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A unified view on Aharanov–Bohm like phases and some applications

C S UNNIKRISHNAN
Gravitation Group, Tata Institute of Fundamental Research, Mumbai 400 005, India
Near-Zero Laboratory, Indian Institute of Astrophysics, Bangalore 560 034, India
Email: unni@tifr.res.in, unni@iiap.ernet.in

Abstract. The analysis of the Aharanov–Bohm phase and other similar physical effects in this paper is motivated by the philosophy that all physical changes, including phase changes, should originate in one of the local physical interactions even if they are described elegantly and concisely as topological or geometric changes. The topological or geometric nature comes about either due to an additional physical principle or due to certain special spatial or temporal property of the fields from the source. Similar remarks apply to rotation or precession of polarization and spin vectors. As a primary example I describe the Aharanov–Bohm phase as arising from the Coulomb interaction of a charge in the electrostatic potential created by other charges. The topological nature comes about because the interaction energy has zero gradient throughout space, except in a compact region enclosed by the quantum paths. This analysis brings out the unifying aspects of the scalar and the vector A–B effects, and the Aharanov–Casher phase. Then I discuss two other related problems with descriptions in the geometrical and the interaction pictures; I discuss how quantum complementarity is realized without the Heisenberg back action on momentum in certain atom interferometry experiments. In the second example, I show that the Thomas precession of the spin results from the local torque in the accelero-magnetic field, a field predicted in analogy with the gravitomagnetic field. I end the discussion with some remarks on the classical nature of fringe shifts in Aharanov–Bohm like phenomena in electromagnetism and gravitation.

Keywords. Aharanov–Bohm phase; locality; atom-interferometry; Thomas precession.

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1. Introduction

Geometrical description of physical phenomena has been a powerful theoretical framework that flourished in the last century with the success of the general theory of relativity and of the gauge theories of fundamental interactions. Closely related is the description of physical quantities in terms of topological properties of either the real or the abstract physical space related to the phenomenon. Apart from imparting elegance and beauty to the theoretical understanding, such geometrical and topological descriptions often greatly simplify the calculations, and aid in significant generalizations. One of the important examples of such descriptions is that of the quantum phases. From the Aharanov–Bohm phase [1] to the Berry–Pancharatnam phases [2,3], in physical situations ranging from simple interferometry to complicated many-body effects [4], geometrical and topological description have been very successful. On the other hand, separation of the total phase into a dynamical part and a geometrical part has also obscured in some cases certain unifying aspects and physical origin of different kinds of quantum phases. I discuss Aharanov–Bohm like phases within the general philosophy that all physical changes, including phase changes, should originate in one of the *local* physical interactions, and should admit a description in terms of the interaction itself. The topological or geometric nature comes about either due to an additional principle (as in the case of the equivalence principle in gravity) or due to certain special spatial or temporal property of the fields from the source. Seen from this angle, many of the debates on certain aspects of the Aharanov–Bohm phases are superfluous, especially the debate regarding the role of fields as opposed to the role of potentials. Both concepts are abstractions on the physical concept of interaction energy which is a more basic physical quantity. The physical origin of the phase change is universally a term of the form $\int E\mathrm{d}t$, where E is the interaction energy. I show that the topological nature of the phase comes from the gradient free nature of the interaction energy in the problem. Since quantum phases and 'polarization' are closely related, these remarks are relevant in the context of rotations of polarization or spin as well.

2. A fresh look at the A–B effect

The surprise factor in the Aharanov–Bohm phase was the fact that the phase change occurred to particle wave functions which did not 'interact with the electromagnetic field' [5,6]. While the differential phase change depends on the local potential, as $\exp(-\frac{ie}{\hbar}\oint A\mathrm{d}x)$, where A is the vector potential, the total phase change depends only on the field flux enclosed by the interfering paths. In the standard configuration in which the interfering electron paths encircle a solenoid with enclosed flux Φ, the phase shift $\Delta\varphi = \frac{e}{\hbar}\Phi$. The flux itself is not 'seen' by the electrons. The spatial nonlocality in this description has even given rise to a debate on the role of nonlocality in such phases. In fact, adherence to a narrow view on the concept of interaction, written classically in terms of the charge and the force field and expressed as the Lorentz force law, has obscured the remarkably simple universality of the A–B like phases. The force law describes changes in the momentum of the particle, and requires a spatial gradient in the interaction energy of the charge. But relative phase changes can occur without a change in the momentum even for classical waves, and this requires only a difference in the interaction energy between two parts of the wave and the force fields are not relevant. This is true for quantum waves as well. The A–B phase change results from the Coulomb interaction energy between the test charge and the charges that create the current and the magnetic field. The standard A–B effect happens to be discussed in a situation where this interaction energy is independent of the spatial position of the test charge with respect to the currents.

The essential picture may be sketched in analogy with an example involving classical electromagnetic waves – a laser beam in a Mach–Zehnder interferometer (see figure 3b for the basic configuration). If a dielectric material is introduced in one of the paths (or two different lengths of the material in the two paths) the interference fringes will shift. Yet, there is no change in the momentum of the photons, and the total phase shift depends only on the difference of the optical path lengths inside the two pieces of materials. There are no 'fields'or forces (momentum changing influences), but there is an 'interaction energy' (gradient free potential), which in this case is a uniform refractive index in some portion of each path. Indeed, if the two optical paths are through two different media such that the change in refractive index is confined to a narrow interface of the two media, the shift of the interference pattern depends on the difference in the refractive indices that is manifest only in a spatial region not 'seen' by the beams. There is nothing more mysterious in the Aharanov–Bohm effect than what is there in this optical example, except that the equivalent of the medium with the uniform refractive index is harder to notice in the A–B effect.

To focus the ideas, let us consider the quantum mechanics of a charged particle moving near an infinite conducting sheet in which there is a uniform current as indicated in figure 1. The conductor is neutral in a reference frame which is stationary with respect to the conductor. But in a moving frame, as in a frame in which the moving particle is at rest, the conductor with a current is charged owing to the different Lorentz transformation factors on the electron density and the lattice ion density [7]. This excess charge density σ is uniform over the surface of the conductor and depends linearly on the current and the velocity of the charged test particle. The sign of σ changes when the velocity of the charged particle or the direction of the current is reversed. The *electrostatic force* on the particle due to this excess charge is same as what is more conveniently denoted as the $\mathbf{v} \times \mathbf{B}$ *magnetic force*. Since we are considering an infinite sheet (or for practical purposes, a conducting sheet which is sufficiently large in dimensions), the force on the charged particle is independent of the distance from the sheet. The charged particle will feel a constant force $F \propto e\sigma$, and also a spatially varying phase change $\delta\phi(x) = E(x)\delta t$ in this situation. Now consider a second sheet at a separation d behind the first sheet, with a current which is equal in magnitude and opposite in direction. The excess charge from this sheet exerts a force $-F$ on the test particle, again independent of the distance of the particle from the sheet. The result is a net zero force on the particle. The Coulomb interaction energy of the charge with each sheet is linear in the distance from the sheet and therefore the phase change is $\{E(x_1) - E(x_2)\}\delta t \propto (x_1 - x_2)\delta t C = \mathrm{d}\delta t C$, where C is a constant proportional to the current. Clearly the phase change is independent of the spatial coordinate and depends linearly on the separation between the conducting sheets. This derivation highlights two aspects of the A–B phase. The apparent spatial nonlocality is just a statement of the fact that two equal and opposite forces are cancelled at the location of the test particle, and the total phase change depends on the difference of the interaction energy which is proportional to the 'flux' or the product of the separation of the sheets and the current. This 'flux' is not in physical contact with the charge. *In effect, the spatial uniformity of the interaction energy allowed us to have a description in terms of a quantity that is geometrical in nature and that is not in contact with the charge.* The topological character of the A–B effect is the property that the Coulomb interaction energy between the test charge and the (Lorentz transformed) charge distributions in the conductor is spatially uniform with zero gradients in the region in which the particle is moving, and changes sign when the sheets are crossed. So, there is a finite difference in the interaction energy when the sheets are crossed and the total phase change is proportional to this difference. Since there is no gradient of energy, there is no force (field).

The situation with an infinite solenoid is exactly the same, though the direct calculation is more complicated. The solenoid may be thought of as being formed by a small corrugation on each sheet, with the two sheets merged together (figure 2). The currents cancel except in the region with the corrugation and we get an infinite solenoid. The interaction energy of the test charge with the charge excess in the solenoid seen by the moving particle is independent of the distance of the particle from the centre of the solenoid, but the sign

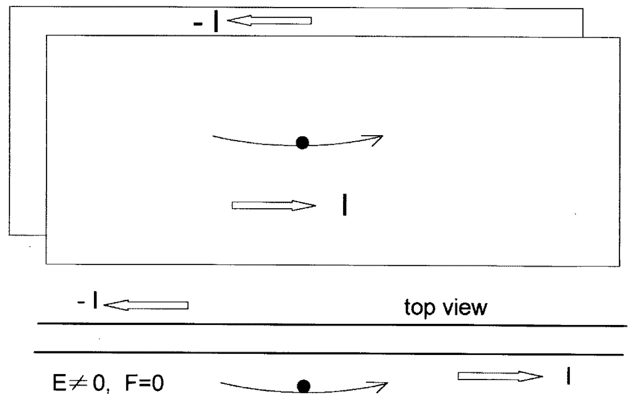

Figure 1. The configuration of conducting sheets and moving charge that clarifies the physical origin and properties of the standard Aharanov–Bohm effect.

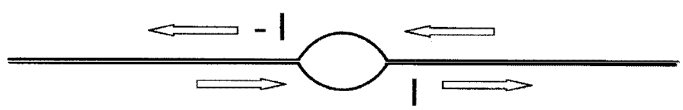

Figure 2. 'Solenoid' formed by two conducting sheets with opposite currents. The Coulomb interaction energy of a charge moving outside the solenoid is nonzero and gradient free.

of the interaction energy changes when the test charge is at a point across the solenoid. There are no forces and the phase change which is proportional to the difference in the interaction energy on the two sides is nonzero, and it is proportional to the 'flux'. This description also brings out the meaning of the gauge invariance of the total phase change. Any additional interaction energy that affect both paths equally, in an integral sense, will not have any effect on the total phase change. Since Coulomb interaction is causal, there is no place for any nonlocality in this problem. It seems that while the various debates have led to some remarkable experiments to avoid criticisms [5], many of the criticisms themselves were based on inadequate appreciation of the physical cause of the A–B phase change.

This analysis shows that the various A–B like phases — the original 'magnetic' A–B effect, the electric A–B effect [1] and the Aharanov–Casher effect [8] — are all manifestation of the same phenomenon, namely that of the phase change associated with an interaction energy which is spatially gradient free. For the magnetic effect and the electric effect, the physical source is the Coulomb interaction energy, and for the A–C effect it is the magnetostatic interaction energy $\mu \cdot \mathbf{B}$. Whether there is force (field) in the region where the particle is moving is irrelevant from a fundamental point of view, though for experiments such a field and force will lead to complications in terms of additional shift of the entire fringe pattern. Clearly, there is no causality problems of any kind in the A–B like effects, since the interaction energy can change only causally. The A–C effect has the same topological status as the A–B effect. They are all birds of the same feather.

3. Complementarity, geometric phase and which-path experiments

One of the remarkable achievements in geometrizing physical effects was the discovery by Pancharatnam [3] in the context of optics and by Berry [2] in the context of quantum mechanics that it was possible to separate the total phase change into a dynamical and a geometrical part. The geometrical phase has properties that allow significant unification and generalizations, apart from significant simplicity in calculations. There is another possible point of view stressing the dynamical physical origin of geometric phases associated with manipulations of states of fundamental particles. The geometric phase associated with spin rotations can be obtained directly from a generalization of the usual dynamical phase $\int p_i \mathrm{d}x^i$, to include the spin as one of the momenta p_i and the angular coordinate as the corresponding $\mathrm{d}x^i$. So, measurements of geometric phase φ_{geom} for fundamental particles is in effect a direct measurement of the spin (projection) s of these particles, through the relation $\varphi_{\mathrm{geom}} = \int s\mathrm{d}\theta$, where $\mathrm{d}\theta$ is the infinitesimal angular rotation on the spin projection [9]. In this view there is a physical meaning to the relative phase between a state and another rotated from the first by an angle which would make the second state orthogonal to the first. (One might ask then why there is no interference between such states though there is coherence. The answer is that there is interference, but the pattern is the overlap of two interference patterns shifted with respect to each other by π [10].)

Now I discuss an important application that has emerged from this point of view.

Most of the familiar discussion on complementarity principle is based on the uncertainty principle for the position and momentum variables, since which path information is essentially position information. But there is a class of atom interferometry experiments employing micromaser detectors in which the path is inferred without measuring the position directly [11]. In the atom interferometry experiment employing excited state atoms and the micromaser which-path detectors, the atomic wave-function splits into two amplitudes at a double slit and then there are two resonant cavities in line with each slit through which the atoms pass (figure 3). The high finesse cavity is tuned to ensure spontaneous emission of a low energy photon into the cavity and the presence of this photon after the atom passes to the detector provides the means for which-path detection. The momentum kick from

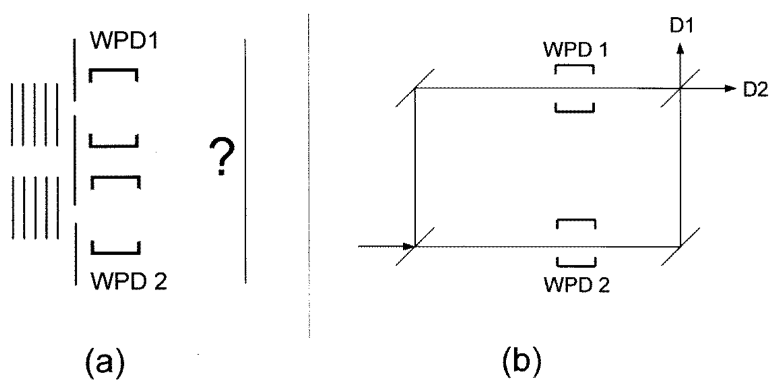

Figure 3. Two experiments with micromaser which-path detectors. (**a**) A two-slit experiment. (**b**) A Mach–Zehnder configuration. In both cases, if the cavities have nearly 100% finesse there will be complete loss of interference. But there is no back action on the momentum.

the emission of such a low energy photon is not sufficient to explain the loss of interference in such experiments. There have been considerable debate [12–15] on the issue and the fact that there is no momentum back-action was confirmed in a recent experiment by Durr, Nonn and Rempe [16]. Mathematically, the loss of interference is attributed to the correlation established between the which-path detector and the atomic wave-function.

The view that complementarity can arise due to correlations is formally correct. But the question remains as to whether there is any physical mechanism that acts on the wave-function to scramble the phase, without changing the momentum. It is important to note that the correlation with a detector takes a finite time to be established and it is physically more reasonable to think that the phase of the amplitudes are altered even before the final correlation is established with a cavity. This local view is important to have consistency in the context of delayed choice experiments.

It is possible to change the phase without introducing a momentum kick, as in the case of the A–B effect, if the physical process involves a rotation of the spinor wave-function of the two-level atom. If the quantum state changes by a process equivalent to a spin flip, the wave-function picks up a phase of $\pi/2$. In an atom interferometry experiment, *if the which-path detectors work on the principle of spontaneous emission into a tuned cavity then a change of state is equivalent to a rotation of the a spinor through π and the interference pattern should shift by $\pi/2$.* Since the direction of the rotation is unspecified in the case of spontaneous emission, there are two distinct sets of interference patterns shifted from a mean position by $\pm\pi/2$ and this results in two overlapping patterns shifted with respect to each other by π. The result is the apparent absence of the interference pattern. There is no momentum back-action. (In ref. [17], the authors have discussed the idea of loss of interference due to spin rotation for the electron in a magnetic field. An example in which the fluctuations in the geometric phase destroys interference in a two slit which-path experiment with photons has been discussed by Bhandari [18] and the similarity between the optics experiment and the atom interferometry experiments has been mentioned.)

The initial state of the atom, before entering the double slit and cavities, can be represented as $\psi_o(r,i) = \chi(r)\,|e\rangle$, where the spatial part and the internal state (denoted by $i = e$ or g, for excited state or ground state) are explicitly written. If there are no cavities, the wave-function after passing through the double slit is

$$\psi(r) = \frac{1}{\sqrt{2}}[\chi_1(r) + \chi_2(r)]\,|e\rangle\,. \tag{1}$$

This coherent superposition gives the interference pattern at the detector plane.

If a photon is registered in the upper detector, then the atom wave-function after passing through the detector would pick up a phase of $\pm\pi/2$. If the photon is emitted in the lower detector there is a similar change in phase. We can write the wave-function of the atom after passing through the cavities as

$$\psi(r) = \frac{1}{\sqrt{2}}[\chi_1(r)\exp(\pm i\pi/2) + \chi_2(r)]\,|g\rangle\,, \tag{2}$$

for the case in which the spontaneous emission takes place in the upper cavity and

$$\psi(r) = \frac{1}{\sqrt{2}}[\chi_1(r) + \chi_2(r)\exp(\pm i\pi/2)]\,|g\rangle\,, \tag{3}$$

for the case in which the emission takes place into the lower cavity. The net result is the overlap of two interference patterns, both of which are shifted with respect to the interference pattern without the cavities by $\pm\pi/2$, and the bright fringes of one pattern will overlap with the dark fringes of the other resulting in an *apparently washed out interference*. This conclusion holds even if there is only one good which-path detector.

At this stage I can state an interesting corollary: *If quantum complementarity is taken as a fundamental principle, then it implies that two-level atoms, neutrons etc. should behave like spinors under rotations*. They should pick up the phase factor $\pi/2$ under rotation through π; otherwise quantum complementarity will be violated in interference experiments [19].

The wave-functions of eq. (2) and (3) can be written only if the quantum state of the cavities can be distinguished in principle before and after the emission. If the occupation number in the cavity is already very large, the two cavities are indistinguishable even after the emission. Then the wave-functions $\chi_1(r)$ and $\chi_2(r)$ are coherent and the normal interference pattern emerges (the visibility will be a function of the photon occupation inside the cavities) [20]. Some authors have also interpreted these results in terms of the uncertainty relation between the occupation number and the phase angle, $\Delta N \Delta\phi \geq 1$, and when ΔN is large due to photon statistical fluctuations, the phase fluctuation is small [18,20]. But the uncertainty relation gives only a bound and not the right solution of two overlapping interference patterns. (One could also ask what is the physical system relevant here on which the uncertainty principle is applied. The number-phase uncertainty refers to the photons in the cavity whereas the phase uncertainty we need is on the atomic wave-function! It turns out that the uncertainty principle becomes relevant not for explaining the loss of interference, but for explaining the impossibility of doing quantum erasure when the occupation number in the cavity is small [19].)

It is important to mention that there is no which-path detection without quantum back-action; but in the experiments we are discussing, the back-action is on the internal angular state rather than on the linear momentum. As an additional remark we may mention that the case when there is only one high finesse cavity is extremely interesting, though we do not discuss it here. There is still 100% path information and corresponding loss of interference, though there is no direct physical effect that can change the phase for 50% of the atoms [21].

The similarity between the Aharanov–Bohm phase and the random spinor phase in the which-path experiment is obvious. In fact, the whole problem can be analysed in terms of the random Aharanov–Bohm phase introduced due to the interaction of the atoms with the effective scalar potential due to the photons in the cavity. The sign of potential and of the resulting phase change is uncertain, but the potential does not change the momentum of the atoms.

The next example I wish to discuss is also one of changes in spin, but I discuss this in the context of classical gravity. Our approach identifies the local field responsible for the Thomas precession of the spin.

The Einstein's equations of general relativity can be written, in the limit of weak gravitational field, in a form resembling the Maxwell's equations of electromagnetism [22,23]. The time-time component of the metric represents the scalar potential, whose gradient gives the standard Newtonian gravitational field $(\mathbf{g})$, and the space-time components defines a vector potential whose curl is the analogue of the magnetic field. The gravitomagnetic field in general relativity is generated by moving and rotating masses, just as the magnetic field is generated by currents and magnetic moments. Angular momentum interacts with the gravito-magnetic field and there are analogues of spin-orbit and spin-spin interactions leading to spin precession [22,23]. It is extremely difficult to study these components of the tensorial gravitational interaction owing to the smallness of the coupling. There is some evidence for the existence of the gravito-magnetic field from lunar laser ranging [24] and the LAGEOS experiments [25]. The space experiment gravity probe-B, an experiment seeking a direct precision measurement of the gravito-magnetic field by monitoring the precession of a gyroscope in the gravitomagnetic field of the Earth, is in preparation [26].

The gravito-electric and magnetic fields have transformation properties similar to that of the electromagnetic fields under Lorentz transformations. The induced gravito-magnetic field in the moving frame of a gyroscope in orbit in the Earth's gravitational field is of the form $B_g \propto (\mathbf{v} \times \mathbf{g})/c$, similar to the relativistic transformation of the electric field into a magnetic field in a frame moving with velocity $\mathbf{v}$. The precession angle of the gyroscope in the induced gravito-magnetic field depends on the local gravitational potential and the rate of precession is proportional to the local gravitational field. Remarkably, the spin precession rate does not depend on the curvature of space-time. By the equivalence principle, *a magnetic like field should arise from the relativistic transformation of the inertial acceleration as well* provided there is a nonzero $\mathbf{v} \times \mathbf{a}$ for the inertial motion, where $\mathbf{a}$ is the acceleration. This is the case, for example, for motion along a curved path. The equivalence principle demands the existence of a magnetic-like inertial force field and the interaction of angular momentum with such an accelero-magnetic field will lead to spin precession.

The equation for the spin precession is

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \frac{1}{2c}\mathbf{s} \times \mathbf{B}_a = \frac{1}{2c}\mathbf{s} \times (\mathbf{v} \times \mathbf{a})/c. \tag{4}$$

The frequency of spin precession then is $\frac{\mathbf{v} \times \mathbf{a}}{2c^2}$. Spin precession with exactly this value for precession frequency is well known in atomic physics. I identify this as the Thomas precession [27].

This shows that the Thomas precession is due to the spin-orbit coupling through the accelero-magnetic field. *The torque that is responsible for the precession of the spin results from a local physical interaction*. (Usually it is treated as resulting from combining boosts in different directions, and no torque is identified as responsible for the change in the angular momentum. Both views are equivalent – one is geometrical, in terms of modification to the Minkowski metric and the other view 'physical', in terms of a potential.) The principle of equivalence ensures that the spin precession in the gravito-magnetic field will have the same characteristics as in the accelero-magnetic field since the curvature does not play a role in the precession rate. *In any metric gravitation theory that obeys the equivalence principle to the accuracy demanded by experiments, Thomas precession implies the existence of the gravito-magnetic field*. This is a significant and important result, especially in the context of the difficulty in studying the gravito-magnetic field directly. Since atomic spectroscopy is a high precision tool and since the doublet separations are measured to remarkable accuracies exceeding 10^{-10}, atomic spectroscopy may be considered as the most precise test of the existence and properties of the gravito-magnetic field, in a domain in which the equivalence principle is valid.

4. The classical nature of A–B fringe shifts

Quantum phase changes in a gravitational field is an interesting topic. The physical effects are obtained by including the Newtonian gravitational potential in the Schrödinger equation. There have been some discussion as to whether there is any violation of the equivalence principle in cases involving gravity and quantum mechanics, or whether there is any 'nonclassical' signatures (like physical quantities depending on the Planck's constant) in such cases [28,29].

As an example let us take the gravitational analogue of the electric A–B effect [30]. If there are two interfering paths, x_1 and x_2, through regions with different potentials ϕ_1 and ϕ_2, the accumulated relative phase shift in the electromagnetic case is given by $\frac{e}{\hbar}\left(\int_{x1}\phi_1 \mathrm{d}t_1 - \int_{x2}\phi_2 \mathrm{d}t_2\right)$. Conceptually the scalar gravitational A–B effect is very similar to the electromagnetic A–B effect. The phase change in the gravitational case is given by $\frac{m}{\hbar}\left(\int_{x1}\phi_1(x)\mathrm{d}t_1 - \int_{x2}\phi_2(x)\mathrm{d}t_2\right)$. Here $\phi(x)$ is the gravitational potential. In effect this phase shift is the same as the gravitational phase shift measured in neutron interferometry experiments [31].

The formal expression for quantum phase change in a gravitational field explicitly contains the Planck's constant and the mass of the particle. At first sight this may be surprising since gravitational phenomena obey the equivalence principle and no measurable quantity should contain the inertial or gravitational mass explicitly. Also, there is no other known laboratory gravitational phenomenon in which the Planck's constant appears explicitly. But the apparent 'quantum signature' is a mirage. It may seem surprising to many if it is stated that neither the gravitational A–B effect *nor the electromagnetic A–B effect* contain any quantum signature in the experimentally measured quantity – in the shift of the interference fringes.

The expressions for the phase shifts is proportional to the charge of the field. Therefore, in the gravitational case, the phase shift is proportional to the gravitational mass. The inertial mass comes into picture through the de Broglie relation connecting the wavelength and the momentum of the particles. *The fringe shift itself, which is the product of the phase shift and the wavelength, is the directly measured quantity and it is independent of the mass if the inertial and gravitational masses are equivalent*. The fringe shift for propagation over length l is $\Delta s = a\frac{m}{\hbar}(\Delta\phi)\lambda t \propto (\Delta\phi)t^2/l$, where a is a scaling factor. This is *independent of the mass and the Planck's constant* [30]. The quantum signature is imprinted only in the fringe spacing. In other words, while the interference pattern is a consequence of the quantum nature of the atoms, the shift in the fringe pattern itself is purely classical. A similar remark applies to the quantum signature in the electromagnetic A–B effect. The fringe shift itself is classical, with no dependence on the Planck's constant [32]. The phase shift can be written as

$$\Delta\varphi = \frac{e}{\hbar}(\phi_1 - \phi_2)t = \frac{e}{\hbar}(\phi_1 - \phi_2)l/v, \tag{5}$$

where l is the length scale of the interfering paths and v the average velocity of the particles. The fringe shift is

$$\Delta s = a\lambda\Delta\varphi = \frac{\hbar}{p}\frac{e(\phi_1 - \phi_2)}{\hbar}\frac{l}{p/m} = \frac{\frac{1}{2}e(\phi_1 - \phi_2)l}{p^2/2m}. \tag{6}$$

I rewrite this as

$$\Delta s = \frac{\frac{1}{2}a(\Delta E_{\rm Potential})l}{E_{\rm Kinetic}}. \tag{7}$$

This is the general expression for the fringe shift in A–B effect and it is applicable to all the A–B like phases we have discussed. In the magnetic A–B case this is not obvious from the standard expression containing the magnetic flux, but our earlier analysis establishing the equivalence between the magnetic and electric A–B effects implies this result for the magnetic A–B effect. The fringe shift looks classical and depends on the ratio of the difference in the potential (interaction) energy over the two paths and the kinetic energy. Curiously, the expression contains a factor of $1/2$, accounting for the *quantum fact* that there is one particle and two paths.

5. Summary

In summary, I have presented a unified view on Aharanov–Bohm like phases motivated by the physical requirement that all physical changes, including those in phases should originate in local physical interactions. This ‘local realistic’ view has clarified many basic issues that have been debated widely. (In fact, pursuing this line of thought lead to a remarkable resolution of the long standing EPR nonlocality puzzle [33].) Phases arising in spin rotation are closely related to the A–B phase. I have presented the solution to the problem of origin of quantum complementarity without Heisenberg momentum back action in atom interferometry which-path experiments. Consideration of the motion of a spin in a gravitational field has lead to the result that the Thomas precession is the result of a spin-orbit coupling with a local accelero-magnetic field, found by applying the equivalence principle. The unified picture also brought out the fact that the shift of the interference fringes in the general class of A–B like effects is ‘classical’, in the sense of not containing the Planck’s constant signature.

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