\theta) = 0 \quad (13)$$

so that

$$x^2 = ((1 + \alpha) \pm [(1 - \alpha)^2 + 4\alpha Sin^4\theta]^{1/2}/2\alpha \quad (14)$$

The above relation is same as given in Jain and Roberts (1991). For the case $\theta = 0$ (parallel propagation) the dispersion relation (11) becomes

$$(1 - \alpha x^2)^{1/2}(Sin^2\gamma_1 - x^2)+$$

$$(1 - x^2)^{1/2}(Sin^2\gamma_2 - \alpha x^2) = 0 \quad (15)$$

which can be simplified to yield

$$x^6(\alpha^2 - \alpha) + x^4(1 - \alpha^2 + 2\alpha Sin^2\gamma_1 - 2\alpha Sin^2\gamma_2)$$

$$+x^2(Sin^4\gamma_2 - \alpha Sin^4\gamma_1 + 2\alpha Sin^2\gamma_2 - 2Sin^2\gamma_1)$$

$$+(Sin^4\gamma_1 - Sin^4\gamma_2) = 0 \quad (16)$$

For $\gamma_1 = \gamma_2$, the above relation reduces to

$$\alpha x^4 - (1 + \alpha)x^2 + Sin^2\gamma(1 + Cos^2\gamma) = 0 \quad (17)$$

so that

$$x^2 = ((1 + \alpha) \pm [(1 - \alpha)^2 + 4\alpha Cos^4\gamma]^{1/2})/2\alpha \quad (18)$$

Whenever θ, γ_1 and γ_2 are such that $(\theta + \gamma_1)$ and $(\theta + \gamma_2)$ is 90°, the dispersion relation reduces to

$$(1 - \alpha x^2)^{1/2}(1 - x^2)+$$

$$(1 - x^2)^{1/2}(1 - \alpha x^2) = 0 \quad (19)$$

This is same as in Roberts (1981) with $l = 0$. This has roots $x = \pm 1, \pm \alpha^{-1/2}$, which do not describe surfaces waves.

In order to derive the relation for the existence of surface modes, we turn to Eq. (3). It is interesting to note that Eq. (3) will have real roots only when ϵ_1 and ϵ_2 are of opposite signs and the decaying constants $\tau_{1,2}$ should both be positive for the roots to represent surface wave propagation. This implies that ω/k should lie in the region

$$min(v_{A1,2} Sin(\theta + \gamma_{1,2})) < \omega/k <$$

$$max(v_{A1,2} Sin(\theta + \gamma_{1,2})) \quad (20)$$

For the case when the lower fluid is field free as considered by Satya Narayanan (1995), the behaviour of the surface waves differ significantly compared to the present study.

3. Discussion of the results

It is evident from the dispersion relation that the phase speed depends on α, γ_1, and γ_2. We are forced to present the results for specific values of these parameters. The normalized phase speed of the surface waves as a function of the propagtion vector θ is presented in Fig. 1 for $\alpha = 0.8$, $\gamma_1 = 60°$ and different values of $\gamma_2 = 50°, 60°, 70°, 75°$. The phase speed has an increasing trend upto $\theta = 30°$ and monotonically decreases.

An increase in the inclination angle γ_2 lowers the phase speed of these waves for $\theta > 30°$.

Fig. 2 presents a similar situation when $\alpha = 0.8$, $\gamma_2 = 40°$ and $\gamma_1 = 30°, 35°, 45°$ and 50°. In contrast to Fig. 1, the phase speed of the wave increases monotonically upto $\theta \approx 50°$ and then decreases. Here also, the phase speed of the wave decreases as

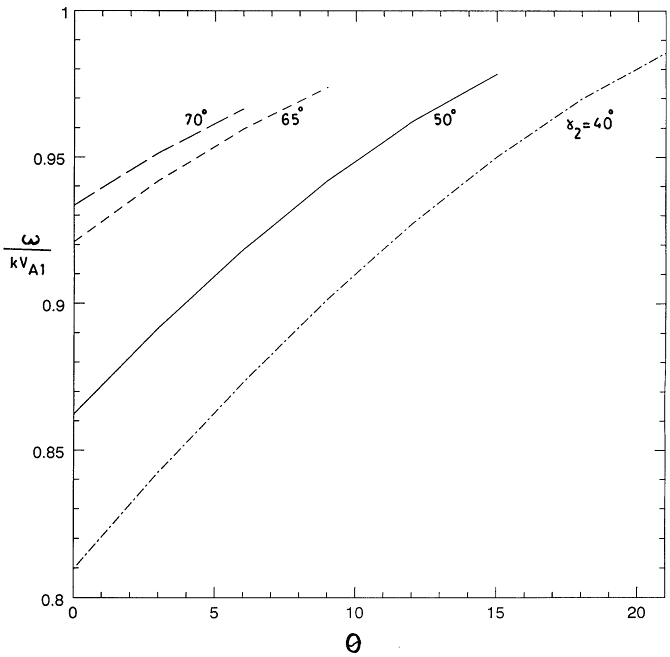

Fig. 1. Normalized phase speed of the surface wave as a function of θ

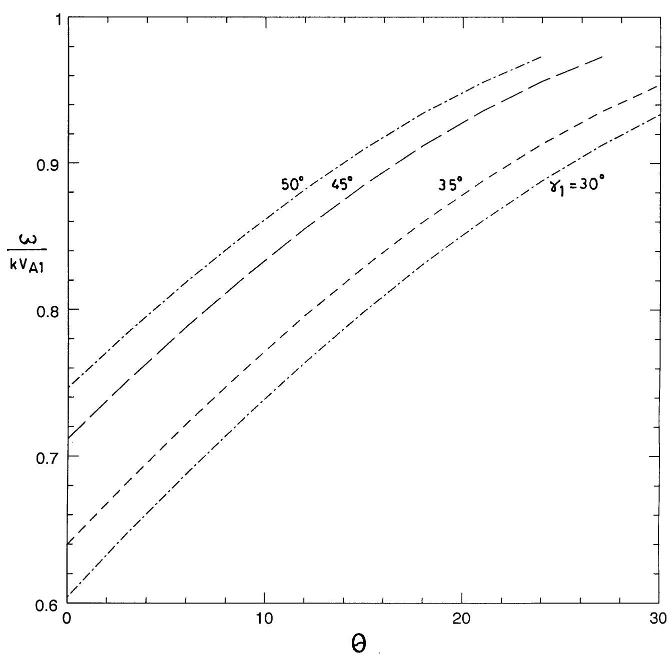

Fig. 2. Same as in Fig. 1, but for different γ

the inclination angle is increased for $\theta > 50°$. The magnitude of the phase speed is less for the waves in Fig. 2 compared to Fig. 1. This clearly indicates the strong dependence of the phase speed on the inclination angles. We have carried out similar calculations for other choice of parametric values. The results are qualitatively similar and we skip the details.

Fig. 3 presents the variation of phase speed for different values of α with γ_1 = 60° and γ_2 = 50°. The phase speed reduces

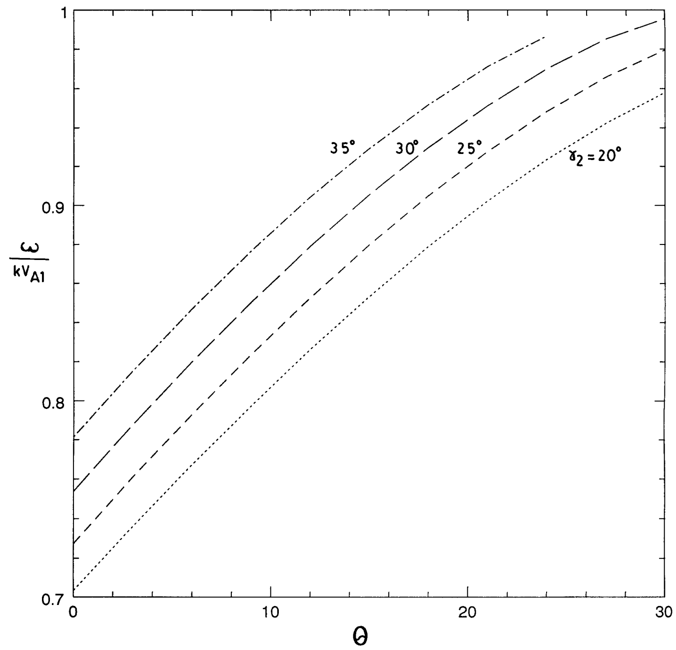

Fig. 3. Variation of the phase speed of the surface wave for different γ_2 with γ_1 = 60°.

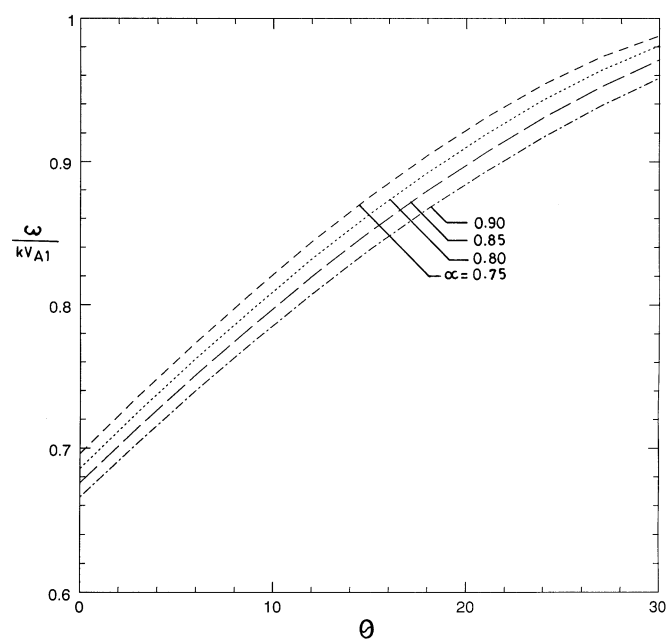

Fig. 4. Variation of the phase speed of the surface wave for different α.

significantly with increasing values of α, the trend being the same. The variation is more for $\theta > 40°$.

4. Conclusions

The dispersive characteristics of the interfacial (surface) waves are greatly altered when the inclination angle of the magnetic field to the interface is different from being parallel or perpendicular. The magnitude of the phase speed decreases with an increase of α. More realistic geometries will have to be studied to get a better picture of these waves. This will be pursued in future.

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