J. Astrophys. Astr. (2000) **21**, 255-258

An Operator Perturbation Method of Polarized Line Transfer V. *Diagnosis of Solar Weak Magnetic Fields*

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Abstract. We present an application of the PALI (Polarized Approximate Lambda Iteration) method to the resonance scattering in spectral lines formed in the presence of weak magnetic fields. The method is based on an operator perturbation approach, and can efficiently give solutions for oriented vector magnetic fields in the solar atmosphere.

Key words. Polarization—magnetic fields, radiative transfer—stars: atmospheres—methods: numerical.

1. Introduction

We refer to polarized spectral lines formed outside the active regions having 'weak' oriented fields ($0 \leq B \leq 300$ G) which cause de-polarization in Stokes-Q, and rotation of the plane of polarization in Stokes-U parameter of the 'resonantly scattered' line radiation. This phenomenon called 'Hanle Effect', is invoked to explain the linear polarization changes observed in lower chromospheric resonance lines such as Ca I 4227 Å, Sr 4607 Å and Sr II 4078 Å (see Bianda *et al.* 1999 for observational Hanle diagnostics). The theoretical interpretation of such data demands the solution of NLTE polarized line transfer equation, for several combinations of independent parameters.

2. Results and discussions

The details of the Hanle scattering problem are described in Nagendra *et al.* (1998, 1999). The coherent superposition of radiatively broadened Zeeman sub-states in weak fields ($\Delta \nu_L \leq \Gamma_R$), is responsible for the Hanle effect. The Stokes-V parameter is negligible in weak fields.

2.1 *The two-parameter polarization diagrams*

Determination of field parameters from observed data can be attempted with the help of 'two-parameter polarization diagrams' showing a network of *iso-strength* and

$D\nu_L = G_R$

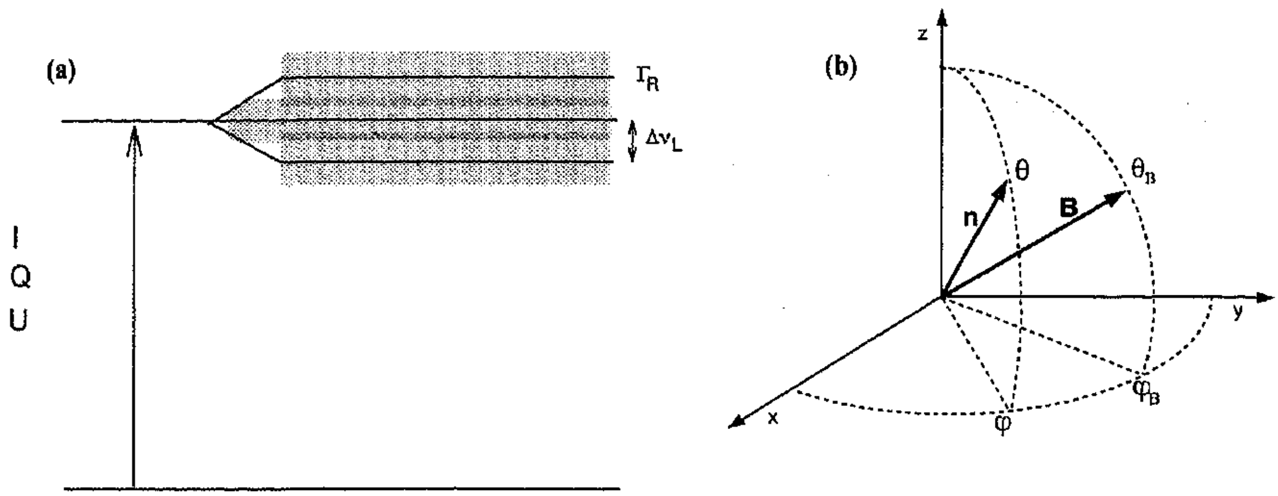

Figure 1. (**a**): Hanle effect in weak magnetic fields (B = 10 – 300 G), when ($\Delta\nu_L \approx \Gamma_R$). Pure Zeeman effect ($B$ = 1000 – 3000 G) corresponds to ($\Delta\nu_L \gg \Gamma_R$); (**b**): Geometry specifying the direction of the magnetic field B and of the line-of-sight **n**. Angles θ and θ_B are the co-latitudes of **n** and **B**, respectively. The azimuthal angles φ and φ_B are measured from the x-axis in an anti-clockwise direction in the xy-plane - that also represents the 1-D plane parallel slab atmosphere. The z-axis is the vertical direction. The light gray bands represent the radiative width Γ_R.

iso-azimuth curves. For a given line of sight, determined by the values of θ and φ (Fig. 1b), we choose a value of θ_B and vary the two other parameters of the vector magnetic field, γ_B and φ_B. The field strength parameter is $\gamma_B = 2\pi\Delta\nu_L g_J/\Gamma_R$, where g_J is the Lande g-factor of the upper level with a radiative width Γ_R, and $\Delta\nu_L$ is the Larmour frequency (Fig. 1a). If an observational data point (*U/I, Q/I*) falls within an interval defined by $[\Delta\gamma_B, \Delta\varphi_B]$, we get upper and lower limits on the possible values of γ_B and φ_B. This approach is useful when an independent estimate of θ_B is available. We now point out the generalization of the relevant equations in Nagendra *et al.* (1998, 1999), where the meaning of mathematical symbols in equations (1) and (2), can be found. For PRD mechanism, $\bar{\mathbf{J}}(\tau, x)$ is given by (when φ_B is constant with depth):

$$\bar{\mathbf{J}}(\tau,x) = \hat{R}(\varphi_B)\bar{\mathbf{J}}^0(\tau,x), \tag{1}$$

where $\bar{\mathbf{J}}_0(\tau, x)$ is the *reduced mean intensity* computed for the special case of field azimuth $\varphi_B = 0$. Notice that one can get the $\bar{\mathbf{J}}(\tau, x)$ for an arbitrary value of φ_B, using the above equation. The corresponding line source vector is:

$$\mathbf{S}_l(\tau,x) = (1-\varepsilon(\tau))\hat{H}_B(\theta_B,\varphi_B,B)\bar{\mathbf{J}}(\tau,x) + \mathbf{S}^{th}(\tau), \tag{2}$$

which gives the *reduced specific intensities* by application of a formal solution of the transfer equation. The iso-azimuth curves are computed by solving the Hanle transfer problem fully, for several values of γ_B. Iso-strength curves however are computed rapidly by taking advantage of the Hanle symmetry mentioned above. The polarization diagrams are easy to construct since one can express I, Q and U in terms of the six components of the 'azimuth independent reduced specific intensity vector' written as I = $(I_I, I_Q, I_{+1}, I_{-1}, I_{+2}, I_{-2})^T$ (see equations (81)-(83) in Nagendra *et al.* 1998).

Details of constructing the well known "two-parameter polarization diagrams" are presented in Nagendra *et al.* (1998). Exactly identical model parameters are employed

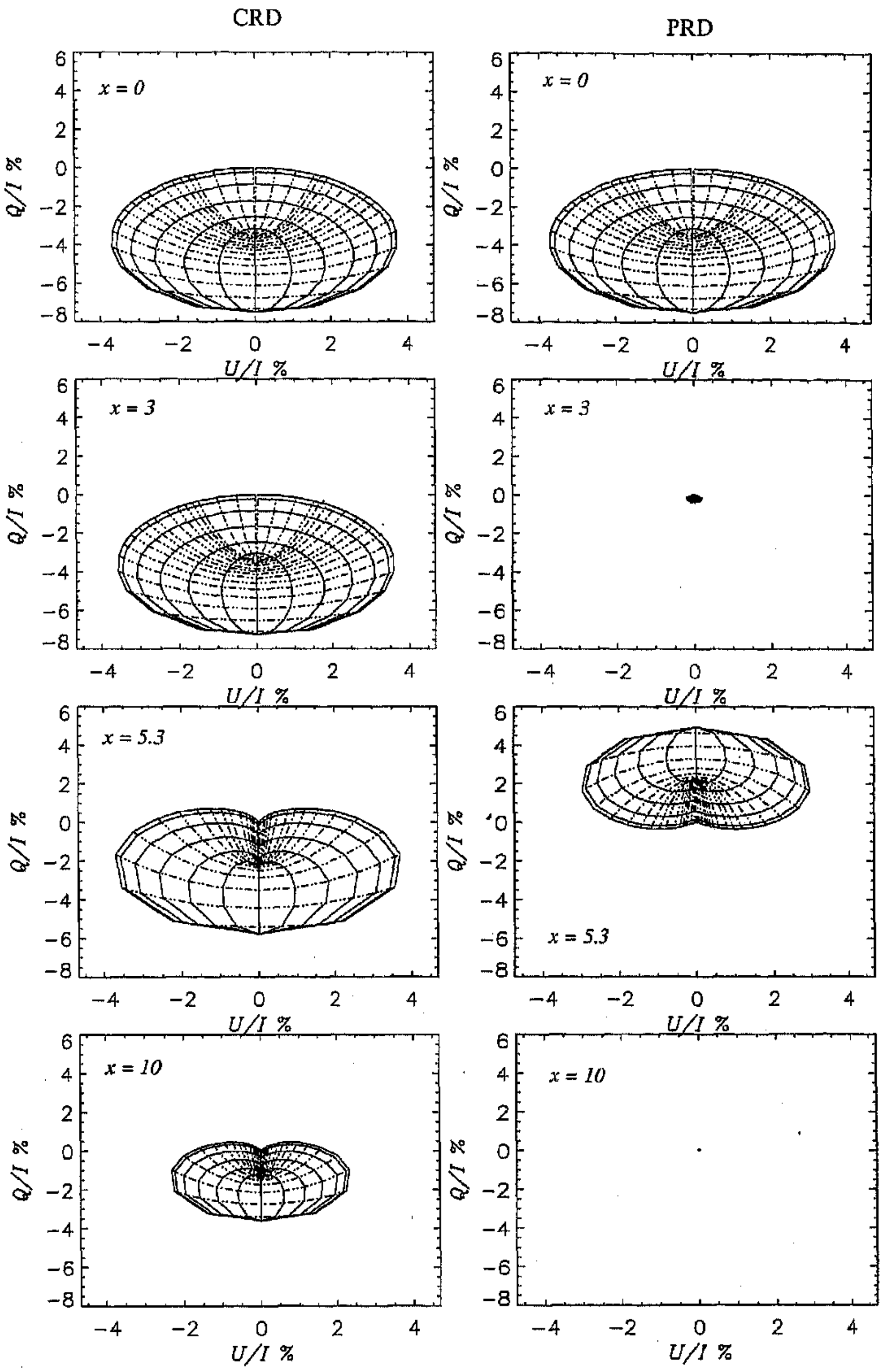

Figure 2. The two-parameter polarization diagrams for Hanle effect in an optically thin line. A comparison of CRD and PRD mechanisms is presented. The magnetic field is assumed to be horizontal ($\theta_B = 90°$). The field strength parameter γ_B and field azimuth φ_B are independent parameters. γ_B varies from 0-100 (dashed horizontal lines moving down to up); and φ_B varies from 0-180° (solid vertical lines moving from left to right). The *iso-azimuth curves* (solid lines) are drawn by fixing φ_B and varying γ_B . The *iso-strength curves* (dashed lines) are drawn by fixing γ_B and varying φ_B. The line of sight (LOS) is fixed at $(0, \varphi) = (90°, 0°)$. The results for four frequencies x= 0,3,5.3 and 10 are presented. The iso-strength curves show the *Hanle de-polarization* and *saturation effects* clearly. The iso-azimuth curves show the effect of *rotation of the plane ofpolarization* when γ_B varies. Iso-azimuth curves meet at the point $\gamma_B = 0$ where $(U/I = 0)$, and for $\varphi_B = (90°, 270°)$, they are parallel to the $(U/I) = 0$ axis. The Hanle de-polarizing ability is maximum for $\theta_B = 90°$.

in computing Fig. 15 of that paper, and the Fig. 2 in this paper. In Fig. 2, the two basic mechanisms of line scattering are considered: the *Partial Frequency Redistribution* (PRD) which is physically more realistic for resonance lines, compared to the *Complete Frequency Redistribution* (CRD). The CRD diagrams uniformly decrease in size (degree of linear polarization (Q/I, U/I)), and reach a constant level for frequencies $x > 10$. The PRD diagrams show a strong sign reversal at $x = 3$, where polarization is ≈ 0 (panel 2); become fully positive (panel 3); and reach zero polarization for large frequencies (panel 4). They follow the well known physical characteristics of CRD and PRD (see Fig. (1b) of Faurobert 1987).

3. Conclusions

The PALI is about 100 times faster than the conventional methods of solving the Hanle scattering problem. It is suitable for realistic modeling of the Hanle effect observations in spectral lines formed on the Sun. The Hanle effect is useful in exploring the spatially unresolved vector magnetic fields in a parameter space where the ordinary Zeeman effect is not practically useful.

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