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SW Lacertae—A quadruple system

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Abstract. Orbital period study using photoelectric times of minima of SW Lac reveals that the period change is caused by the orbital motion of a triple system around a fourth body. The orbital periods of the third and the fourth body are 19.67 yr and 70.25 yr respectively, and their masses are close to 1 $M_{\odot}$ each.

Key words : SW Lac — multiple star systems

1. Introduction

SW Lac is a W UMa-type eclipsing binary. The masses of the primary and secondary are 1.05 $M_{\odot}$ and 0.89 $M_{\odot}$ respectively (Giannone & Giannuzzi 1974). Their spectral type has been identified as G3p + G3p (Koch *et al.* 1963). Many detailed investigations have been made on the nature of its orbital period. Schilt (1923) analysed the available data including his own photographic minima. He expected that the observed change in the period would be easily explained on the basis of the motion of the eclipsing binary around a third body. Chou (1963) reserved his comment on the cause of the period change in SW Lac on account of meagre data available then. Lang & Vetesnik (1966) found that there was no 'monotonous' change in the period, only discontinuous changes. They explained this feature by means of 'sudden changes in the distribution of matter in the system', or by 'loss of matter out of the system'. In addition Faulkner (1978), Brownlee (1957), Kreiner & Frasinka (1977), Bookmyer (1965) and Semeniuk (1971) have made period studies of this system.

In order to understand clearly the actual nature of the variation of the orbital period of this system, we preferred only photoelectric times of minima to other types to avoid unnecessary scattering of points in the (O-C) diagram. The present analysis of SW Lac based on the available data in the literature neither supports mass exchange hypothesis nor does it confirm Woodward's (1956) suspicion that the changes of the period might be due to the so-called rotation of the apsidal line. But we modify Schilt's (1923) idea of a triple system into that of a quadruple system. In other words we believe that the observed changes in the period of SW Lac can be explained convincingly on the basis of light time effect of a triple system around a fourth body. Also, the masses of the third and fourth components turn out to be reasonable.

2. Period analysis

A plot of the time residuals of all the available photoelectric minima, listed in Table 1, using the light elements of Kreiner & Frasinka (1977) is shown in Figure 1. Retaining the initial epoch of the above authors we used a different period which is 0.89 s longer than the one given by them. Figure 2 corresponds to this modified ephemeris :

$$\text{Heliocentric minimum } I = JD\ 2442697.404 + 0^{d}.320724716E \quad \dots (1)$$

In Figure 2 it may be seen that the points follow a curve which is a combination of two sine curves of different frequencies. Physically this means that the eclipsing pair is moving around a third component while this triplet itself is in motion around a fourth body. Therefore SW Lac can be rightly called a quadruple system. We assumed a third body period of 19.67 yr (22400 binary cycles) and the fourth body period of 70.25 yr (80000 binary cycles); the latter is of course less certain than the former. The orbits of these bodies have been taken to be circular so that (O-C) can be represented by equation of the type

$$\text{(O-C)} = \frac{a \sin i}{c} \sin \frac{360}{P_n} (E + E_0) \quad \dots (2)$$

where P_n stands for the long term period.

First, a sine curve shown in solid line in Figure 2 was fitted by the method of least squares using points between -14000 and -23000 cycles, for the third body orbital period $P_1 = 19.67$ yr. It is represented along with standard errors, by

$$\text{(O-C)}_I = \underset{\pm 7}{0.0426} - \underset{\pm 11}{0.0202} \sin \frac{360}{22400} [E + \underset{\pm 1.2}{1749.3}]. \quad \dots (3)$$

The deviations of all the points from this sine curve C_1 were found and plotted in Figure 3(a). Secondly, another sine curve, represented by

$$\text{(O-C)}_{II} = \underset{\pm 8}{-0.0398} - \underset{\pm 45}{0.0409} \sin \frac{360}{80000} [E - \underset{\pm 3.3}{2414.4}] \quad \dots (4)$$

was fitted through the points in Figure 3(a) for the fourth body orbital period $P_2 =$ 70.25 yr. Therefore the theoretical representation of the points in the (O-C) diagram turns out to be the algebraic sum of equations (3) and (4) :

$$\text{(O-C)}_{III} = \underset{\pm 11}{0.0028} - \underset{\pm 11}{0.0202} \sin \frac{360}{22400} [E + \underset{\pm 1.6}{1749.3}] - \underset{\pm 45}{0.0409} \sin \frac{360}{80000} [E - \underset{\pm 3.3}{2414.4}] \quad \dots (5)$$

which is shown by a dashed line in Figure 2. The deviations of points Δ(O-C) = (O-C)$-$(O-C)$_{III}$, calculated from equation (5) are given in column 5 of Table 1 and

Table 1. Photoelectric times of minima of SW Lac

Sl. No.	J D⊙		E	(O-C)*	Δ(O-C)	Reference
1	243 3923. 5446	II	—27357	+0.04630	—0.00176	1
2	3928. 51748		—27341	+0.04794	—0.00010	1
3	3931. 40381		—27332	+0.04775	—0.00028	1
4	3993. 30416		—27139	+0.04823	+0.00048	1
5	4271. 37095		—26272	+0.04669	+0.00081	1
6	4600. 58995	II	—25246	+0.04177	—0.00087	1
7	4600. 59008	II	—25246	+0.04190	—0.00074	2
8	4637. 7950	II	—25130	+0.04275	+0.00053	3
9	4658. 8024		—25064	+0.04268	+0.00071	3
10	4660. 7272		—25058	+0.04313	+0.00118	3
11	4663. 7730	II	—25049	+0.04205	+0.00013	3
12	4664. 5766		—25046	+0.04384	+0.00193	3
13	4664. 7367	II	—25046	+0.04358	+0.00167	3
14	4665. 6976	II	—25043	+0.04230	+0.00040	3
15	4665. 8575		—25042	+0.04184	—0.00005	3
16	4666. 8216		—25039	+0.04376	+0.00188	3
17	4667. 7833		—25036	+0.04329	+0.00142	3
18	4668. 7451		—25033	+0.04292	+0.00106	3
19	4680. 6115		—24996	+0.04250	+0.00078	3
20	4681. 5737		—24993	+0.04253	+0.00082	3
21	4681. 7348	II	—24993	+0.04327	+0.00156	3
22	5036. 6130		—23886	+0.03957	+0.00227	3
23	5037. 5748		—23883	+0.03919	+0.00190	3
24	5037. 7350	II	—23883	+0.03903	+0.00174	3
25	5040. 6180	II	—23874	+0.03551	—0.00174	4
26	5040. 6182	II	—23874	+0.03571	—0.00154	4
27	5055. 5357		—23827	+0.03951	+0.00245	4
28	5055. 6940	II	—23827	+0.03745	+0.00039	4
29	5055. 6945	II	—23827	+0.03795	—0.00089	3
30	5057. 6190		—23821	+0.03810	+0.00107	3
31	5374. 4900	II	—22832	+0.03307	+0.00009	5
32	5379. 4638		—22817	+0.03564	+0.00272	5
33	5390. 3683		—22783	+0.03550	+0.00272	5
34	5390. 5266	II	—22783	+0.03344	+0.00066	5
35	6037. 5829		—20765	+0.02763	+0.00162	6
36	6045. 4384	II	—20741	+0.02537	—0.00058	6
37	6045. 43843	II	—20741	+0.02540	—0.00055	2
38	6045. 6005		—20740	+0.02711	+0.00117	6
39	6045. 60053		—20740	+0.02714	+0.00120	2
40	6046. 4020	II	—20738	+0.02680	+0.00086	6
41	6046. 56336		—20737	+0.02780	+0.00186	2
42	6046. 5634		—20737	+0.02784	+0.00190	6
43	6048. 4870		—20731	+0.02709	+0.00117	6
44	6048. 6470	II	—20731	+0.02673	+0.00081	6
45	6049. 4490		—20728	+0.02691	+0.00100	6
46	6049. 4493		—20728	+0.02721	+0.00130	6
47	6049. 6091	II	—20728	+0.02665	+0.00074	6
48	6049. 60913	II	—20728	+0.02668	+0.00077	2
49	6050. 41146		—20725	+0.02720	+0.00129	2
50	6050. 4115		—20725	+0.02724	+0.00133	6
51	6050. 57086	II	—20725	+0.02624	+0.00033	2

Continued

Table 1—*Continued*

Sl No.	J D⊙			E	(O-C)*	Δ(O-C)	Reference
52	243 6050.	5709	II	—20725	+0.02628	+0.00037	6
53	6461.	4147	II	—19444	+0.02172	—0.00198	7
54	6462.	3761	II	—19441	+0.02094	—0.00275	7
55	6463.	3391	II	—19438	+0.02177	—0.00192	7
56	6480.	3361	II	—19385	+0.02036	—0.00328	7
57	6843.	3967	II	—18253	+0.02058	—0.00293	7
58	6844.	3593	II	—18250	+0.02101	—0.00250	7
59	6844.	3590	II	—18250	+0.02071	—0.00280	7
60	6847.	4066		—18240	+0.02142	—0.00210	7
61	6848.	3695		—18237	+0.02215	—0.00137	7
62	7173.	7460	II	—17223	+0.02342	—0.00145	2
63	7191.	7065	II	—17167	+0.02334	—0.00164	2
64	7192.	8301		—17163	+0.02440	—0.00059	8
65	7194.	7552		—17157	+0.02515	—0.00015	8
66	7194.	9139	II	—17157	+0.02349	—0.00151	8
67	7201.	6485	II	—17136	+0.02287	—0.00217	8
68	7202.	7726		—17132	+0.02443	—0.00062	8
69	7220.	5720	II	—17077	+0.02361	—0.00156	2
70	7220.	7335		—17076	+0.02475	—0.00042	2
71	7225.	5445		—17061	+0.02488	—0.00032	2
72	7225.	7039	II	—17061	+0.02392	—0.00128	2
73	7233.	5629		—17036	+0.02516	—0.00010	2
74	7258.	5795		—16958	+0.02523	—0.00020	2
75	7262.	5873	II	—16946	+0.02398	—0.00148	2
76	7556.	6958	II	—16029	+0.02791	—0.00013	2
77	7572.	5719		—15979	+0.02814	—0.00006	2
78	7572.	7328	II	—15979	+0.02868	+0.00048	2
79	7573.	6952	II	—15976	+0.02890	+0.00069	2
80	7577.	7038		—15963	+0.02844	+0.00018	2
81	7578.	6660		—15960	+0.02847	+0.00020	2
82	7611.	7012		—15857	+0.02902	+0.00040	2
83	7614.	5879		—15848	+0.02920	+0.00055	2
84	7615.	5502		—15845	+0 02932	+0 00066	2
85	7619	5582	II	—15833	+0 02827	—0.00043	2
86	7845.	5115		—15128	+0.03100	—0.00035	4
87	7846.	4740		—15125	+0.03133	—0.00003	4
88	7846.	4760		—15125	+0.03333	+0.00197	4
89	7869.	4062	II	—15054	+0.03171	+0.00006	4
90	7871.	4904		—15047	+0.03120	—0.00048	4
91	7875.	3397		—15035	+0.03180	+0.00007	22
92	7875.	4994	II	—15035	+0.03114	—0.00059	4
93	7879.	5097		—15022	+0.03238	+0.00060	4
94	7876.	3014		—15032	+0.03133	—0.00041	22
95	7876.	4683	II	—15031	+0.03786	+0.00612	22
96	7877.	4234	II	—15028	+0.03079	—0.00097	22
97	7878.	3846	II	—15025	+0.02981	—0.00196	22
98	7879.	3476	II	—15022	+0.03064	—0.00114	22
99	7879.	6662	II	—15021	+0.02851	—0.00328	22
100	7881.	2744	II	—15016	+0.03309	+0.00128	22
101	7881.	4339		—15016	+0.03228	+0.00042	22
102	7883.	3590		—15010	+0.03298	+0.00115	22

Continued

Table 1—*Continued*

Sl. No.	J D⊙			E	(O-C)*	Δ(O-C)	Reference
103	243 7885.	4419	II	—15003	+0.03117	—0.00069	22
104	7886.	4045	II	—15001	+0.03160	—0.00027	4
105	7887.	3671	II	—14998	+0.03203	+0.00015	4
106	7887.	5276		—14997	+0.03217	+0.00029	4
107	7890.	4144		—14988	+0.03244	+0.00052	4
108	7897.	4698		—14966	+0.03190	—0.00011	4
109	7903.	4034	II	—14948	+0.03209	0	4
110	7916.	7153		—14906	+0.03392	+0.00166	9
111	7919.	7603		—14897	+0.03203	—0.00027	9
112	7926.	6559		—14875	+0.03205	—0.00034	9
113	7926.	8139		—14875	+0.02969	—0.00270	9
114	7929.	7008		—14866	+0.03007	—0.00236	9
115	7929.	8627		—14865	+0.03160	—0.00083	9
116	7940.	6019	II	—14832	+0.02653	—0.00604	9
117	7941.	4091		—14829	+0.03191	—0.00067	4
118	7959.	3705		—14773	+0.03273	—0.00009	4
119	7961.	2945		—14767	+0.03280	—0.00004	4
120	8235.	5180		—13912	+0.03625	—0.00037	10
121	8670.	4214		—12556	+0.03693	—0.00584	4
122	8670.	4252		—12556	+0.04073	—0.00204	4
123	8708.	2710		—12438	+0.04102	—0.00227	10
124	8709.	3930	II	—12435	+0.04048	—0.00282	10
125	9039.	42074	II	—11406	+0.04249	—0.00496	11
126	9040.	38393	II	—11403	+0.04351	—0.00395	11
127	9041.	34579	II	—11400	+0.04319	—0.00428	11
128	9041.	50748		—11399	+0.04452	—0.00295	11
129	9059.	4680		—11343	+0.04442	—0.00326	10
130	9393.	18563	II	—10303	+0.04802	—0.00282	11
131	9393.	34719		—10302	+0.04921	—0.00163	11
132	9443.	3810		—10146	+0.04997	—0.00124	10
133	244 0035.	4224**		—8300	+0.03354	...	12
134	0110.	4923		—8066	+0.05386	+0.00129	13
135	0128.	4540		—8010	+0.05497	+0.00246	12
136	0202.	2197		—7780	+0.05399	+0.00179	12
137	0373.	4866		—7246	+0.05389	+0.00277	12
138	0419.	3502		—7103	+0.05386	+0.00311	12
139	0441.	4806		—7034	+0.05425	+0.00370	13
140	0467.	4591		—6953	+0.05405	+0.00374	13
141	0497.	2869		—6860	+0.05445	+0.00442	12
142	0515.	2460		—6804	+0.05297	+0.00312	12
143	0542.	3457	II	—6720	+0.05143	+0.00187	14
144	0836.	4479	II	—5803	+0.04907	+0.00335	15
145	0842.	3820		—5784	+0.04976	+0.00413	15
146	0843.	5030	II	—5781	+0.04822	+0.00261	15
147	0848.	4758		—5765	+0.04979	+0.00426	15
148	1167.	4277	II	—4770	+0.04095	+0.00115	23
149	1172.	4009		—4755	+0.04292	+0.00321	23
150	1192.	7656	II	—4692	+0.04161	+0.00231	16
151	1192.	9264		—4691	+0.04204	+0.00275	16
152	1249.	3737		—4515	+0.04179	+0.00368	23
153	1583.	4060	II	—3473	+0.03929	+0.00886	24

Continued

Table 1—*Continued*

Sl No.	$JD\odot$		E	(O-C)*	Δ(O-C)	Reference
154	244 1598. 3161		—3427	+0.03570	+0.00563	24
155	1598. 3154		—3427	+0.03500	+0.00493	24
156	1683. 3055		—3162	+0.03305	+0.00511	24
157	1900. 4293		—2485	+0.02622	+0.00393	17
158	2361. 7776	II	—1046	+0.01201	+0.00219	25
159	2369. 7962	II	—1021	+0.01249	+0.00288	25
160	2630. 3750		—209	+0.00247	—0.00025	18
161	2723. 8271	II**	+82	—0.03668	...	25
162	2724. 7881	II**	+85	—0.03786	...	25
163	2738. 7390**		+129	—0.03848	...	25
164	2697. 4039		0	—0.00010	—0.00110	18
165	2768. 2821		+221	—0.00206	—0.00127	19
166	3013. 7899	II	+986	—0.00903	—0.00240	25
167	3049. 8707		+1099	—0.00976	—0.00231	25
168	3398. 4899		+2186	—0.01833	—0.00384	20
169	3411. 4804	II	+2226	—0.01718	—0.00246	21
170	3411. 6343		+2227	—0.02364	—0.00891	21
171	3459. 7476		+2377	—0.01905	—0.00348	21
172	3460. 5490	II	+2379	—0.01946	—0.00388	21
173	3487. 6504		+2464	—0.01930	—0.00326	21
174	3488. 6128		+2467	—0.01907	—0.00302	21
175	3802. 5987		+3446	—0.02267	—0.00210	26

Note : *Obtained using $JD\odot\ I = 2442697.404 + 0^{d}.320724716\ E$.
**not included in the diagram as well as in analysis.

References: 1. Kwee (1958), 2. Bookmyer (1965), 3. Brownlee (1957), 4. Lang & Vetesnik (1966), 5. Hinderer (1960), 6. Broglia (1962), 7. Widorn (1962), 8. Muthsam & Rakos (1974), 9. Chou (1963), 10. Pohl (1967), 11. Rucinski (1968), 12. Kizilirmak & Pohl (1970), 13. Semeniuk (1971), 14. Muthsam (1972), 15. Pohl & Kizilirmak (1971), 16. Meyer (1972), 17. Baldinelli (1973), 18. Kizilirmak & Pohl (1976), 19. Baldinelli & Ghedini (1976), 20. Kizilirmak *et al.* (1978), 21. Faulkner (1979), 22. Kalchaev *et al.* (1968), 23. Pohl & Kizilirmak (1972), 24. Pohl & Kizilirmak (1974), 25. Skillman (1977), 26. Faulkner & Booknyer (1980).

shown in Figure 3(b). Even a casual observation of Figure 3(b) will reveal that there is a very small fluctuation of points. This fluctuation is rather small when compared with other W UMa type binaries, say, 441 Bootis B.

3. Masses of third and fourth components

The mass functions for the triple and quadruple systems can be written respectively as

$$f(m)_3 = \frac{(a_{12} \sin i_1)^3}{P_1^2} = \frac{m_{12} \sin^3 i_1}{\alpha_1 (1 + \alpha_1)^2} \qquad \ldots (6)$$

and

$$f(m)_4 = \frac{(a_{123} \sin i_2)^3}{P_2^2} = \frac{m_{123} \sin^3 i_2}{\alpha_2(1 + \alpha_2)^2} \qquad \ldots (7)$$

where $\alpha_1 = \dfrac{m_{12}}{m_3}$, $\alpha_2 = \dfrac{m_{123}}{m_4}$,

m_{12} = total mass of the eclipsing binary,
m_3, m_4 = individual massəs of third and fourth components of the system,
m_{123} = total mass of the triple system,
i_1 = angle of inclination of the third body orbit,
i_2 = angle of inclination of the fourth body orbit,
a_{12} = semi major axis of the third body orbit,
a_{123} = semi major axis of the fourth body orbit.

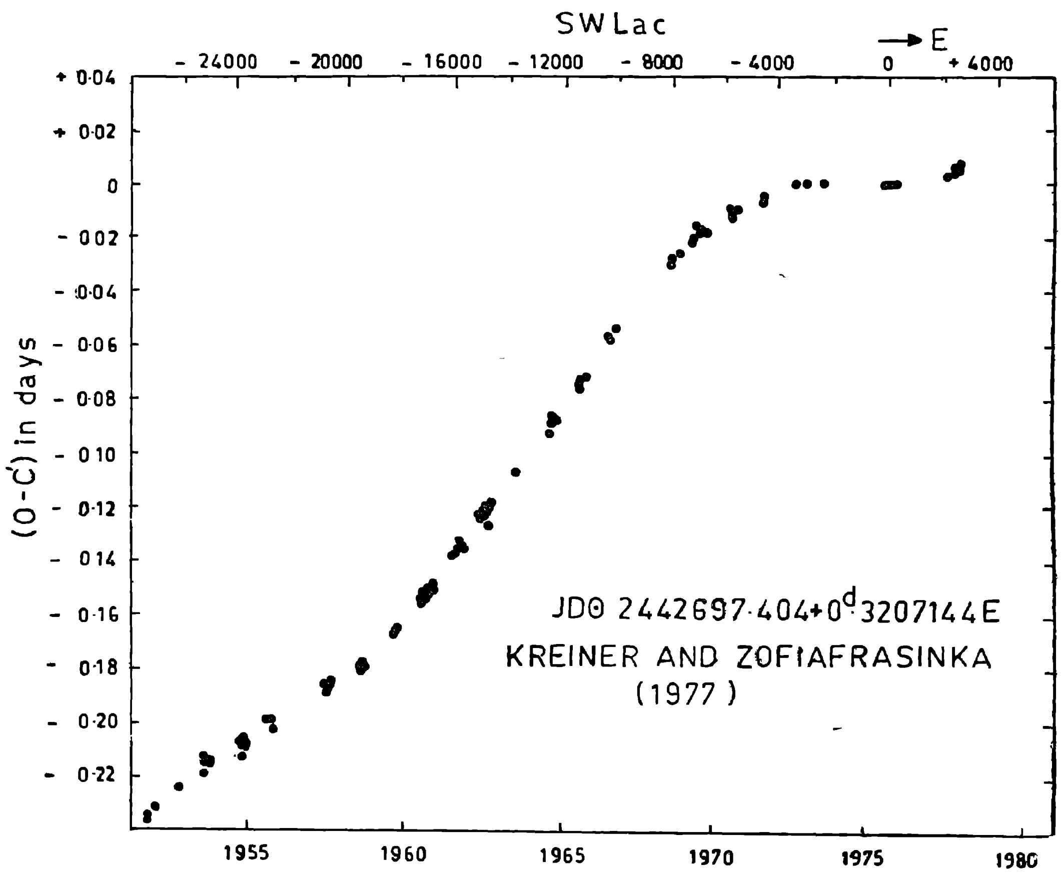

Figure 1. Time residuals (O-C′) versus the number of cycles elapsed *E*, based on the light elements given by Kreiner and Frasinka (1977).

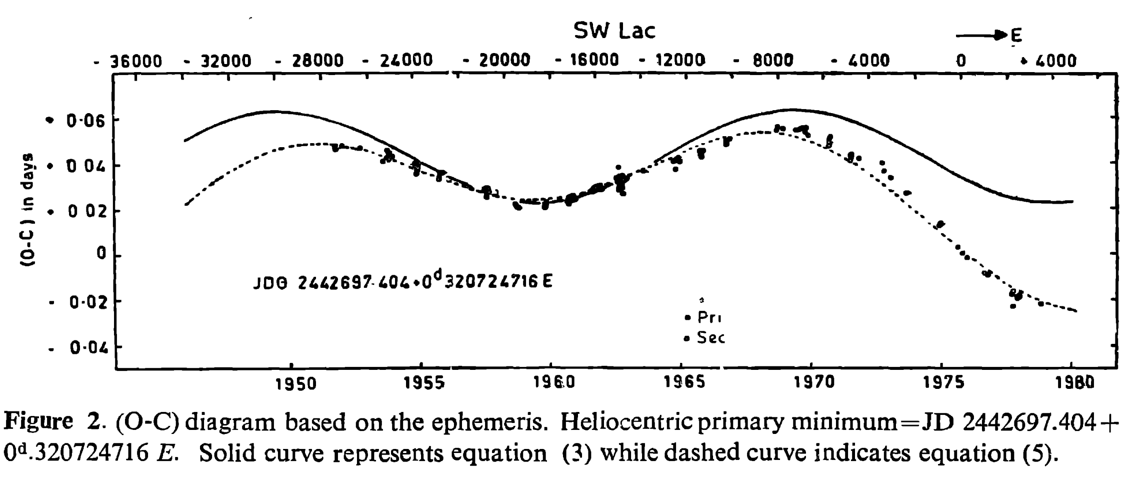

Figure 2. (O-C) diagram based on the ephemeris. Heliocentric primary minimum=JD 2442697.404+ $0^d.320724716$ *E*. Solid curve represents equation (3) while dashed curve indicates equation (5).

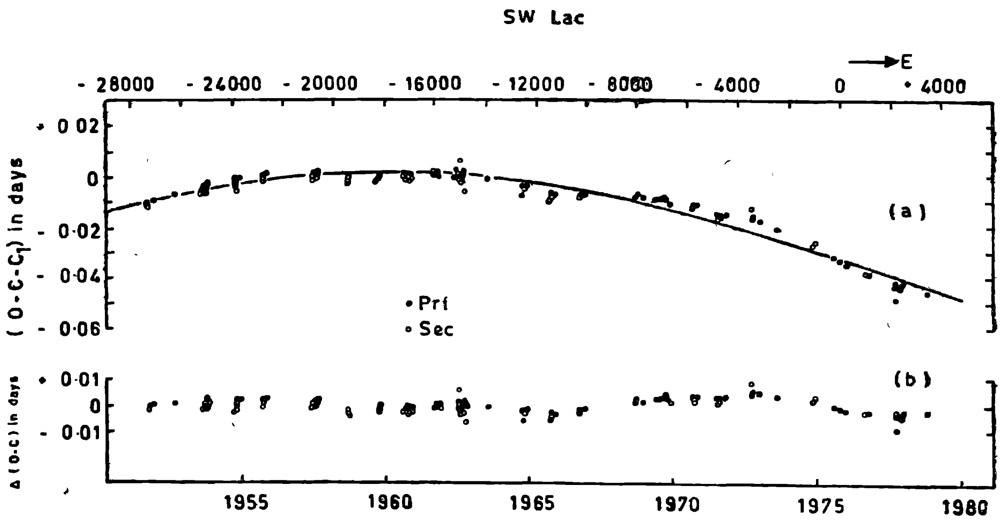

Figure 3. (a) Time residuals after the effect due to third body has been eliminated. Solid curve represents equation (4). (b) Time residuals after the effects due to third and fourth bodies have been eliminated.

The last two quantities multiplied by $(\sin i)/c$ are given by the amplitude coefficients of their respective Fourier terms. From equations (3) and (4) we obtain $a_{12} \sin i_1 = 3.51$ a.u. and $a_{123} \sin i_2 = 7.11$ a.u. Then equations (6) and (7) give

$$f(m)_3 = \frac{(3.51)^3}{(19.67)^2} = 0.112\, M_\odot$$

$$f(m)_4 = \frac{(7.11)^3}{(70.25)^2} = 0.073\, M_\odot$$

If we take the total mass of the eclipsing system to be 1.94 $M_\odot$ (Giannone & Giannuzzi 1974) the third mass m_3 turns out to be 0.99 $M_\odot$ for $i = 90°$ and 1.19 $M_\odot$ for $i = 60°$. In case we assume that the third mass is 1 $M_\odot$, the mass of the fourth body comes out to be 1.05 $M_\odot$ for $i_2 = 90°$, and $m_4 = 1.26\, M_\odot$ for $i_2 = 60°$. It may be observed that the masses of the third and fourth components are almost below the Chandrasekhar mass limit, for moderate values of the angle of inclination $90° \geqslant i_1\,(i_2) \geqslant 60°$. Hence they are likely to be white dwarfs with low luminosity which makes them undetectable by other means. A more reliable estimates for these quantities can be made if the entire cycle of the triple system around the fourth body is covered with accurate observations, and astrometric studies made to determine i_1 and i_2.

4. Conclusion

A convincing interpretation for the observed period change in SW Lac is that of the light-time effect of a triple system around a fourth body. The third and fourth bodies in SW Lac are of equal mass of about 1 $M_\odot$. The short-term period variation in this system is not so conspicuous as in other W UMa systems.

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