EFFECT OF NEWTONIAN COOLING ON WAVES IN A MAGNETIZED ISOTHERMAL ATMOSPHERE

DIPANKAR BANERJEE and S. S. HASAN
Indian Institute of Astrophysics, Sarjapur Road, Bangalore 560034, India

J. CHRISTENSEN-DALSGAARD
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, and
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark

(Received 15 July 1996; in revised form 17 December 1996)

Abstract. We examine the influence of nonadiabatic effects on the modes of an isothermal stratified magnetic atmosphere. The present investigation is a continuation of earlier work by Hasan and Christensen-Dalsgaard (1992) and Banerjee, Hasan, and Christensen-Dalsgaard (1995, 1996), where the interaction of various elementary modes in a stratified magnetized atmosphere was studied in the purely adiabatic limit. The inclusion of radiative dissipation based on Newton's law of cooling demonstrates the importance of this effect in the study of magnetoatmospheric waves. We analyze the physical nature of magnetoacoustic gravity (or MAG) oscillations in the presence of Newtonian cooling and find that the eigenfrequency curves in the diagnostic diagram, as in the previous analysis, undergo *avoided crossings*. However, the qualitative nature of the mode interaction is strongly influenced by radiative dissipation, which leads to strong mode damping in the avoided-crossing regions. We demonstrate this effect for the interaction between the Lamb mode and a magnetic mode. Our results could be important in the analysis of waves in flux tubes on the Sun.

1. Introduction

Observations of oscillations in the solar atmosphere provide a powerful diagnostic tool to study the nature of magnetic structures. It is well known that the solar photosphere is permeated with vertical magnetic fields, usually in the form of flux tubes with different scales. Observation of oscillations in these magnetic structures have been widely reported. The high-resolution imaging observations (Berger and Title, 1996) suggest that the strong component of the magnetic field outside of sunspots and pores is concentrated in isolated flux tubes of about 200 km in diameter with kilogauss field strength. There is further evidence of weak-field components at the center of supergranular cells (Zirin, 1987). Estimate of the strength of this field component vary between 100−500 G (Keller *et al.*, 1994). Apart from well-known umbral oscillations, recent simulations (Steiner *et al.*, 1994) predict that the effect of granular convection is to induce oscillations of the flux tubes with a characteristic time scale of about 5 min. Hence it is natural to investigate the theoretical nature of the various wave modes that can be present in these magnetic structures. The purpose of this short contribution is to provide some further insight into the properties of magnetoatmospheric oscillations.

The present investigation is a continuation of earlier work by Hasan and Christensen-Dalsgaard (1992, Paper I) and Banerjee, Hasan, and Christensen-

Solar Physics **172:** 53–60, 1997.

Dalsgaard (1995 (Paper II), 1996), where the interaction of various elementary modes in a stratified magnetized atmosphere was studied in the purely adiabatic limit. A detailed study of the general case of radiation hydrodynamics was provided by Mihalas and Mihalas (1984), while Bogdan and Knölker (1989) considered compressive waves in a homogeneous radiating magnetized fluid. We consider the slightly more complex problem of an isothermal, stratified atmosphere but simplify the treatment of radiative effects, approximating them by Newton's law of cooling (e.g., Spiegel, 1957; Mihalas and Mihalas, 1984). It was pointed out by Bünte and Bogdan (1994) that Newtonian cooling can be incorporated in the solution of any isothermal magneto-atmospheric wave problem by replacing γ, the ratio of specific heats, by a complex frequency-dependent quantity. This procedure permits one to generalize easily the previous calculations to include radiative dissipation. Bünte and Bogdan treated a planar, isothermal and stratified atmosphere in the presence of a horizontal magnetic field, whereas in this paper we consider a vertical magnetic field.

2. The Wave Equation with Newtonian Cooling

We shall confine our attention to an isothermal atmosphere with a vertical magnetic field which is unbounded in the horizontal direction. We assume the Lagrangian displacement ξ to vary as $\xi \sim \mathrm{e}^{i(\omega t - kx)}$, where ω is the angular frequency and k is the horizontal wave number. Assuming Newton's law of cooling, and taking the vertical dimension of the perturbation to be small compared with the local scale height, the relation between the Lagrangian perturbations in pressure p and density ρ is approximately given by $\delta p/p \simeq \gamma^* \delta\rho/\rho$, where

$$\gamma^*(\omega) = \frac{1 + \mathrm{i}\omega\tau_{\mathrm{R}}\gamma}{1 + \mathrm{i}\omega\tau_{\mathrm{R}}} \; ; \tag{1}$$

here

$$\tau_{\mathrm{R}} = \frac{\rho c_v}{16\chi\sigma T^3} \tag{2}$$

is the radiative relaxation time, χ being the mean linear absorption coefficient per unit length, T the temperature, c_v the specific heat per unit volume, and σ the Stefan-Boltzmann constant. For simplicity, we assume that τ_{R} is constant over the atmosphere. With these assumptions, the linearized equations for MAG waves are given by a system of two differential equations,

$$\left[v_{\mathrm{A}}^2 \frac{\mathrm{d}^2}{\mathrm{d}z^2} - (\tilde{\gamma} c_{\mathrm{S}}^2 + v_{\mathrm{A}}^2)k^2 + \omega^2\right] \xi_x - \mathrm{i}k \left[\tilde{\gamma} c_{\mathrm{S}}^2 \frac{\mathrm{d}}{\mathrm{d}z} - g\right] \xi_z = 0 \, , \tag{3}$$

$$\left[\tilde{\gamma} c_{\mathrm{S}}^2 \frac{\mathrm{d}^2}{\mathrm{d}z^2} - \tilde{\gamma}\gamma g \frac{\mathrm{d}}{\mathrm{d}z} + \omega^2\right] \xi_z - \mathrm{i}k \left[\tilde{\gamma} c_{\mathrm{S}}^2 \frac{\mathrm{d}}{\mathrm{d}z} - (\tilde{\gamma}\gamma - 1)g\right] \xi_x = 0 \, , \tag{4}$$

where ξ_z and ξ_x are the amplitudes of the vertical and horizontal displacement, g is the acceleration due to gravity, and $\tilde{\gamma}$ is a dimensionless parameter given by $\tilde{\gamma} = \gamma^*/\gamma$. The adiabatic sound speed and Alfvén speed are given, respectively, by

$$c_{\mathrm{S}} = \sqrt{\frac{\gamma p}{\rho}} \quad \text{and} \quad v_{\mathrm{A}} = \frac{B}{\sqrt{4\pi\rho}} . \tag{5}$$

Equations (3) and (4) have the same structure as the linearized wave equation for adiabatic perturbations (see Paper I), apart from the appearance of the parameter $\tilde{\gamma}$, which incorporates Newtonian cooling. In the limit $\tau_{\mathrm{R}} \to \infty$ i.e. in the limit of adiabatic perturbations, $\gamma^* = \gamma$ and $\tilde{\gamma} = 1$. Letting $\tilde{\gamma} = 1$ in Equations (3) and (4) we recover the linearized equations given in Paper I. In the limit $\tau_{\mathrm{R}} \to 0$, corresponding to isothermal perturbations, $\gamma^* = 1$.

In an isothermal atmosphere ρ has the following height dependence

$$\rho = \rho_0 \mathrm{e}^{-z/H} , \tag{6}$$

where H is the scale height of the atmosphere. It is convenient to work in terms of dimensionless wave number and frequency parameters defined by

$$K = kH , \quad \text{and} \quad \Omega = \frac{\omega H}{c_{\mathrm{S}}} , \tag{7}$$

and the dimensionless vertical coordinate

$$\theta = \frac{\omega H}{v_{\mathrm{A}}} = \frac{c_{\mathrm{S}}}{v_{\mathrm{A},0}} \, \Omega \, \mathrm{e}^{-z/(2H)} , \tag{8}$$

where $v_{\mathrm{A},0}$ is the Alfvén speed at $z = 0$. Using the definition of dimensionless frequency the complex gamma can be written as

$$\gamma^* = \frac{1 + \mathrm{i}\Omega\tilde{\tau}_{\mathrm{R}}\gamma}{1 + \mathrm{i}\Omega\tilde{\tau}_{\mathrm{R}}} , \tag{9}$$

where the dimensionless relaxation time scale is given by $\tilde{\tau}_{\mathrm{R}} = (c_{\mathrm{S}}/H)\tau_{\mathrm{R}}$. In terms of the variables defined by Equations (7) – (9), Equations (3) and (4) can be combined into a fourth-order differential equation for ξ_x whose general solution can be expressed in terms of Meijer functions (Zhugzhda, 1979).

3. Solutions for $K \to 0$

For physical reasons and also for the purpose of mode classification it is instructive to consider first the limiting case $K \to 0$. From Equations (3) and (4), we find that in the limit $K = 0$, ξ_x and ξ_z become decoupled. It is fairly straightforward to see that as $K \to 0$, there are two sets of solutions (Paper I):

$$\xi_x = c_1 J_0(2\theta) + c_2 Y_0(2\theta)\ , \qquad \xi_z = 0\ , \tag{10}$$

$$\xi_z = c_3 \theta^{-1+\mathrm{i}\alpha} + c_4 \theta^{-1-\mathrm{i}\alpha}\ , \qquad \xi_x = 0\ , \tag{11}$$

where $\alpha = (4\Omega^2/\tilde{\gamma} - 1)^{1/2}$, c_i $(i = 1, 2, \ldots)$ are constants, and J_0 and Y_0 are the Bessel functions. The asymptotic limit of Equation (10) corresponds to the slow MHD waves for arbitrary field strength and this solution is the same as in the case of purely adiabatic propagation. Thus in this limit the radiative diffusion does not affect the slow MHD waves discussed in Papers I and II. On the other hand, Equation (11) represents a vertically propagating wave when, approximately, $|\Omega|$ exceeds the cut-off frequency Ω_c (which in dimensionless units is $\sqrt{|\tilde{\gamma}|}/2$). In the limit $\tau_\mathrm{R} \to \infty$, Ω_c reduces to the adiabatic acoustic cut-off frequency Ω_a and the solutions of Equation (11) represent adiabatic acoustic modes in an unmagnetized plasma. In the general case of finite cooling time, the solutions of Equation (11) have been affected by the radiation field compared with the purely adiabatic case.

4. $K - \Omega$ Diagram for Weak Field

The normal modes of a stratified atmosphere with a weak magnetic field in the purely adiabatic limit were extensively studied in Papers I and II. We present here the numerical results for the nonadiabatic case. As before we consider a cavity of thickness d, which permits standing-wave solutions. The behaviour of the MAG waves is reflected in their properties in the $K - \Omega$ diagram. We focus on zero-displacement boundary conditions at both ends of the cavity:

$$\xi_x = \xi_z = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d\ . \tag{12}$$

Figures 1(a) and (b) show respectively the variation of the real and imaginary part of the complex frequency with horizontal wave number K, for $\tilde{\tau}_\mathrm{R} = 0.05$. The solutions were obtained by solving the Equations (3) and (4) numerically, using a complex version of the Newton–Raphson–Kantorovich scheme (Cash and Moore, 1980) with the above boundary conditions. The atmosphere is characterized by $D = 1, \epsilon = 0.01$, and $\gamma = 5/3$, where $D = d/H$ is the dimensionless height and $\epsilon = v_{\mathrm{A},0}/c_\mathrm{S}$ is a measure of magnetic field strength. The figures depict a region of the $K - \Omega$ diagram where the magnetic modes are strongly influenced by the Lamb mode, shown by the dashed curve in Figure 1(a) in the purely adiabatic limit.

Comparison of Figure 1(a) with Figure 7 of Paper I (purely adiabatic case) enables us to identify the elementary wave modes and discern the effect of radiative heat exchange on the general properties of the modes. Firstly, the real parts of the magnetic-mode frequencies are essentially unaffected by the inclusion of radiative losses, as already indicated by Equation (10). However, the avoided crossings have shifted because the pure Lamb mode has been modified. Another interesting feature, shown in Figure 1(b), is that we have a substantial increase in the imaginary part

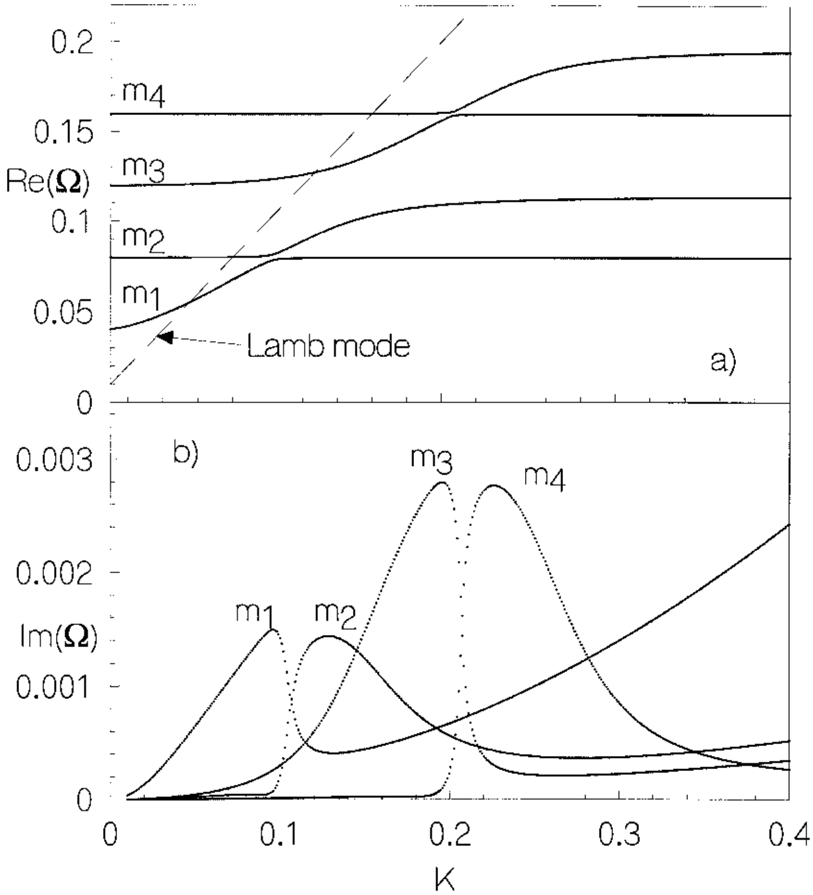

Figure 1. Part of the diagnostic diagram showing the interaction between the magnetic modes with Lamb mode, for a dimensionless radiative relaxation time $\tilde{\tau}_R = 0.05$. Variation with K of the (a) real (solid lines) and (b) imaginary (dotted lines) parts of the frequency. In panel (a) the dashed line marks the location of the adiabatic Lamb mode.

of the frequency in that portion of the $K - \Omega$ diagram where there is an avoided crossing between the modified Lamb mode and the magnetic modes. It appears that damping is enhanced in those parts of the $K - \Omega$ plane where the modes behave predominantly as the Lamb mode. The alternation between narrow and broad avoided crossings of the real part of Ω in Figure 1(a) gives rise to an asymmetry in the behaviour of the imaginary part of Ω as a given mode changes between being predominantly of a m-mode and a Lamb-mode nature. This is closely analogous to the results of the asymptotic analysis of Paper I, concerning the behaviour in the vicinity of nearly degenerate modes: in the present case of nonadiabatic wave propagation, we again find that ‘even’ crossings are typically much narrower than the ‘odd’ crossings.

A striking difference, compared with the adiabatic case, is the lack of an upturn in Ω as a function of K for the m_1 mode at the highest K considered (compare with Figure 5 of Paper I). As discussed in Paper I, this upturn probably arises because of interaction with the lowest-order g mode; otherwise, the low-frequency g modes, with frequencies below the m_1 mode, appear to be eliminated. With Newtonian cooling, the effective Brunt-Väisälä frequency, given by $\tilde{\Omega}_{\rm BV}^2 = (\gamma^* - 1)/\gamma\gamma^*$ (in dimensionless units), decreases with decreasing τ_R and is zero for $\tau_R = 0$. The same is therefore true of the frequencies of pure (i.e., non-magnetic) g modes. Indeed, in the case of a non-magnetic atmosphere it was shown (Mihalas and Mihalas, 1984; Bünte and Bogdan, 1994) that if cooling occurs on a sufficiently short time scale gravity waves cannot exist. Physically, this is not surprising: the buoyancy force that drives gravity waves arises solely from horizontal temperature fluctuations, which vanish when $\tau_{\rm R} \rightarrow 0$. This explains the lack of effect of the g-mode interaction in Figure 1(a); indeed, we note that the real part of $\tilde{\Omega}_{\rm BV}$, evaluated at $\Omega = 0.5$, is about 0.072 for this value of $\tilde{\tau}_{\rm R}$. We conjecture, however, that the comparatively rapid increase with K of the imaginary part of the frequency of the m_1 mode might be related to a beginning influence of the low-frequency g modes for larger values of K.

The interaction between the magnetic mode and the modified Lamb-mode is illustrated in Figure 2(a) for different values of $\tilde{\tau}_{\rm R}$. As $\tilde{\tau}_{\rm R}$ increases, i.e., as we approach the adiabatic limit, the avoided crossing shifts towards the left and the modified magnetic mode approaches the dashed line (behaves more as a pure adiabatic Lamb mode). Hence the general effect of a finite cooling time is the shift of the avoided intersection point.

It is instructive to consider a single mode, which we choose as the first magnetic mode m_1 and to follow its behaviour as it interacts with the Lamb mode. Figure 2(b) depicts the variation with K of the imaginary part of the complex frequency, for different values of $\tilde{\tau}_{\rm R}$. The figure clearly reveals that for $\tilde{\tau}_{\rm R} > 5$ and $\tilde{\tau}_{\rm R} < 0.1$ the imaginary part of Ω approaches zero, which further indicates that in these limits there is no dissipation. However, for intermediate values of $\tilde{\tau}_{\rm R}$ there is mode damping. As seen above, the damping is largest where the modes take on the nature of a Lamb mode in the avoided-crossing regions. Another interesting feature is that in the range $0.1 < \tilde{\tau}_R < 5$ we have a considerable increase in the imaginary part of Ω for large values of K; as before, this might indicate a growing coupling between g modes and the m modes which allows the modes to decay faster.

5. Conclusions

We have presented new solutions for magnetoatmospheric waves in an isothermal atmosphere with a vertical magnetic field in the presence of radiative heat exchange based on Newton's law of cooling. This radiative heat exchange gives rise to a temporal decay of oscillations with a characteristic dimensionless decay time

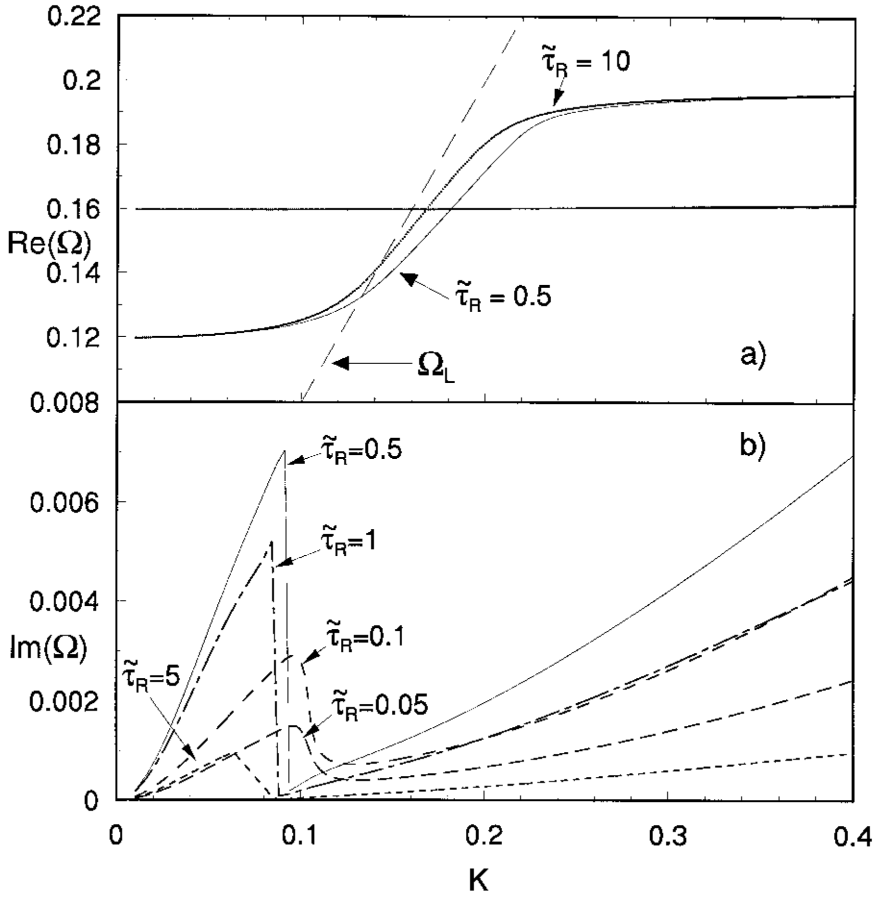

Figure 2. Region of interaction between m- and Lamb-type solutions for different values of $\tilde{\tau}_R$ as labeled. Variation of (a) real part and (b) imaginary part of the frequencies with K. The dashed line in (a) shows the adiabatic Lamb mode.

$\tilde{\tau}_D = 1/\ \Omega_I$, where Ω_I is the imaginary part of Ω. Depending on the value of the radiative relaxation time $\tilde{\tau}_R$, the modes are effectively damped by the radiative dissipation in as short a time as two oscillation period; however, in the limits of very large or very small $\tilde{\tau}_R$, corresponding to nearly adiabatic or nearly isothermal oscillations, the modes are essentially undamped. The existence of mode damping in the presence of radiative exchange is hardly surprising; however, a new feature of our analysis is that the damping is significantly enhanced by the mode coupling in the regions of avoided crossing. For small-scale magnetic structures on the Sun such mechanisms might be very important for wave leakage.

We expect this study to contribute to the investigation of heating in active regions. The present analysis allows us to understand qualitatively the behaviour of the normal modes in the presence of radiative heat exchange. A more comprehensive treatment based on an asymptotic analysis similar to the one in Paper I is needed, however, to shed further light on details of the spectrum and interaction of the modes. This will be attempted in a subsequent investigation.

Acknowledgements

This work was partly supported by the Danish National Research Foundation through the establishment of the Theoretical Astrophysics Center. Support from the Smithsonian Institution through a foreign currency grant No. 9411FR00007 is gratefully acknowledged.

References

Banerjee, D., Hasan, S. S., and Christensen-Dalsgaard, J.: 1995, *Astrophys. J.* **451**, 825. (Paper II).
Banerjee, D., Hasan, S. S., and Christensen-Dalsgaard, J.: 1996, *Bull. Astron. Soc. India* **24**, 325.
Berger, T. E. and Title, A. M.: 1996, *Astrophys. J.* **463**, 365.
Bogdan, T. J. and Knölker, M.: 1989, *Astrophys. J.* **339**, 579.
Bünte, M. and Bogdan, T. J.: 1994, *Astron. Astrophys.* **283**, 642.
Cash, J. R. and Moore, D. R.: 1980, *BIT* **20**, 44.
Hasan, S. S. and Christensen-Dalsgaard, J.: 1992, *Astrophys. J.* **396**, 311 (Paper I).
Keller, C. U., Deubner, F. L., Egger, U., Fleck, B., and Povel, M. P.: 1994, *Astron. Astrophys.* **286**, 626.
Mihalas, D. and Mihalas, B. W.: 1984, *Foundations of Radiation Hydrodynamics*, Oxford University Press, New York.
Spiegel, E. A.: 1957, *Astrophys. J.* **126**, 202.
Steiner, O., Grossman-Doerth, V., Knölker, M., and Schüssler, M.: 1994, in G. Klare (ed.), *Rev. Mod. Astron.* **7**, 17.
Zirin, H.: 1987, *Solar Phys.* **110**, 101.
Zhugzhda, Yu. D.: 1979, *Soviet Astron.* **23**, 353.