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On the frequency shift in radiation from a source orbiting a blackhole

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Abstract. An analytical study has been carried out of the non-tangential emission of photons in the orbital plane from a monochromatic source of radiation orbiting a blackhole, using the method of geometrical optics. The formulation is applicable to eccentric orbits. The frequency shift in radiation, which is a combination of Doppler effect and gravitational redshift, is studied for various angles of emission. For compact orbits, the analysis predicts, purely on geometrical grounds, features which may be of astrophysical interest, namely, the spectral line broadening with an asymmetrical profile in the case of a stellar-mass blackhole and peculiar line oscillations in the case of a supermassive blackhole.

Key words : Schwarzschild metric—null geodesics—frequency shifts—blackhole

1. Introduction

A few years ago, a theoretical study was made by us of the gravitational searchlight phenomenon occurring in the vicinity of a Schwarzschild blackhole (Chitre *et al.*, 1974, 1975). The radiation which is emitted in a forward narrow cone by charged particles in highly relativistic circular orbits around a blackhole can be blueshifted because of the supersession of the strong gravitational redshift in the vicinity of the event horizon by relativistic Doppler effect. The work was concerned with photons emitted by charged particles in the instantaneous direction of their motion (tangential emission). The present work explores the case of non-tangential emission of radiation in the orbital plane from a source orbiting a blackhole, applicable also to an eccentric orbit. As will become apparent, these considerations give rise to a few interesting consequences when the orbits are compact enough.

The source of photons could be an ordinary star orbiting a supermassive blackhole, or a hot spot or a flare produced in an accretion disk around one (stellar/supermassive) due to generation and growth of instabilities which can last for many revolutions, or a stellar outburst that may take place in the 'vicinity' of the blackhole.

It is reasonable to expect eccentric orbits in some of the cases mentioned above. The emission of the gravitational waves however can circularize the orbit in a period shorter than $\sim 10^{10}$ yr depending on the masses involved and their separation. The time of circularization of an eccentric orbit is given by (Lightman *et al.* 1975)

$$\tau = \frac{15}{304} \frac{a^4 F(e_1, e_2)}{m_1 m_2 (m_1 + m_2)}\,; \quad F(e_1, e_2) = \int_{e_1}^{e_2} \frac{(1 - e^2)^{5/2}\, de}{e(1 + (121/304)\, e^2)}. \qquad \ldots(1)$$

Here and elsewhere m_1 and m_2 are the masses of bodies in geometrical units (*i.e.* $G = c = 1$), e_1 and e_2 are initial and final eccentricities respectively, a the semimajor axis of the orbit and $F(e_1, e_2) \sim 1$ for $e_1 \simeq 1$ and $e_2 \simeq 0$. Thus, for instance, for a system consisting of an $M_1 = 10^9\, M_\odot$ blackhole and an $M_2 = 10\, M_\odot$ star, a distance about $10\, m_1$ apart, $\tau \sim 10^8$ yr if $e_1 \simeq 1$ and $e_2 \simeq 0$. In fact when the orbit is compact enough, a circular orbit also can shrink because of the emission of gravitational waves, according to relations

$$a(t) \simeq a_0 \left(1 - \frac{t}{T_s}\right)^{1/4}; \quad T_s = \frac{5 a_0^4}{256 m_1^2 m_2}, \qquad \ldots(2)$$

where T_s is the spiral-in time. Choosing the same parameters as in the example above, $T_s \sim 10^7$ yr. It is obvious that for smaller ratios of m_1/m_2, τ and T_s decrease appreciably. Liebes (1964), Campbell & Matzner (1973) and Cunningham & Bardeen (1973) have worked out in detail the optical appearance of a source in the background of a gravitating mass and in orbit round a Schwarzschild or a Kerr blackhole. In what follows, we shall be mainly concerned with the frequency shifts. The geometrical optics approximation is used, neglecting the backscatter caused by the presence of the spacetime curvature.

2. Null geodesics : the case of non-tangential emission

The spacetime exterior to a nonrotating neutral blackhole is given by the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{(1 - 2mr^{-1})} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \qquad \ldots(3)$$

where $m = GM/c^2$ is implied. The general equations for geodesic motion are

$$\frac{d^2 X^\mu}{ds^2} + \Gamma^\mu_{\nu\sigma} \frac{dX^\nu}{ds} \frac{dX^\sigma}{ds} = 0 \quad (\mu, \nu, \sigma = 0, 1, 2, 3). \qquad \ldots(4)$$

Note that $X^0 = t$, $X^1 = r$, $X^2 = \theta$, $X^3 = \varphi$.

Now we proceed to write the equations of motion of the source and the photons it emits. The formulation is exact as long as the geometrical optics and the test particle approximation (source mass $\ll$ blackhole mass) can be considered valid. Let us suppose that the eccentric orbit is specified by an eccentricity e and a semi-latus rectum a. Let us take $d\theta/ds = 0$ and assume that the observer stays far away from the hole at $r = R$ ($\gg 2m$) in the plane of the orbit $\theta = (\pi/2)$. According to McVittie (1970), metric (3) and the equation of geodesic motion (4) solved together give

$$\frac{dr}{ds} = \pm\left[\gamma^2 - \left(1 - \frac{2m}{r}\right)\left(1 + \frac{h^2}{r^2}\right)\right]^{1/2}, \qquad ...(5)$$

$$\frac{d\theta}{ds} = 0, \qquad ...(6)$$

$$\frac{d\varphi}{ds} = \frac{h}{r^2} = \frac{(ma)^{1/2}}{r^2}\left[1 - \frac{(3+e^2)\,m}{a}\right]^{-1/2} \qquad ...(7)$$

and

$$\frac{dt}{ds} = \frac{\gamma}{(1-2mr^{-1})} = \frac{[1 - 4ma^{-1} + 4(1-e^2)\,m^2a^{-2}]^{1/2}}{(1-2mr^{-1})\,[1-(3+e^2)\,ma^{-1}]^{1/2}}. \qquad ...(8)$$

Here γ is the energy per unit rest mass of the source and h its orbital angular momentum per unit rest mass as measured at infinity. For a circular orbit, $e = 0$, $r = a$ and $dr/ds = 0$ also. The angular velocity of the source about the hole is defined as $n = d\varphi/dt$.

For the photons, the various components of the 4-momentum are

$$\frac{dr}{d\lambda} = \gamma\left[1 - \frac{(1-2mr^{-1})}{r^2}\,\frac{h^2}{r^2}\right]^{1/2}, \qquad ...(9)$$

$$\frac{d\theta}{d\lambda} = 0, \qquad ...(10)$$

$$\frac{d\varphi}{d\lambda} = \frac{h}{r^2} \qquad ...(11)$$

and

$$\frac{dt}{d\lambda} = \frac{\gamma}{(1-2mr^{-1})}. \qquad ...(12)$$

The source emits photons in all directions but only those emitted in the plane of the orbit will be able to reach the distant observer. In view of equations (5)–(12), we write 4-velocity of the source as

$$u^\alpha(s) = \left[\frac{\gamma}{(1-2mr^{-1})},\ \frac{dr}{ds},\ 0,\ \frac{d\varphi}{ds}\right], \qquad ...(13)$$

that of the observer as

$$u^\alpha(0) = (1, 0, 0, 0), \qquad ...(14)$$

and direction of the photon at r as

$$p_\beta = \left[\gamma,\ -\frac{1}{(1-2mr^{-1})}\,\frac{dr}{d\lambda},\ 0,\ -r^2\frac{d\varphi}{d\lambda}\right]. \qquad ...(15)$$

The net frequency shift can be calculated by using the following general expression due to Schroedinger (1950):

$$\frac{\nu_0}{\nu} = 1 + z = \frac{\vec{u} \cdot \vec{p} \mid_{\text{source}}}{\vec{u} \ \vec{p} \mid_{\text{observer}}}. \qquad \ldots(16)$$

Here ν_0 is the frequency of emission and ν that at reception. Hence, if r_0 refers to the instantaneous location of the source with respect to the origin of the system of coordinates, then from equation (16), we can write the frequency shift as

$$1 + z = \frac{\gamma}{(1 - 2mr^{-1})} \left[1 + \frac{1}{\gamma^2} \frac{dr}{ds} \frac{dr}{d\lambda} + nq \right] \Big|_{r=r_0}, \qquad \ldots(17)$$

where q, defined as the impact parameter of the photon, is given by $q = h/\gamma$. In order to evaluate q, let us consider the two physical components υ_r and υ_φ of the photon velocity as according to an observer at rest in the Schwarzschild field with respect to his proper reference frame:

$$\upsilon_r = \left(\frac{-g_{11}}{g_{00}} \right)^{1/2} \frac{dr/d\lambda}{dt/d\lambda} = \frac{1}{\gamma} \frac{dr}{d\lambda}, \qquad \ldots(18)$$

$$\upsilon_\varphi = \left(\frac{-g_{33}}{g_{00}} \right)^{1/2} \frac{d\varphi/d\lambda}{dt/d\lambda} = \frac{(1 - 2mr^{-1})^{1/2}}{r} q, \qquad \ldots(19)$$

such that

$$\upsilon_r^2 + \upsilon_\varphi^2 = 1. \qquad \ldots(20)$$

Therefore, it is reasonable to write

$$\upsilon_r = \cos \delta, \quad \upsilon_\varphi = \sin \delta. \qquad \ldots(21)$$

where δ is the angle at which the photon is emitted with respect to the radius vector of the source through the origin of the system of coordinates; δ increases in the direction opposite to that of motion of the source. Thus, for a radially ingoing photon $\delta = \pi$ whereas for one going outwards $\delta = 0$. In the case of gravitational searchlight (tangential emission), $\delta = 3\pi/2$ so that $\upsilon_r = 0$ and $\upsilon_\varphi = -1$. However, in the case of an eccentric orbit δ is not necessarily $\pi/2$ or $3\pi/2$ for tangential emission. In view of equations (9) – (12), the line element (1) for $ds^2 = 0$ gives

$$q = q_0 \left[1 - \frac{1}{\gamma^2} \left(\frac{dr}{d\lambda} \right)^2 \right]^{1/2} = q_0 \sin \delta. \qquad \ldots(22)$$

where

$$q_0 = \frac{r_0}{(1 - 2mr_0^{-1})^{1/2}} \qquad \ldots(23)$$

characterizes a photon emitted at $\delta = \pi/2$, in the direction opposite to that of motion of the source. The frequency shift is therefore

$$1 + z = \frac{\gamma}{(1 - 2mr^{-1})} \left[1 \pm \frac{\cos \delta}{\gamma} \frac{dr}{ds} + nq \right] \Big|_{r=r_0}, \qquad \ldots(24)$$

$$r_0 = a(1 + e \cos \varphi)^{-1}.$$

Here, $\pm$ sign refer to $dr/ds \gtrless 0$. For $\delta = 3\pi/2$ and $e = 0$, this equation reduces to equation (18) of Chitre *et al.* (1975) for the gravitational searchlight. For large circular orbits, equation (24) approximates to

$$z \simeq \left(\frac{m}{a}\right)^{1/2} \sin \delta. \qquad \ldots(25)$$

It can be noted that Doppler blueshift overcomes the gravitational redshift for forward emission ($\pi < \delta < 2\pi$) from a large circular orbit.

3. Propagation and frequency shifts of photons emitted at different angles

Photons emitted by the source at different angles not only suffer different frequency shifts but also different amount of gravitational bending. A photon emanating from the source at an instantaneous position $r_0 > 3m$ will be captured by the black-hole unless the impact parameter satisfies the condition

$$q > q_{\mathrm{lim}} = 3.3^{1/2} m \qquad \ldots(26)$$

(see, for instance, Misner *et al.* 1973). A photon emitted at an impact parameter slightly in excess of q_{lim} would make numerous rounds of the central blackhole before arriving at a distant detector. Consequently, such a photon takes a longer time to arrive there compared to the one emitted with a q greater than q_{lim}. The limiting angle corresponding to $q = q_{\mathrm{lim}}$ is, therefore,

$$\delta_0 = \sin^{-1} \frac{3.3^{1/2} m}{q_0}. \qquad \ldots(27)$$

Photons emitted at an angle $\delta < \pi - \delta_0$ and $\delta > \pi + \delta_0$ escape whereas those emitted at a certain δ may get captured when r_0 is such that q falls below its limiting value and may escape at another instant when r_0 becomes such that q exceeds q_{lim}. Table 1 gives the values of the limiting angle δ_0 corresponding to different values of eccentricity and semi-latus rectum, when the source is at the apastron and periastron respectively of its orbit.

Table 1. Limiting angle variation with semi-latus rectum and eccentricity

a/m	$e = 0$ δ_0 (deg)	$e = 0.5$ $\delta_{0\,\max}$ (deg)	 $\delta_{0\,\min}$ (deg)	$e = 1$ $\delta_{0\,\max}$ (deg)
100	2.949	4.403	1.481	5.844
40	7.274	10.801	3.677	14.269
10	27.695	40.701	14.269	53.609
6	45.000	66.716	23.284	90.000
4	66.716	$r_{0\,\min} < 3m$	34.229	—
3	90.000	,,	45.000	—

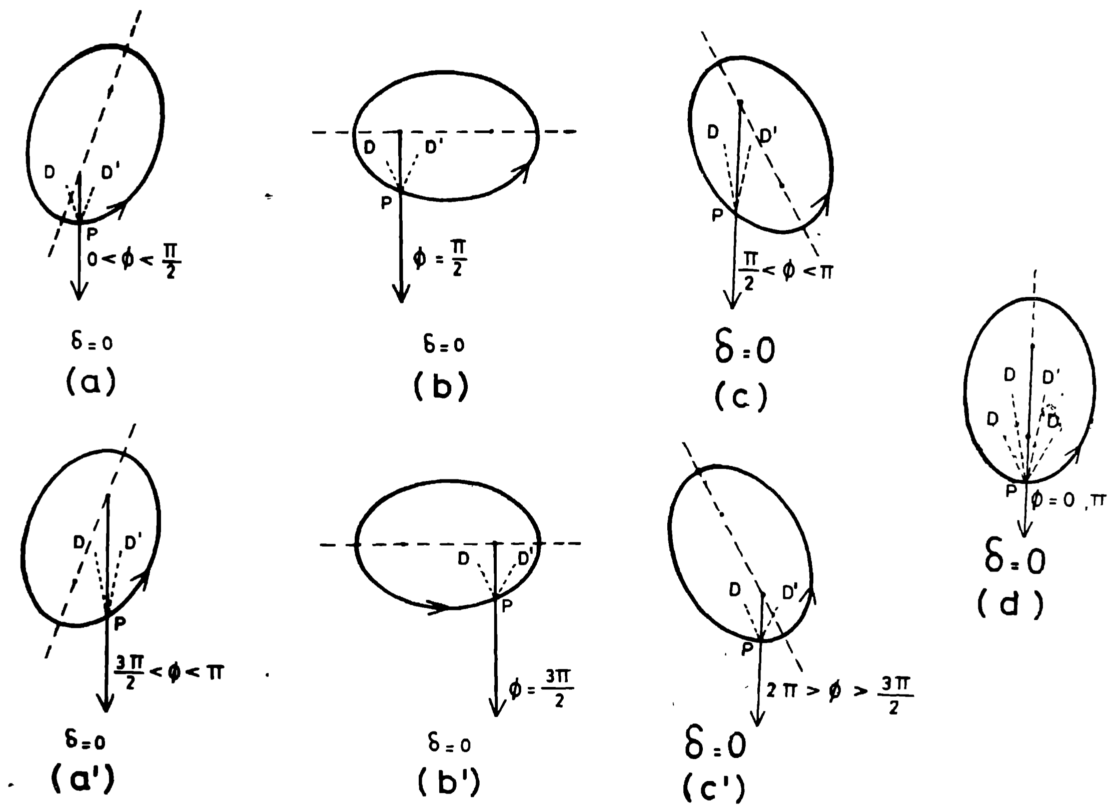

Figure 1. Various possible orientations of the orbit with respect to the line of sight for the cases of perfect alignment of the blackhole, the source and the observer.

Let us focus our attention on the emission of photons by the source at different values of δ. To start with, we consider the simplest case *i.e.* one of perfect alignment. The centre of the blackhole, the source and the observer fall on the same line (line of sight). Figure 1 depicts various possible orientations of the orbit with respect to the line of sight. Photons emitted by the source within a cone D'PD are captured by the blackhole. The apex angle of the cone, δ_0, varies with r_0 in accord with equation (27). Photons emitted at $\delta = 0$ are received by the remote observer after every complete revolution, with a frequency shift amounting to

$$1 + z(\delta = 0) = \frac{\gamma}{(1 - 2mr_0^{-1})}\left[1 \pm \frac{1}{\gamma}\left\{\gamma^2 - \left(1 - \frac{2m}{r_0}\right)\left(1 + \frac{h^2}{r_0^2}\right)\right\}^{1/2}\right]. \qquad \ldots(28)$$

In this equation, $\pm$ sign refer to $dr/ds \gtrless 0$ and

$$\gamma^2 \geqslant \left(1 - \frac{2m}{r_0}\right)\left(1 + \frac{h^2}{r_0^2}\right) \qquad \ldots(29)$$

corresponds to the timelike trajectory of the source. In this equation, the sign of equality refers to the circular case, or to the source at its periastron or apastron, so that

$$1 + z(\delta = 0) = \frac{1}{(1 - 3ma^{-1})^{1/2}} = \text{constant} > 1. \qquad ...(30)$$

Obviously, these photons are always redshifted and are received whenever the source comes in between the blackhole and the observer such that $dr/ds = 0$. However, in general $1 + z(\delta = 0)$ may be $>$ or < 1, *i.e.* the $\delta = 0$ photons may be red, or blueshifted according as $dr/ds < 0$ or > 0.

In the case of an eccentric orbit we have to take into account also the fact that the periastron advances significantly when the orbit is compact. The periastron advance would introduce a secular change in the frequency shift of a photon emitted at a certain $\delta = 0$ in the case under consideration. Suppose, to start with, the major axis of the orbit coincides with the line of sight and that the source is at its apastron and in between the blackhole and the observer. Let T_p be the time taken by the periastron to shift by 2π. Generally $T_p \gg$ orbital period. Then in an interval $T_p/2$, the blackhole, the source and the observer are once again exactly aligned, the source now being at its periastron and in between the blackhole and the observer. Since $dr/ds = 0$ at $r_{0\,\max}$ and $r_{0\,\min}$, the respective frequency shifts would be

$$1 + z_{\min}(\delta = 0) = \frac{\gamma}{(1 - 2mr^{-1}_{0\,\max})}; \; 1 + z_{\max}(\delta = 0) = \frac{\gamma}{(1 - 2mr^{-1}_{0\,\min})}, \qquad ...(31)$$

the magnitude of the difference between $z_{\min}$ and $z_{\max}$ amounting to

$$| \Delta z_1 | = \frac{4me\gamma}{a\,[1 - 4ma^{-1} + 4(1 - e^2)m^2a^{-2}]}. \qquad ...(32)$$

For instance, for $e = 0.5$ and $a = 10m$, $z_{\min} = 0.08$, $z_{\max} = 0.39$. Hence, a 6000 Å line will be shifted by amounts 480 Å and 2340 Å toward the red for $z_{\min}$ and $z_{\max}$ respectively. Δz_1 is larger if e is larger. For a large value of a and/or e, $\gamma \rightarrow 1$ and

$$| \Delta z_1 | \simeq \frac{4me}{a}\left(1 + \frac{4m}{a}\right). \qquad ...(33)$$

How much does the frequency shift of a photon emitted at an angle $\delta = 0$ change, after the periastron has advanced by an amount $\Delta\varphi$ in one complete revolution ? In order to find this out, let us refer to Figure 2. There are in fact two cases to be considered : (a) when the source is at its periastron to start with ($\varphi = 0$), the semimajor axis coinciding with the line of sight; after once going round the blackhole, the source intersects the line of sight again, its radius vector now making an angle $\varphi = -\Delta\varphi$ with the semimajor axis, (b) when the respective angles are $\varphi = \pi$ (source at its apastron) and $\varphi = \pi - \Delta\varphi$. We notice that the change in the frequency shift in case (a) is not the same as that in the case (b). These are given by

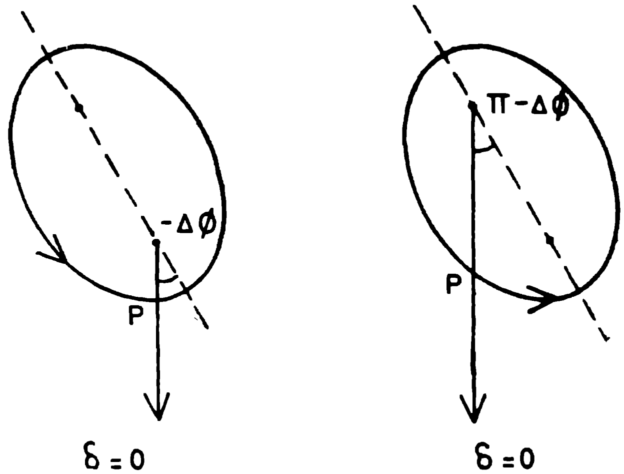

Figure 2. Shows the cases of perfect alignment after one revolution when the periastron advances by a small angle.

$$1 + z_{\max}(\varphi = 0) = \frac{\Upsilon}{(1 - 2mr_{0\,\min}^{-1})};$$

$$1 + z(\varphi = -\Delta\varphi) = \frac{\Upsilon}{(1 - 2mr_0^{-1})}\left[1 \pm \frac{1}{\Upsilon}\frac{dr}{ds}\bigg|_{r=r_0}\right] \qquad \ldots(34)$$

for case (a), and by

$$1 + z_{\min}(\varphi = \pi) = \frac{\Upsilon}{(1 - 2mr_{0\,\max}^{-1})};$$

$$1 + z(\varphi = \pi - \Delta\varphi) = \frac{\Upsilon}{(1 - 2mr_0^{-1})} \times \left[1 \pm \frac{1}{\Upsilon}\frac{dr}{ds}\bigg|_{r=r_0}\right] \qquad \ldots(35)$$

for case (b). To the first order in $\Delta\varphi$, we therefore have

$$|\Delta z_2|_{\pm} = \frac{em\Delta\varphi}{[ma - 2m^2(1 \pm e)]^{1/2}\,[1 - (3 + e^2)\,ma^{-1}]^{1/2}}. \qquad \ldots(36)$$

In equation (36), + sign refers to case (a) whereas − sign to case (b). Furthermore, $|\Delta z_2|_+ > |\Delta z_2|_-$. For example, for $e = 0.5$ and $a = 10m$, $|\Delta z_2|_+ \simeq 0.23\Delta\varphi$ and $|\Delta z_2|_- \simeq 0.20\Delta\varphi$. In the derivation of the foregoing equations, we have neglected any change in e and/or a due to gravitational radiation.

As is well known, the photons emitted at various angles, as well as those emitted close to limiting angle, can give rise to the formation of luminous concentric rings around the blackhole. This happens whenever the blackhole, the source and the observer are exactly aligned; the observer can receive the photons in any plane which includes the line of sight. One, therefore, sees a number of concentric rings about the central blackhole. When the source is in between the blackhole and the observer, it appears as a red or a blue point [vide equation (28) and the following discussion] at the centre of the ring system. The innermost ring corresponds to photons which were emitted at δ close to $\pi \pm \delta_0$. They suffer the largest amount of bending and also take the longest time to reach the distant observer. Following Chitre *et al.* (1975), it is then easy to show that the angular radius of the innermost ring is

$$\phi \simeq \frac{q}{R} = \frac{q_0 \sin \delta_0}{R} = \frac{3.3^{1/2} m}{R}, \qquad \ldots(37)$$

where R is the location of the observer ($\gg 2m$). Here, we would like to stress that the ring system so formed is not as simple as that formed in the case where the source is stationary and not gravitationally bound to the blackhole. In the latter case the rings have a uniform width, intensity and colour. In the present case, the additive and subtractive contributions of the Doppler effect and the photon emission off the plane of the orbit render the appearance of the ring system complex. The ring system consists of redshifted as well as blueshifted photons. For the innermost ring, the difference in the frequency shifts amounts to

$$| \Delta z_3 | = \frac{2n\gamma \,.\, 3.3^{1/2} m}{\left(1 - \frac{2m}{r_0}\right)}, \qquad \ldots(38)$$

which varies with r_0. When the orbit is a compact one, the frequency shift difference is large and consequently the intensity of the rings would be mainly contributed by the blueshifted photons. As the source moves off the line of sight, the rings start degenerating into a number of distorted ghost images of the source appearing around the blackhole, though asymmetrically placed and nonuniform in colour. When the source goes behind the blackhole, the rings appear again, with the central red/blue point now missing.

For a compact orbit, the sensitive dependence of the frequency shift and the gravitational bending on δ give rise to an interesting effect, *viz.*, spectral line broadening in the case of a stellar-mass blackhole, and, peculiar oscillations of a number of lines across the spectrum in the case of a supermassive blackhole. To illustrate this, let us suppose that the source moves in a circular orbit, emitting monochromatic radiation at frequency ν_0. Now, wherever the source be in its orbit, owing to a large gravitational bending there would always be some photons emitted at various angles in the forward as well as backward direction reaching the observer at a given instant. As the source moves in its orbit, photons emitted at progressively changing values of δ are received at $r = R$. The net result is the oscillations of a number of spectral lines about ν_0 in a peculiar manner. In the case of a stellar-mass blackhole, the orbital period of a source in a compact orbit is $\ll 1$ s. Therefore, the source goes round the blackhole so quickly that the spectral region between $z(\pi/2)$ and $z(-\pi/2)$ is apparently occupied, giving the impression of a broadened spectral line. The wings of the spectral line correspond to emissions at $\delta = \pm \pi/2$, separated by an amount

$$| \Delta z_4 | = 2 \left[\frac{ma}{(a - 3m)(a - 2m)} \right]^{1/2}. \quad \text{...(39)}$$

For instance, if $\lambda_0 = 6000$ Å, $a = 20m$, the width, $\Delta\lambda = | \Delta z_4 | \lambda_0$, would be equal to 3068 Å. It can be noted that $| \Delta z_4 | > 1$ for $a < 8.275m$. According to the Liouville theorem, the observed (I_ν) and the emitted (I_{ν_0}) monochromatic intensities are related through

$$I_\nu = \left(\frac{\nu}{\nu_0} \right)^3 I_{\nu_0} \quad \text{...(40)}$$

so that we can express the ratio of the intensities near the wings as

$$\frac{I_{\text{blue}}}{I_{\text{red}}} = \left(\frac{1 + nq_0}{1 - nq_0} \right)^3. \quad \text{...(41)}$$

For instance, when $a = 20m$, $I_{\text{blue}}/I_{\text{red}} \simeq 4$ whereas for $a = 10m$, $I_{\text{blue}}/I_{\text{red}} \simeq 9$. The ratio thus gets steeper as a decreases. Therefore, for a compact orbit, the line profile would be highly asymmetrical. In the case of the source orbiting a supermassive blackhole, the situation is rather complicated; the orbital period is comparatively much larger and one expects peculiar oscillations of a number of spectral lines across the spectrum, the bluer lines being comparatively the brighter.

4. Conclusions

In this paper we have examined the frequency shift in the non-tangential emission of radiation in the orbital plane from a monochromatic source going round a blackhole in an eccentric orbit. The analytical treatment is independent of the blackhole mass. For compact orbits, we find a sensitive dependence of the frequency shift and gravitational bending on the angle of emission, the latter being measured with respect to the radius vector. This leads to an interesting consequence, namely, peculiar spectral line oscillations in the spectrum. The line oscillations are detectable by a remote observer in the case of a blackhole of mass much exceeding 10^4 or $10^5 M_\odot$ as the orbital period in a high energy orbit would generally be $\gg 1$ s. In the case when the blackhole mass $\lesssim 10^4$ or $10^5 M_\odot$, the orbital period in a high energy orbit would be $\lesssim 1$ s. The line oscillations on either side of the emission frequency would, therefore, be so fast as to produce the impression of a broadened spectral line on the photographic plate in the spectrograph. The broadened line is asymmetrically placed with respect to the emission frequency as can be verified from equation (24), and has an asymmetrical line profile as a consequence of Liouville's theorem. The broadening will be the same if the point source is replaced by a ring of emitting particles; the broadening is genuine this time, regardless of the blackhole mass.

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