Singlet and triplet differential cross sections for $pp \rightarrow pp\pi^0$

P. N. Deepak*
Department of Studies in Physics, University of Mysore, Mysore-570 006, India

G. Ramachandran
Indian Institute of Astrophysics, Koramangala, Banglore-560 034, India

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The singlet and triplet differential cross sections for $pp \rightarrow pp\pi^0$ have been estimated for the first time at 325, 350, 375, and 400 MeV using the results of the recent experimental measurements [Phys. Rev. C **63**, 064002 (2001)] of Meyer *et al.*

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The study of the reaction $\vec{p}\vec{p} \rightarrow pp\pi^0$ has excited considerable experimental interest [1–4], since the transition is mainly to the final Ss state and is completely spin dependent at threshold energies up to 300 MeV and the large values of momentum transfer involved probes the interaction at very short distances. As the energy increases, the transitions to Ps and Pp states are expected to contribute [1,2] below 400 MeV although pionic d-wave effects have been reported [5] even at a beam energy of 310 MeV. Taking, therefore, transitions to Sd and Ds states also into account, an analysis of the measurements [4] on a complete set of polarization observables have been presented recently for energies below 400 MeV. Attention has also been drawn [6] to the large longitudinal analyzing power characterizing this reaction. It is of interest to note [7] that the difference ${}^3d^2\sigma_{+1} - {}^3d^2\sigma_{-1}$ is indeed proportional to the longitudinal analyzing power, where ${}^{2s_i+1}d^2\sigma_m$ denotes the differential cross section for the process $pp \rightarrow pp\pi^0$ as a function of five independent kinematic variables characterizing the three-body final state when the reaction is initiated in the spin state $|s_i,m\rangle$, $m=-s_i,\ldots,+s_i$ and $s_i=0$, 1 corresponding to singlet and triplet states. Measurements have also been reported [8] at energies up to 425 MeV that were analyzed in terms of the above-mentioned partial waves, where evidence for Ds state was seen through the presence of a $\cos^4\theta$ term even at 310 MeV. In this context, it is of interest to note that analysis of the total cross section σ into its singlet and triplet components ${}^{2s_i+1}\sigma_m$ have been attempted in several $\vec{p}\vec{p} \rightarrow pp\pi^0$ measurements [1–3] using the theoretical results of Bilenky and Ryndin [9], which, however, are not applicable at the differential level. On the other hand, it may also be noted that the irreducible tensor approach [10] to pion production in NN collisions leads to elegant formulas for the differential cross sections in terms of irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$ of rank $\lambda=|s_i-s_f|$ to (s_i+s_f), where s_i,s_f denote the initial and final channel spins. This approach, moreover, is not limited by the number of final partial waves. The purpose of this report is, therefore, to present estimates, using [7,10] for the singlet and triplet cross sections at the differential level itself based on the recently reported measurements of Meyer *et al.* [4].

We first of all note that the experimentally measured differential cross section for $\vec{p}\vec{p} \rightarrow pp\pi^0$ may be written, using the same notations as in [4], as

$$\frac{d\sigma(\theta_p,\varphi_p,\theta_q,\varphi_q,\epsilon,\vec{P},\vec{Q})}{d\Omega_p\, d\Omega_q\, d\epsilon} \equiv \sigma(\xi,\vec{P},\vec{Q}) = \mathrm{Tr}(\mathbf{M}\rho\mathbf{M}^\dagger), \tag{1}$$

where ρ denotes the initial spin density matrix

$$\rho = \tfrac{1}{2}\,(1+\vec{\sigma}_1\cdot\vec{P})(1+\vec{\sigma}_2\cdot\vec{Q}) \tag{2}$$

and $\mathbf{M}$ has the form [10,7]

$$\mathbf{M} = \sum_{s_i,s_f=0,1}\ \sum_{\lambda=|s_i-s_f|}^{s_i+s_f} (S^\lambda(s_f,s_i)\cdot M^\lambda(s_f,s_i)), \tag{3}$$

in terms of the irreducible tensor operators $S^\lambda_\mu(s_f,s_i)$ of rank λ, as defined in [11]. The irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$ may be expressed following [10] in terms of the partial wave reaction amplitudes $\mathsf{M}^J_{l_q(l_ps_f)j;ls_i}(s,s_{12})$, employing the same notations for the angular momenta and the Mandelstam variables as in [4], through

$$\begin{aligned} M^\lambda_\mu(s_f,s_i) = &\sum_{l,l_p,l_q,L_f,j,J} W(ls_iL_fs_f;J\lambda)C(L_fl\lambda;\mu 0\mu) \\ &\times \mathsf{M}^J_{l_q(l_ps_f)j;ls_i}(s,s_{12})(Y_{l_p}(\theta_p,\varphi_p) \\ &\otimes Y_{l_q}(\theta_q,\varphi_q))^{L_f}_\mu . \end{aligned} \tag{4}$$

We next express ρ in the channel spin representation in the form

$$\rho = \sum_{s_i,s_i'=0}^{1}\ \sum_{k=|s_i-s_i'|}^{(s_i+s_i')} (I^k(s_i,s_i')\cdot S^k(s_i,s_i')), \tag{5}$$

where

$$I^k_q(s_i,s_i') = \frac{1}{2}[s_i']\sum_{k_1=0}^{1}\sum_{k_2=0}^{1}(-1)^{k_1+k_2-k}[k_1][k_2] \times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_i \\ \frac{1}{2} & \frac{1}{2} & s_i' \\ k_1 & k_2 & k \end{Bmatrix} (P^{k_1}\otimes Q^{k_2})^k_q, \tag{6}$$

*Electronic address: pndeepak@sancharnet.in

with $P_0^0 = Q_0^0 = 1$.

Using the known properties [11] of the irreducible tensor operators S_q^k, the differential cross section given by Eq. (1) may then be expressed in the form

$$\sigma(\xi,\vec{P},\vec{Q}) = \sum_{s_i,s_i'=0}^{1} \sum_k (I^k(s_i,s_i') \cdot \mathcal{B}^k(s_i,s_i')), \qquad (7)$$

where the irreducible tensors

$$\mathcal{B}_q^k(s_i,s_i') = [s_i] \sum_{s_f} [s_f]^2 \sum_{\lambda,\lambda'} (-1)^\lambda [\lambda] \times [\lambda'] W(s_i' k s_f \lambda; s_i \lambda')(M^\lambda(s_f,s_i) \otimes M^{\dagger\lambda'}(s_f,s_i'))_q^k \qquad (8)$$

are bilinear in terms of the irreducible tensor amplitudes. They can be explicitly evaluated using Eq. (4) for transitions to Ss, Ps, Pp, Sd, and Ds states.

The unpolarized differential cross section $\sigma_0(\xi)$ for the process is then given by

$$\sigma_0(\xi) = \sum_{s_i=0,1} \sum_{m=-s_i}^{s_i} {}^{2s_i+1}\sigma_m(\xi), \qquad (9)$$

where

$${}^1\sigma_0(\xi) = \frac{1}{4}\mathcal{B}_0^0(0,0) \qquad (10a)$$

$${}^3\sigma_0(\xi) = \frac{1}{4}\left[\frac{1}{3}\mathcal{B}_0^0(1,1) - \frac{\sqrt{2}}{3}\mathcal{B}_0^2(1,1)\right] \qquad (10b)$$

$${}^3\sigma_{\pm 1}(\xi) = \frac{1}{4}\left[\frac{1}{3}\mathcal{B}_0^0(1,1) \pm \frac{1}{\sqrt{6}}\mathcal{B}_0^1(1,1) + \frac{1}{3\sqrt{2}}\mathcal{B}_0^2(1,1)\right]. \qquad (10c)$$

Using standard Racah algebra, we also have explicitly

$$\mathcal{B}_0^0(0,0) = A_1 \cos^2\theta_p + A_2 \sin^2\theta_p, \qquad (11a)$$

$$\mathcal{B}_0^0(1,1) = B_0 + B_1(3\cos^2\theta_q - 1) + B_2(3\cos^2\theta_p - 1) + B_3(3\cos^2\theta_q - 1)(3\cos^2\theta_p - 1) + B_4 \sin 2\theta_p \sin 2\theta_q \cos\Delta\varphi + B_5 \sin^2\theta_p \sin^2\theta_q \cos 2\Delta\varphi, \qquad (11b)$$

$$\mathcal{B}_0^1(1,1) = C_1 \sin 2\theta_p \sin 2\theta_q \sin\Delta\varphi + C_2 \sin^2\theta_p \sin^2\theta_q \sin 2\Delta\varphi, \qquad (11c)$$

$$\mathcal{B}_0^2(1,1) = D_0 + D_1(3\cos^2\theta_q - 1) + D_2(3\cos^2\theta_p - 1) + D_3(3\cos^2\theta_q - 1)(3\cos^2\theta_p - 1) + D_4 \sin 2\theta_p \sin 2\theta_q \cos\Delta\varphi + D_5 \sin^2\theta_p \sin^2\theta_q \cos 2\Delta\varphi. \qquad (11d)$$

The $\mathcal{B}_q^k(s_i,s_i')$ may then be related to the A_{ij}'s defined in [4] through

$$\mathcal{B}_0^0(0,0) = \sigma_0(\xi)[1 - A_\Sigma(\xi) - A_{zz}(\xi)], \qquad (12a)$$

$$\mathcal{B}_0^0(1,1) = \sigma_0(\xi)[3 + A_\Sigma(\xi) + A_{zz}(\xi)], \qquad (12b)$$

$$\mathcal{B}_0^1(1,1) = \sqrt{6}\sigma_0(\xi)[A_{z0}(\xi) + A_{0z}(\xi)], \qquad (12c)$$

$$\mathcal{B}_0^2(1,1) = 2\sqrt{2}\sigma_0(\xi)[A_{zz}(\xi) - \tfrac{1}{2} A_\Sigma(\xi)]. \qquad (12d)$$

These relations enable us to identify the coefficients A_i, B_i, C_i, D_i in Eqs. (11) in terms of the E, F_k, H_k^{ij}, I, K of [4] for which numerical values have been deduced and given in Table IV of [4]. It needs to be mentioned here that the E, F_k, H_k^{ij}, I, K given in Table IV of [4] are dimensionless as they have been normalized by a common factor of $(8\pi^2)/(\sigma_{\text{tot}})$, where σ_{tot} denotes the spin-averaged total cross section given in Table V of [4]. As such, they have to be multiplied

TABLE I. Values of the coefficients occurring in Eqs. (14) at the four bombarding energies based on the results of Meyer *et al.* [4].

	325 MeV		350 MeV		375 MeV		400 MeV	
	Value	Error	Value	Error	Value	Error	Value	Error
α_1	0.166	0.082	0.361	0.087	0.665	0.034	0.717	0.054
α_2	2.996	0.349	2.218	0.364	1.622	0.142	1.378	0.233
α_3	0.083	0.041	0.181	0.043	0.333	0.017	0.359	0.027
α_4	1.498	0.174	1.109	0.182	0.811	0.071	0.689	0.116
β_1	0.045	0.099	0.133	0.104	0.206	0.038	0.273	0.07
β_2	−0.034	0.12	−0.102	0.128	−0.16	0.04	−0.21	0.056
β_3	−0.054	0.423	−0.163	0.427	−0.257	0.403	−0.336	0.41
β_4	−0.04	0.431	−0.122	0.437	−0.194	0.404	−0.252	0.426
β_5	0.029	0.108	0.053	0.109	0.059	0.101	0.061	0.107
β_6	0.004	0.004	0.012	0.013	0.018	0.019	0.008	0.027
β_7	−0.024	0.023	0.009	0.021	−0.018	0.009	0	0.014
β_8	−0.059	0.008	−0.173	0.024	−0.270	0.038	−0.306	0.04
F_1	0.168	0.021	0.265	0.022	0.262	0.007	0.297	0.013

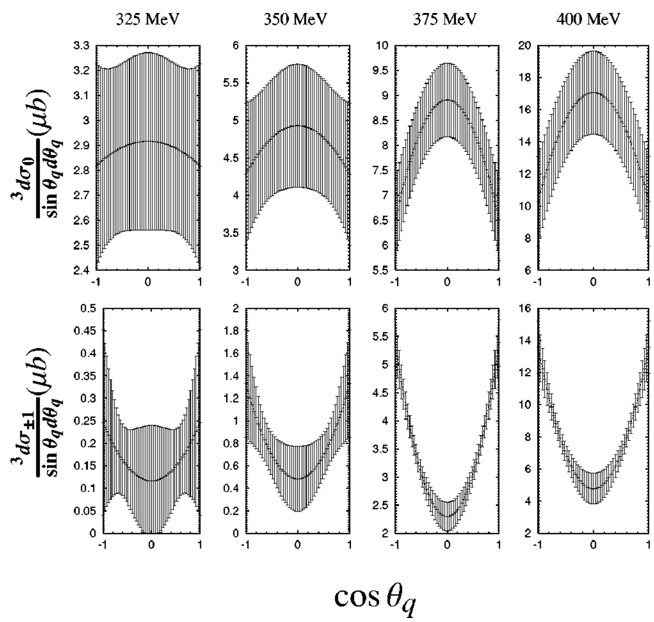

FIG. 1. A plot of $(^3d\sigma_m)/(\sin\theta_q\, d\theta_q)$, $m=\pm 1$, 0 as functions of θ_q based on the results of Meyer *et al.* [4].

by $(\sigma_{\rm tot})/(8\pi^2)$ before estimating the A_i, B_i, C_i, D_i of Eq. (11) numerically and hence the $^{2s_i+1}\sigma_m(\xi)$ of Eq. (10). The one-dimensional differential cross sections are then defined as

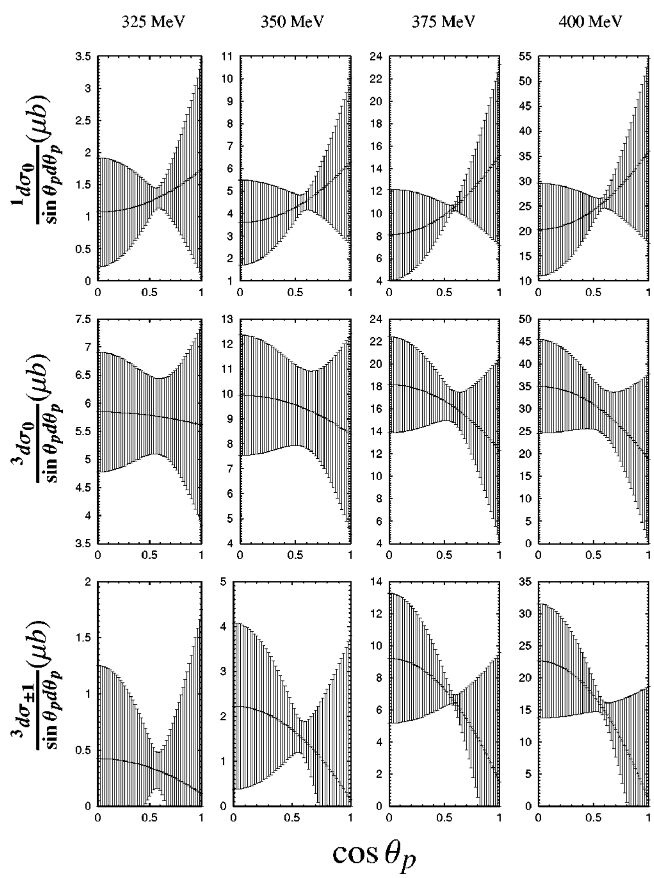

FIG. 2. A plot of $(^{2s_i+1}d\sigma_m)/(\sin\theta_p\, d\theta_p)$, $m=\pm 1$, 0 as functions of θ_p based on the results of Meyer *et al.* [4].

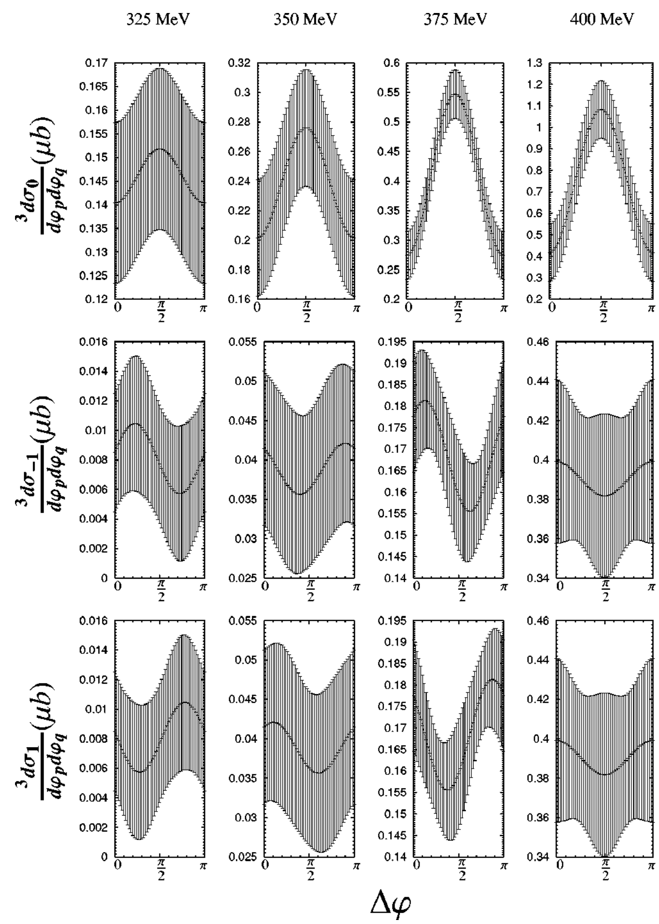

FIG. 3. A plot of $(^3d\sigma_m)/(d\varphi_p\, d\varphi_q)$, $m=\pm 1$, 0 as functions of $\Delta\varphi$ based on the results of Meyer *et al.* [4].

$$\frac{^{2s_i+1}d\sigma_m}{\sin\theta_q\, d\theta_q}=\int\ {}^{2s_i+1}\sigma_m(\xi)d\Omega_p\, d\varphi_q d\epsilon, \tag{13a}$$

$$\frac{^{2s_i+1}d\sigma_m}{\sin\theta_p\, d\theta_p}=\int\ {}^{2s_i+1}\sigma_m(\xi)d\Omega_q\, d\varphi_p d\epsilon, \tag{13b}$$

$$\frac{^{2s_i+1}d\sigma_m}{d\varphi_p\, d\varphi_q}=\int\ {}^{2s_i+1}\sigma_m(\xi)d(\cos\theta_p)d(\cos\theta_q)\, d\epsilon. \tag{13c}$$

They may now be given explicitly in terms of the trigonometric functions of the respective angles as

$$\frac{^3d\sigma_{\pm 1}}{\sin\theta_q\, d\theta_q}=\frac{\sigma_{\rm tot}}{8}\left[\alpha_1+\beta_1(3\cos^2\theta_q-1)\right], \tag{14a}$$

$$\frac{^3d\sigma_0}{\sin\theta_q\, d\theta_q}=\frac{\sigma_{\rm tot}}{8}\left[\alpha_2+\beta_2(3\cos^2\theta_q-1)\right], \tag{14b}$$

$$\frac{^1d\sigma_0}{\sin\theta_q\, d\theta_q}=\frac{\sigma_{\rm tot}}{2}F_1, \tag{14c}$$

$$\frac{^3d\sigma_{\pm 1}}{\sin\theta_p\, d\theta_p}=\frac{\sigma_{\rm tot}}{4}\left[\alpha_1+\beta_3(3\cos^2\theta_p-1)\right], \tag{14d}$$

TABLE II. Numerical estimates deduced for the (angle independent) $({}^{1}d\sigma_0)/(\sin\theta_q\, d\theta_q)$ and $({}^{1}d\sigma_0)/(d\varphi_p\, d\varphi_q)$ at the four bombarding energies based on the results of Meyer *et al.* [4].

	325 MeV		350 MeV		375 MeV		400 MeV	
	Value (μb)	Error	Value (μb)	Error	Value (μb)	Error	Value (μb)	Error
${}^{1}d\sigma_0/(\sin\theta_q\, d\theta_q)$	0.647	0.081	2.253	0.187	5.24	0.140	12.771	0.559
${}^{1}d\sigma_0/(d\varphi_p\, d\varphi_q)$	0.033	0.004	0.114	0.010	0.266	0.007	0.647	0.028

$$\frac{{}^{3}d\sigma_0}{\sin\theta_p\, d\theta_p}=\frac{\sigma_{\rm tot}}{4}[\alpha_2+\beta_4(3\cos^2\theta_p-1)], \tag{14e}$$

$$\frac{{}^{1}d\sigma_0}{\sin\theta_p\, d\theta_p}=\sigma_{\rm tot}\,[F_1+\beta_5(3\cos^2\theta_p-1)], \tag{14f}$$

$$\frac{{}^{3}d\sigma_{\pm 1}}{d\varphi_p\, d\varphi_q}=\frac{\sigma_{\rm tot}}{8\pi^2}\,[\alpha_3+\beta_6\cos 2\Delta\varphi\pm\beta_7\sin 2\Delta\varphi], \tag{14g}$$

$$\frac{{}^{3}d\sigma_0}{d\varphi_p\, d\varphi_q}=\frac{\sigma_{\rm tot}}{8\pi^2}\,[\alpha_4+\beta_8\cos 2\Delta\varphi], \tag{14h}$$

$$\frac{{}^{1}d\sigma_0}{d\varphi_p\, d\varphi_q}=\frac{\sigma_{\rm tot}}{4\pi^2}F_1, \tag{14i}$$

where the coefficients α_i, β_i are once again dimensionless since $\sigma_{\rm tot}$ occurs as a common factor in each of the Eqs. (14). It may be noted that F_1 is the same as in [4]. The numerical values of all these coefficients together with their errors are presented in Table I. Of the total 16 partial wave amplitudes taken into consideration, it is clear that 11 amplitudes: $\mathsf{M}^0_{1(11)1;11}$, $\mathsf{M}^1_{1(11)0;11}$, $\mathsf{M}^1_{1(11)1;11}$, $\mathsf{M}^1_{1(11)2;11}$, $\mathsf{M}^2_{1(11)1;11}$, $\mathsf{M}^2_{1(11)2;11}$, $\mathsf{M}^2_{1(11)1;31}$, $\mathsf{M}^2_{1(11)2;31}$, $\mathsf{M}^3_{1(11)2;31}$, $\mathsf{M}^0_{0(11)0;00}$, $\mathsf{M}^2_{0(11)2;20}$ lead to $s_f=1$ and 5 amplitudes: $\mathsf{M}^0_{0(00)0;11}$, $\mathsf{M}^2_{0(20)2;11}$, $\mathsf{M}^2_{0(20)2;31}$, $\mathsf{M}^2_{2(00)0;11}$, $\mathsf{M}^2_{2(00)0;31}$ lead to $s_f=0$. It is also clear from Eq. (8) that these two sets do not mutually interfere. This has been noted also in [4]. It may also be noticed that only two among the 16 viz., $\mathsf{M}^0_{0(11)0;00}$ and $\mathsf{M}^2_{0(11)2;20}$ are the singlet amplitudes. Both of them lead to $l_q=0$. Hence, it follows that the singlet differential cross section $({}^{1}d\sigma_0)/(\sin\theta_q\, d\theta_q)$ is independent of θ_q, which is clear from Eqs. (10a) and (11a). From these two equations it can also be seen that $({}^{1}d\sigma_0)/(d\varphi_p\, d\varphi_q)$ is independent of $\Delta\varphi$. However, the singlet differential cross section $({}^{1}d\sigma_0)/(\sin\theta_p\, d\theta_p)$ varies with θ_p since $l_p=1$ in either of the singlet partial wave amplitudes. The dependence on the angle θ_p is explicitly seen from Eqs. (10a) and (11a). The estimates deduced for the triplet differential cross sections $({}^{3}d\sigma_m)/(\sin\theta_q\, d\theta_q)$ are given in Fig. 1, while $({}^{1}d\sigma_0)/(\sin\theta_p\, d\theta_p)$ and $({}^{3}d\sigma_m)/(\sin\theta_p\, d\theta_p)$ are shown in Fig. 2, whereas $({}^{3}d\sigma_m)/(d\varphi_p\, d\varphi_q)$ as a function of $\Delta\varphi$ are shown in Fig. 3 for the four bombarding energies 325, 350, 375, 400 MeV. The estimates for $({}^{1}d\sigma_0)/(\sin\theta_q\, d\theta_q)$ and $({}^{1}d\sigma_0)/(d\varphi_p\, d\varphi_q)$ that are independent of θ_q and $\Delta\varphi$ respectively are given in Table II for the bombarding energies 325, 350, 375, and 400 MeV. It may be noted that $\mathcal{B}^1_0(1,1)$ vanishes on integration with respect to $d\varphi_q\, d\varphi_p$. Therefore, $({}^{3}d\sigma_1)/(\sin\theta_q\, d\theta_q)=({}^{3}d\sigma_{-1})/(\sin\theta_q\, d\theta_q)$, $({}^{3}d\sigma_1)/(\sin\theta_p\, d\theta_p)=({}^{3}d\sigma_{-1})/(\sin\theta_p\, d\theta_p)$ whereas $({}^{3}d\sigma_1)/(d\varphi_p\, d\varphi_q)\neq({}^{3}d\sigma_{-1})/(d\varphi_p\, d\varphi_q)$.

The numerical values for E, F_k, H_k^{ij}, I, K given in Table IV of [4] together with $\sigma_{\rm tot}$ given in Table V of [4] are adequate to deduce the above estimates for the differential cross sections ${}^{2s_i+1}\sigma_m$ at the one-dimensional level. It may, however, be noted from Eqs. (10), (11), and (12) that one also needs estimates (based on experiments) for H_3^{00}, H_3^{Σ}, H_3^{zz} in addition to those given in Table IV of [4] in order to estimate ${}^{2s_i+1}\sigma_m(\xi)$ at the double differential level. It would, therefore, be desirable to experimentally determine the H_3^{00}, H_3^{Σ}, H_3^{zz}. We may point out further that the double differential cross sections ${}^{2s_i+1}\sigma_m(\xi)$ may also be directly determined from experimental measurements as suggested in [7]. Therefore, we would like to encourage further measurements of the double differential cross sections for $\vec{p}\vec{p}\to pp\pi^0$ on the above lines.

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