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An estimation of the amount of heating in solar coronal loops II. Cooling through conduction-driven evaporation*

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Abstract. In this paper a theory of cooling of solar coronal loops through conduction driven evaporation in presence of a source of heating is presented. No attempt has been made to specify the nature of the source. The theory has been applied to estimate the amount of heating for the flare kernel of 1973 September 1 for different breadths of the source of heating. These results are then compared with those of Elwert & Narain (1980).

Key words : coronal loop—evaporative cooling—heating estimate

1. Introduction

Elwert & Narain (1980) estimated the amount of heating for a few solar coronal loops under conduction-dominated cooling. They neglected the exchange of plasma between the loop and the underlying atmospheric layers. This corresponds to a static case. Antiochos & Sturrock (1976, 1978) and Krieger (1978) have pointed out that all static models predict the transmission of large heat fluxes to the transition zone and to the upper chromosphere, for which no observational evidence seems to be available (Krieger 1978). Antiochos & Sturrock (1978) have further shown that this discrepancy can be removed by taking conduction-driven evaporation into account. We present here a theory which does this. This theory has been applied to estimate the amount of heating for the flare kernel of 1973 September 1.

2. Theory

Following are the simplifying assumptions on which the present theory is based.

(i) Temperatures are sufficiently large so that the gravitational effects can be ignored.
(ii) Conduction dominates radiation.
(iii) All the variables can be written as a product of two functions, one depending on time and the other on arc-length (s) along the loop.

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(iv) Flow of energy and mass takes place along the magnetic field lines which are current-free above the chromosphere while the flow across the field lines may be ignored.

(v) The velocities generated by the conduction-driven evaporation of the material from the upper chromosphere are small in comparison with the velocity of sound so that no shocks are present.

Under these assumptions the energy equation to be solved is

$$\rho \frac{d}{dt}\left(\frac{3nkT}{\rho}\right) - \frac{p}{\rho}\frac{d\rho}{dt} = \frac{1}{A}\frac{\partial}{\partial s}\left(A\kappa\frac{\partial T}{\partial s}\right) + K_{\rm H}(t)\exp\left(-s^2/\gamma^2R^2\right), \quad \ldots(1)$$

where p, ρ, T are plasma pressure, mass density and temperature respectively, A the area of cross-section parameter and R the vertical height of the loop (Antiochos 1976, Antiochos & Sturrock 1976, Elwert & Narain 1980), n the number density of electrons and k the Boltzmann constant. κ is the well known coefficient of thermal conductivity given by

$$\kappa = \alpha T^{2.5} \quad (\alpha \approx 10^{-6}) \quad \ldots(2)$$

$K_{\rm H}(t)$ denotes the time dependent part of the source of heating and γ the extension of the source (Elwert & Narain 1980).

The solar corona is predominantly made up of fully ionized hydrogen. The scale height of this hot plasma is $h \simeq 2kT/m_p g$. With $T = 8.0 \times 10^6$ K, $k = 1.38 \times 10^{-16}$ erg K^{-1}, $m_p = 1.67 \times 10^{-24}$ gm and $g = 2.7 \times 10^4$ cm s^{-2}, h turns out to be $\simeq 4.8 \times 10^{10}$ cm, which is much larger than the height of the loops ($\sim 10^8$ cm) under consideration. Consequently, one can assume

$$\frac{\partial p}{\partial s} \approx 0,$$

which implies that the plasma pressure is a function of time only and is given by

$$p = 2nkT. \quad \ldots(3)$$

In a time dependent problem, the pressure inhomogeneities are smoothed out within the time taken by a sound wave to traverse the length of the system. In the present case it turns out to be $\lesssim 10$ s, much smaller than the observed cooling time (450 s). Hence, the pressure and the velocity can be taken to be uniform.

Using

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v\frac{\partial}{\partial s}, \quad \ldots(4a)$$

the equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial}{\partial s}(A\rho v) = 0, \quad \ldots(4b)$$

and equation (1), one obtains

$$\frac{1}{A}\frac{\partial}{\partial s}\left[A\left(2.5\,pv - \kappa\frac{\partial T}{\partial s}\right)\right] = K_{\rm H}(t)\exp\left(-s^2/\gamma^2R^2\right) - 1.5\frac{dp}{dt}. \quad \ldots(5)$$

From equations (1) – (4) and the gas equation we obtain

$$-\frac{dp}{dt} + 2.5\,\frac{p}{T}\,\frac{\partial T}{\partial t} + 2.5\,\frac{pv}{T}\,\frac{\partial T}{\partial s}$$

$$= \frac{1}{A}\,\frac{\partial}{\partial s}\left(A\kappa\,\frac{\partial T}{\partial s}\right) + K_{\mathrm{H}}(t)\exp\left(-s^2/\gamma^2 R^2\right). \qquad ...(6)$$

Equation (5) relates the velocity v of evaporation (mass motion) to the heat flux. It can be integrated to yield

$$A\left(2.5pv - \kappa\,\frac{\partial T}{\partial s}\right) = K_{\mathrm{H}}(t)\int \exp\left(-s^2/\gamma^2 R^2\right) A\,ds$$

$$- 1.5\,\frac{dp}{dt}\int A\,ds + K_1, \qquad ...(7)$$

in which K_1 is the constant of integration which is zero with the boundary conditions $v = 0$, heat flux $= 0$, at $s = 0$.

Depending on the area of cross-section parameter A two cases arise.

Case 1—*Plane parallel geometry*

Here the area of cross-section of the loop is throughout the same, *i.e.*,

$$A(s) = 1.$$

Now Equation (7) reduces to

$$2.5\,pv - \kappa\,\frac{\partial T}{\partial s} = K_{\mathrm{H}}(t)\,I_1(\gamma s) - 1.5\,\frac{dp}{dt}\,s, \qquad ...(8)$$

with

$$I_1(\gamma s) = \int_0^s \exp\left(-s^2/\gamma^2 R^2\right) ds. \qquad ...(9)$$

Equations (8) and (9) when combined with equation (6) lead to

$$-\frac{dp}{dt} + 2.5\,\frac{p}{T}\,\frac{\partial T}{\partial t} + \frac{\kappa}{T}\left(\frac{\partial T}{\partial s}\right)^2 + \frac{K_{\mathrm{H}}(t)}{T}\frac{\partial T}{\partial s}\,I_1(\gamma s) - 1.5\,\frac{dp}{dt}\,\frac{\partial T}{\partial s}\,s$$

$$= \frac{\partial}{\partial s}\left(\kappa\,\frac{\partial T}{\partial s}\right) + K_{\mathrm{H}}(t)\exp\left(-s^2/\gamma^2 R^2\right). \qquad ...(10)$$

In view of the assumption (iii), we may write

$$T(s, t) \equiv T_{\mathrm{M}} T_t(t)\, T_s(s). \qquad ...(11)$$

Equations (10) and (11) together then lead to an equation which on division by $\alpha\, T_{\mathrm{M}}^{3\cdot 5}\ T_t^{3\cdot 5}$ takes the form

$$\frac{1}{\alpha T_M^{3\cdot5}\ T_t^{3\cdot5}}\left[-\frac{dp}{dt}+\frac{2.5\,p}{T_t}\ \frac{dT_t}{dt}\right]+T_s^{1\cdot5}\left(\frac{dT_s}{ds}\right)^2$$

$$+\frac{K_H(t)\,I_1(\gamma s)}{\alpha T_t^{3\cdot5}\ T_M^{3\cdot5}\ T_s}\ \frac{dT_s}{ds}-\frac{1.5\,s}{\alpha T_M^{3\cdot5}\ T_t^{3\cdot5}\,T_s}\ \frac{dp}{dt}\ \frac{dT_s}{ds}$$

$$=\frac{d}{ds}\left(T_s^{2\cdot5}\ \frac{dT_s}{ds}\right)+\frac{K_H(t)\exp\left(-s^2/\gamma^2R^2\right)}{\alpha T_M^{3\cdot5}\ T_t^{3\cdot5}}. \quad ...(12)$$

It is not possible to solve equation (12) analytically without making some additional assumptions. However, it can be solved by numerical integration.

Some insight into this problem can be had by making the following additional assumptions :

$$K_H(t)\,T_t^{-3\cdot5}=\text{constant}=Q_{\max} \quad ...(13)$$

and

$$\frac{dp}{dt}=0 \quad \text{or } p=\text{constant}=p_0. \quad ...(14)$$

These assumptions make equation (12) separable. The time dependent part has the form

$$-\frac{2.5\,p_0}{\alpha T_M^{3\cdot5}}\ \frac{dT_t^{3\cdot5}}{dt}=-k_p^2\,. \quad ...(15)$$

Here $-k_p^2$ is the constant of separation. Under the boundary condition $T_t=1$ at $t=0$, equation (15) integrates to

$$T_t=\left(1+\frac{t}{\tau_p}\right)^{-2/7}, \quad ...(16)$$

with

$$\tau_p=\frac{2.5p_0}{\alpha T_M^{3\cdot5}\,k_p^2} \quad ...(17)$$

The part depending on arc-length s has the form

$$\frac{d^2\psi}{ds^2}-\frac{2}{7\psi}\left(\frac{d\psi}{ds}\right)^2-\frac{Q_{\max}}{\alpha T_M^{3\cdot5}}\ \frac{I_1(\gamma s)}{\psi}\ \frac{d\psi}{ds}$$

$$+\frac{3.5\,Q_{\max}\exp\left(-s^2/\gamma^2R^2\right)}{\alpha T_M^{3\cdot5}}=-k_p^2\,, \quad ...(18)$$

where

$$\psi \equiv T_s^{3.5}. \qquad ...(19)$$

Under the boundary conditions

$$\psi = 1, \quad \frac{d\psi}{ds} = 0 \quad \text{at} \quad s = 0 \quad \text{and} \quad \psi = 0 \quad \text{at} \quad s = s_b, \qquad ...(20)$$

one can solve equation (18) numerically to obtain the amount of heating, temperature- and flux-distribution etc.

Case 2—Line dipole geometry

In this case the flux-tube (loop) has a larger area of cross-section at the top than at its ends (base). Following Antiochos (1976) and Antiochos & Sturrock (1976) we have

$$s = R\theta, \qquad ...(21)$$

and

$$A = \cos^2 \theta. \qquad ...(22)$$

Here θ is the angle which a line joining a point on the loop to the centre of dipole makes with the vertical. In view of equations (21) and (22), the relation (8) between the velocity and heat flux assumes the form

$$2.5pv - \kappa \frac{\partial T}{\partial s} = \frac{RK_{\rm H}(t)\, I_2(\gamma\theta)}{\cos^2\theta} - \frac{0.75R}{\cos^2 \theta} \frac{dp}{dt} (\theta + 0.5 \sin 2\theta) + K_1', \qquad ...(23)$$

where

$$I_2(\gamma\theta) = \int_0^\theta \exp(-\theta^2/\gamma^2) \cos^2 \theta \, d\theta, \qquad ...(24)$$

and K_1' is the constant of integration. Proceeding in the same way as in the constant cross-section case the equations (6), (11), (13), (14) and (23) lead to

$$-\frac{2.5p_0}{\alpha T_{\rm M}^{3.5}} \frac{dT_t^{3.5}}{dt} = -k_l^2, \qquad ...(25)$$

where $-k_l^2$ is the constant of separation for the line dipole geometry. Now equation (25) integrates to

$$T_t = \left(1 + \frac{t}{\tau_l}\right)^{-2/7}, \qquad ...(26)$$

with

$$\tau_l = 2.5p_0/\alpha T_{\rm M}^{3.5} k_l^2, \qquad ...(27)$$

where the boundary condition of the previous case together with equation (21) has been used.

The θ-dependent part may be written as

$$\frac{d^2\psi}{d\theta^2} - \frac{2}{7\psi}\left(\frac{d\psi}{d\theta}\right)^2 - 2\tan\theta\,\frac{d\psi}{d\theta} - \frac{Q_{\max}\,R^2 I_2(\gamma\theta)}{\alpha T_M^{3\cdot5}\,\psi\cos^2\theta}\,\frac{d\psi}{d\theta}$$

$$+\frac{3.5\,Q_{\max}\,R^2}{\alpha T_M^{3\cdot5}}\exp\left(-\theta^2/\gamma^2\right) = -k_l^2\,R^2. \qquad \ldots(28)$$

Equation (28) has to be solved numerically under similar boundary conditions as given by equation (20) in order to get the amount of heating.

The source term, in view of equations (13), (16) and (26), assumes the form

$$Q(\theta, t) = Q_{\max}\,(1 + (t/\tau))^{-1}\exp\left(-\theta^2/\gamma^2\right), \qquad \ldots(29)$$

where τ stands for τ_p or τ_l and

$$Q_{\max} = Q(\theta = 0,\ t = 0). \qquad \ldots(30)$$

Following Elwert & Narain (1980), the total amount of heating may be estimated by integrating (29) over the entire length of the loop and the characteristic cooling time τ of the loop. Thus,

$$Q_T = Q_{\max}\int_0^{\tau}(1 + (t/\tau))^{-1}\,dt\int_{-s_b}^{s_b}\exp\left(-\theta^2/\gamma^2\right)FA\,ds, \qquad \ldots(31)$$

in which F is the actual area of cross section of the flux-tube (loop) at its top $(\theta = 0)$.

3. Method of calculation and results

Equation (30) shows that $Q_{\max}$ is the amount of heating at the top of the loop at the beginning of the cooling phase. For its determination one needs to solve equation (18) for plane parallel geometry and equation (28) for the line dipole geometry. These equations can be solved numerically provided the constants of separation $-k_p^2$ and $-k_l^2$ are known. These constants are defined by equations (17) and (27). If the plasma pressure p_0, the temperature T_M at the top of the loop in the beginning of the cooling phase and the cooling times τ_p and τ_l are known then the constants $-k_p^2$ and $-k_l^2$ become determined.

For the flare kernel of 1973 September 1 the amount of heating at the top has been determined by using the following set of data (Krieger 1978) :

$n = 2.0 \times 10^{11}\ \mathrm{cm^{-3}}$	$2s_b = 1 = 1.5 \times 10^8$ cm
$T_M = 8.0 \times 10^6$ K	$R = 5.6 \times 10^7$ cm
$\Gamma_{\max} = 19$	$\tau_{obs} = 4.5 \times 10^2$ s

Here τ_{obs} is the cooling time calculated from the soft x-ray intensities measured by the Skylab mission. We set $\tau_{obs} = \tau_p = \tau_l$ in order to calculate Q_{max} needed to remove the discrepancy between the observed and the calculated cooling times. Γ_{max} is the maximum value of the compression factor (Krieger 1978) and is a measure of the narrowing of the flux-tube in the case of line dipole geometry. l is the total length of the loop.

The equations (18) and (28) have been solved numerically by Runge-Kutta method. The integrals (9) and (24) have been evaluated by making use of the Simpson rule.

Three representative values $\gamma = 1.0, 0.5, 0.1$ have been chosen. They correspond, respectively, to the heating of full, half and one-tenth of the total length of the loop. For any other value of this parameter a similar procedure can be adopted. The results of calculation are exhibited in Tables 1 and 2.

Table 1. Q_{max} (in erg cm^{-3} s^{-1})

γ	Without evoporation* (static case)		With evaporation (non-static case)	
	$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$
0.1	1107	348	1150	350
0.5	222	78	280	80
1.0	118	51	185	55

*Taken from Elwert & Narain (1980).

Table 2. Q_T^* (in erg)

γ	Without evaporation† (static case)		With evaporation (non-static case)	
	$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$
0.1	2.1×10^{27}	6.7×10^{26}	2.5×10^{27}	7.6×10^{26}
0.5	2.1×10^{27}	6.6×10^{26}	3.1×10^{27}	7.8×10^{26}
1.0	2.2×10^{27}	6.6×10^{26}	3.8×10^{27}	8.2×10^{26}

†Taken from Elwert & Narain (1980).

4. Discussion and conclusions

Tables 1 and 2 show that in static as well as in non-static (present) cases the amount of the heating is of the same order of magnitude. However the heating is slightly greater in the non-static case which is quite expected. This amount of energy is used in moving the gas from upper chromosphere to corona. For temperatures below 2×10^4 K the scale height h comes out to be of the same order as the size of the loops ($\sim 10^8$ cm). Above this temperature the gravitational effects can be neglected. Thus the assumption (i) is in order. The assumption (ii) becomes questionable when the temperature falls below 10^6 K. Now the radiation begins to play a significant role in the further cooling of the coronal plasma. In some cases the radiation cannot be ignored from the very beginning (Moore 1978, Underwood *et al.* 1978). The present approach is then not valid. The assumption (iii) seems justified in the sense that in the post-flare phase the temperature and heating both begin to decline. Consequently, the physical variables are not expected to have strong correlation in their dependence on space and time coordinates. Observations indicate that the loops of temperature $> 10^6$ K can be seen in the immediate vicinity of H_α loops whose temperatures are $\sim 10^4$ K. In general a flare region consists of several loops. Adjacent loops are thermally insulated by the magnetic field. This supports our assumption (iv) (see *e.g.* Antiochos 1976). The assumption (v) is justified on the ground that we are dealing with the decay phase of the flare. Shocks develop when the impulsive heating takes place *i.e.* in the rise phase of the flare. In the decay phase any temperature increase due to the source of heating degenerates into a smooth temperature distribution, because of the large thermal conductivity of the material in the loops, symmetric about the top of the loop. The time-dependence of the source of heating comes from the additional assumption expressed through equation (13). Apparently it does not lead to violation of any of the physical laws although it comes purely on the ground that the energy equation becomes separable. Since the radiative cooling time agrees with the observed cooling time within permissible limits, the source of heating should be such that it compensates the conductive losses exactly (Krieger 1978). This would bring the observed and calculated cooling times into agreement. The observational evidence of continual heating in x-ray emitting coronal loops is now becoming available (Levine & Withbroe 1977, Gerassimenko *et al.* 1978).

The additional assumption expressed through equation (14) is not quite correct. Actually the pressure decreases as the time progresses, but not drastically. This may lead to some overestimation of the amount of heating. $Q_{\max}$ is the amount of heating at the top of the loop per unit volume per unit time. This cannot be compared with the observations directly. However, the total amount of heating Q_T, given by equation (31) can be compared with the total thermal energy of plasma contained in the flux-tube. It can be evaluated provided the actual area of cross-section F at the top of the flux-tube is known. Unfortunately F is not known and therefore an additional assumption is needed for its estimation. Following Elwert & Narain (1980) we assume that the radius r of the flux-tube at its top is one-tenth of its length l. Then

$$Q_T = \pi Q_{\max} \int_0^{\tau} (1 + (t/\tau))^{-1}\, dt\, (0.1l)^2 \int_{-s_b}^{s_b} \exp(-\theta^2/\gamma^2)\, A\, ds \left(\frac{r}{0.1l}\right)^2$$

$$= Q_T^* \, (r/0.1l)^2. \qquad \ldots(32)$$

The normalized amount of heating Q_T^* may now be evaluated and the same has been exhibited in Table 2. Assuming constant cross-section for the flux-tube its total volume is $V = \pi(0.1l)^2\, l \sim 10^{23}\,\mathrm{cm}^2$. The total thermal energy of the plasma contained in the flux-tube is $3nkTV \sim 10^{26}$ erg. This is an order of magnitude smaller than the heat supplied by the source. This discrepancy may be due to our assumption of keeping pressure independent of time. The upper limit of integration in time may not be quite appropriate in view of the assumption (ii).

A close examination of Table 2 shows that the difference in the amount of heating in static and non-static cases is more for plane parallel geometry than for line dipole geometry. This is due to the fact that the curved geometry not only constricts the flow of heat but also the flow of matter as well.

It is worthwhile to mention two recent articles (Krall *et al.* 1978; Antiochos & Krall 1979) related to the present problem. In the former the authors solve the three partial differential equations of mass, momentum and energy in a constant cross-section geometry taking conduction, evaporation and radiation into account whereas in the latter it has been done in line dipole geometry also. Both the articles do not include a source of heating.

In the next and last article of the series the author is trying to investigate the the problem without making the assumptions (ii) and (iii), by solving the three partial differential equations numerically in the presence of a source of heating and allowing for all the cooling mechanisms, *viz.*, conduction, conduction-driven evaporation and radiation. Nagai (1980) has recently made an extensive study in constant cross section geometry with heating and aforesaid cooling mechanism.

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