Cherenkov radiation by massless neutrinos

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Abstract

Due to their weak interactions, neutrinos can polarize a medium and acquire an induced charge. We consider the Cherenkov radiation emitted by neutrinos due to their effective electromagnetic interactions as they pass through a polarizable medium. The effect exists even for massless, chiral neutrinos, where no physics beyond the standard model needs to be assumed.

The study of the electromagnetic properties of neutrinos, in vacuum as well as in a medium, has been a subject of great interest over the years because of its intrinsic interest and also because of its potentially important consequences in a variety of physical, astrophysical and cosmological contexts. Recently, several authors have considered the Cherenkov radiation emitted by neutrinos as they pass through a medium [1], and also the transition radiation produced when they cross the interface between two media with different dielectric properties [2, 3]. In these works, the authors have assumed that the neutrino has an intrinsic magnetic and/or electric dipole moment (and hence also a mass), which are responsible for the electromagnetic interactions with the medium.

In a paper by two of the present authors [4], it was pointed out that neutrinos acquire an induced charge as they propagate through a medium as a consequence of their weak interactions with the background particles [5]. That observation was based on the 1-loop calculation of the effective electromagnetic vertex of the neutrino, which was performed using the methods of "Quantum Statistical Field Theory"[1] [6]. The effective charge was found to be nonvanishing even for a massless neutrino, where no physics outside the standard model needs to be assumed.

Here we point out that, because of the induced electromagnetic interactions of the neutrinos, they can emit Cherenkov radiation (and transition radiation in the case that they cross the interface between two media), even if they are massless and do not have intrinsic electromagnetic dipole moments. In this article we consider these effects, following a method similar to that of Ref. [4], and using the results of Ref. [6] as the basis of the calculations.

1 Kinematics

Our aim is to calculate the rate for the process

$$\nu(k) \to \nu(k') + \gamma(q)\,, \tag{1}$$

where, in the frame where the medium is at rest,

$$k^\lambda = (\omega, \vec{K}), \qquad k'^\lambda = (\omega', \vec{K}')\,, \tag{2}$$

denote the initial and final momenta of the neutrino, and

$$q \equiv k - k' = (\Omega, \vec{Q}) \tag{3}$$

[1] We prefer this name to the more often used "Finite Temperature Field Theory", since the methods apply also for zero temperature but finite density.

denotes the momentum of the emitted photon. Since we are interested in the contribution to the Cherenkov radiation due to the effective $\nu\nu\gamma$ interaction, for our purposes it is sufficient to consider the case in which the neutrinos are strictly massless in the vacuum. Thus, we assume that the on-shell conditions for the neutrinos is [7]

$$k^2 = 0, \qquad k'^2 = 0\,. \tag{4}$$

Using $k' = k - q$, the second equation can be rewritten in the form

$$\cos\theta \equiv \frac{\vec{K}\cdot\vec{Q}}{KQ} = \frac{2\omega\Omega - \Omega^2 + Q^2}{2KQ}\,. \tag{5}$$

Since $-1 \leq \cos\theta \leq 1$, this implies

$$\omega\Omega - KQ \leq \frac{1}{2}(\Omega^2 - Q^2) \leq \omega\Omega + KQ\,. \tag{6}$$

It is easy to see that these conditions cannot be satisfied for $Q < \Omega$. Thus, we must have

$$Q > \Omega \tag{7}$$

in which case the right inequality of Eq. (6) is automatically satisfied, while the left inequality implies the subsidiary condition

$$KQ - \omega\Omega > 0\,. \tag{8}$$

Since we are assuming the vacuum on-shell relation for the neutrino ($\omega = K$), the second condition is equivalent to the first one.

The condition in Eq. (7) shows that for the photon we cannot take the vacuum disperstion relation. Rather, it is important that we take into account that its energy-momentum relation in an isotropic medium is given by the solutions of

$$\frac{Q^2}{\Omega^2} = \varepsilon_t(\Omega, Q) \pm \varepsilon_p(\Omega, Q) \tag{9}$$

for the transverse modes, and

$$\varepsilon_l(\Omega, Q) = 0 \tag{10}$$

for the longitudinal one. The functions $\varepsilon_{t,l,p}$ are the components of the dielectric response function of the system, which are related to the components of the photon self-energy by [8]

$$1 - \varepsilon_t = \pi_T/\Omega^2\,, \qquad 1 - \varepsilon_l = \pi_L/q^2\,, \qquad \varepsilon_p = \pi_P/\Omega^2\,. \tag{11}$$

$\pi_{T,L,P}$ are defined by writing the photon self-energy in the form

$$\pi_{\mu\nu}(q) = \pi_T R_{\mu\nu} + \pi_L Q_{\mu\nu} + \pi_P P_{\mu\nu}\,. \tag{12}$$

with

$$R_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} - Q_{\mu\nu}\,, \tag{13}$$

$$Q_{\mu\nu} = -\frac{q^2}{Q^2}\left(v_\mu - \frac{\Omega q_\mu}{q^2}\right)\left(v_\nu - \frac{\Omega q_\nu}{q^2}\right)\,, \tag{14}$$

$$P_{\mu\nu} = \frac{i}{Q}\epsilon_{\mu\nu\alpha\beta}q^\alpha v^\beta\,, \tag{15}$$

where $v^\mu = (1, \vec{0})$ is the center-of-mass velocity of the medium. Several useful properties of the tensors R, Q and P are given explicitly in Ref. [8], to which we refer the reader. The solutions to Eqs. (9) and (10), which we denote by $\Omega_s(Q)$, give the energy-momentum relation for the three possible polarization modes of the photon. For non-chiral media π_P arises only through parity violation in weak interactions and hence must be small. Therefore, in what follows, we will neglect its effects, thereby assuming that the two transverse degrees of freedom of the photon are degenerate [9].

For future purposes, it is useful to recall that the Eqs. (9) and (10) for the dispersion relations are equivalent to

$$\Omega_{t,l}^2 - Q^2 = \pi_{T,L}\,. \tag{16}$$

In the literature, it is customary to use the indices of refraction, which is yet another way of experssing the dispersion relations. Introducing the functions

$$n_{T,L}(\Omega, Q) = \sqrt{1 - \frac{\pi_{T,L}}{\Omega^2}}\,, \tag{17}$$

solving Eq. (16) is then equivalent to solve

$$n_{T,L} = Q/\Omega_{t,l}\,. \tag{18}$$

The indices of refraction are defined by

$$\begin{aligned} n_{t,l} &\equiv Q/\Omega_{t,l}(Q) \\ &= n_{T,L}(\Omega_{t,l}, Q)\,, \end{aligned} \tag{19}$$

so that the condition of Eq. (7) is expressed as $n_{t,l} > 1$. Eq. (9) implies the familiar relation $n_t = \sqrt{\varepsilon_t(\Omega_t(Q), Q)}$.

2 The $\nu\nu\gamma$ vertex

In the following we follow closely the arguments and results given in Ref. [4]. We define the $\nu\nu\gamma$ amplitude by

$$M = -i\sqrt{N_s}\epsilon_\mu^{(s)*}(q)\overline{u}(k')\Gamma^\mu u(k)\,, \tag{20}$$

where $\epsilon_\mu^{(s)}(q)$ is the polarization vector of the emitted photon and the index s indicates its polarization, with $s = 1, 2$ for the two (degenerate) transverse modes and $s = 3$ for the longitudinal one. The factor $\sqrt{N_s}$ is necessary because the normalization of the photon wavefunction in the medium is not the same as in the vacuum[10]. Assuming that the polarization vectors are normalized such that

$$\sum_{s=1,2} \epsilon_\mu^{(s)}\epsilon_\nu^{(s)} = -R_{\mu\nu} \tag{21}$$

and[2]

$$\epsilon_\mu^{(3)}\epsilon_\nu^{(3)} = Q_{\mu\nu}\,, \tag{22}$$

where $R_{\mu\nu}$ and $Q_{\mu\nu}$ have been defined in Eqs. (13) and (14), one obtains [8, 11]

$$N_{t,l} = \left(\frac{2\Omega}{\frac{\partial}{\partial\Omega}(\Omega^2 - \pi_{T,L})}\right)\Bigg|_{\Omega=\Omega_{t,l}}\,, \tag{23}$$

which, in terms of the index of refraction $n_{T,L}$, reads

$$N_{t,l} = \frac{n_{t,l}^{-1}}{n_{t,l} + \Omega_{t,l}\left(\frac{\partial n_{T,L}}{\partial\Omega}\right)\Big|_{\Omega=\Omega_{t,l}}}\,, \tag{24}$$

[2] In Ref. [8], for example, the right hand side of Eq. (22) has an extra minus sign, which is the correct relation for timelike photons. The present form is appropriate for spacelike ones.

In Ref. [4], it has been shown that the vertex function Γ_μ is given to leading order in the Fermi constant by

$$\Gamma_\mu = -\frac{\sqrt{2}G_F}{e}\gamma^\rho L(\mathcal{A}\pi_{\mu\rho} + \mathcal{B}\pi^5_{\mu\rho}) \,, \tag{25}$$

where $L = \frac{1}{2}(1-\gamma_5)$ is the projection operator for left chirality, $\pi_{\mu\nu}$ is the photon self-energy and $\pi^5_{\mu\nu}$ is a similarly defined function. As shown in Ref. [4], Eq. (25) is valid in all orders of the electromagnetic interactions, where the most general form for $\pi_{\mu\nu}$ is given in Eq. (12) while for $\pi^5_{\mu\nu}$ it is

$$\pi^5_{\mu\rho} = \pi^5 P_{\mu\rho} \,. \tag{26}$$

Finally, the constants $\mathcal{A}$ and $\mathcal{B}$ appearing in Eq. (25) are defined by writing the four-fermion interaction between the neutrino and the electron in the form

$$\mathcal{L}^{(\rm weak)}_{\rm int} = -\sqrt{2}G_F\;[\bar\nu\gamma^\rho L\nu]\;[\bar f\gamma_\rho(\mathcal{A}+\mathcal{B}\gamma_5)f] \,, \tag{27}$$

where f stands for the electron field. In the standard model of electroweak interactions,

$$\mathcal{A} = \begin{cases} 2\sin^2\theta_W + \frac{1}{2} & \text{for } \nu_e \\ 2\sin^2\theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \,. \end{cases} \tag{28}$$

$$\mathcal{B} = \begin{cases} -\frac{1}{2} & \text{for } \nu_e \\ +\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \,. \end{cases} \tag{29}$$

3 Calculation of the rate

The emission rate for transverse or longitudinal photons in a momentum range from Q_1 to Q_2 is given by

$$R_{t,l} = \frac{1}{16\pi\omega^2}\int_{Q_1}^{Q_2} dQ\,\frac{Q}{\Omega_{t,l}(Q)}\left(\sum_s |M|^2\right) \tag{30}$$

$$= \frac{1}{16\pi\omega}\int_{\xi_1}^{\xi_2} d\xi\; n_{t,l}\left(\sum_s |M|^2\right) \,, \tag{31}$$

where in the last step, we have defined a new dimensionless variable

$$\xi \equiv \frac{Q}{\omega} \,. \tag{32}$$

The limits of the integral in Eq. (31) are determined either by our range of interest (e.g., we may be interested in just the optical range), or by the range for which propagating photon modes exist. If, however, propagating modes are available for the entire range of momenta allowed by kinematics, the total rate will be given by

$$R_{t,l}^{(\mathrm{tot})} = \frac{1}{16\pi\omega} \int_0^{\xi_{\max}} d\xi \; n_{t,l} \left(\sum_s |M|^2 \right) . \tag{33}$$

Here, the upper limit of the integral should be determined from the left inequality in Eq. (6), and is given by

$$\xi_{\max} = \frac{2n_{t,l}}{n_{t,l} + 1} . \tag{34}$$

In general $n_{t,l}$ is a function of Q and therefore of ξ, so that Eq. (34) becomes an implicit equation from which $\xi_{\max}$ is determined.

Whether we are interested in the total rate or the rate in any particular range of momenta, the result will be different for transverse and longitudinal photons since they have different dispersion relations. Moreover, the polarization sums in $\sum |M|^2$ will also involve different polarization states. Thus, we carry out the calculation of the rates for transverse and longitudinal photons separately.

3.1 Transverse photons

Using the relation in Eq. (21) for the polarization sum, it then follows that

$$\begin{aligned} \sum_{s=1,2} |M|^2 \;\; &= \;\; \frac{G_F^2}{4\pi\alpha}\, \omega^2 N_t \left(1 - \frac{1}{n_t^2}\right) \left\{ (\mathcal{A}^2|\pi_T|^2 + \mathcal{B}^2|\pi^5|^2) \left[\left(2 - \frac{\xi}{n_t}\right)^2 + \xi^2 \right] \right. \\ & \left. \qquad - \, 4\mathcal{A}\mathcal{B}\mathrm{Re}\, \pi_T^* \pi^5 \left(2\xi - \frac{\xi^2}{n_t}\right) \right\} , \end{aligned} \tag{35}$$

where we have neglected the contribution from π_P, as already stated. It must be remembered that in Eq. (35) the functions π_T and π^5 are evaluated at the correct photon dispersion relation, e.g., $\pi_T = \pi_T(\Omega_t(Q), Q)$. In order to carry out the integral indicated in Eq. (33) we need the explicit expressions for the functions π_T and π^5. We now argue as follows. For an electron gas, the

1-loop formulas for these two functions can be inferred from the calculations of Ref. [6]. In particular, it follows that $\pi^5(0,0) = 0$, and motivated by this we neglect the contribution from this function in Eq. (35). Then we use

$$\pi_T = -\xi^2\omega^2\left(1 - \frac{1}{n_t^2}\right), \tag{36}$$

which follows from Eqs. (16) and (19), to obtain finally

$$R_t = \frac{G_F^2\mathcal{A}^2}{64\pi^2\alpha}\omega^5 F_t\,, \tag{37}$$

where

$$F_t \equiv \int_{\xi_1}^{\xi_2} d\xi\; \xi^4 n_t N_t \left(1 - \frac{1}{n_t^2}\right)^3 \left[\left(2 - \frac{\xi}{n_t}\right)^2 + \xi^2\right]. \tag{38}$$

Eq. (37) is useful for making numerical estimates since all the unknown aspects are contained in the single function F_t. The evaluation of this function is involved because, as mentioned above, the refractive index is a function of Q, and therefore of ξ. However, to obtain a rough estimate of the rates involved, we can pretend that n_t is a constant over the range of integration. In this case we can perform the integral in Eq. (38) for any given value of n_t.

As an illustration, let us consider the case of non-magnetic matter, i.e., materials for which the magnetic permeability $\mu = 1$, or equivalently $\varepsilon_t = \varepsilon_l \equiv \varepsilon$. For large frequencies [12],

$$\varepsilon(\Omega) = \varepsilon_\infty - \frac{\Omega_p^2}{\Omega^2}\,, \tag{39}$$

where the asymptotic value ε_∞ and the plasma frequency Ω_p can be expressed in terms of the imaginary part of the dielectric function $\mathrm{Im}\,\varepsilon$. An important point to notice is that the condition $\mathrm{Im}\,\varepsilon > 0$ for $\Omega > 0$, which follows from fundamental physical requirements [13], implies that $\varepsilon_\infty > 1$ and, therefore, at high frequencies, $n_t \approx \sqrt{\varepsilon_\infty} > 1$. Thus, for example, if we assume that n_t is constant at the value $\sqrt{\varepsilon_\infty}$ within the range of integration for which $\xi_2 = 1$ and $\xi_1 \ll \xi_2$, Eq. (38) gives $F_t = (1 - n_t^{-2})^3 \times \left(\frac{33}{35 n_t} - \frac{2}{3n_t^2} + \frac{1}{7n_t^3}\right)$. For other values of ξ_2 the result can be read from Fig. 1. As can be seen from that figure, F_t becomes negligibly small for any value of n_t if ξ_2 is less than about 0.4. The function increases rapidly as the value of ξ_2 increases

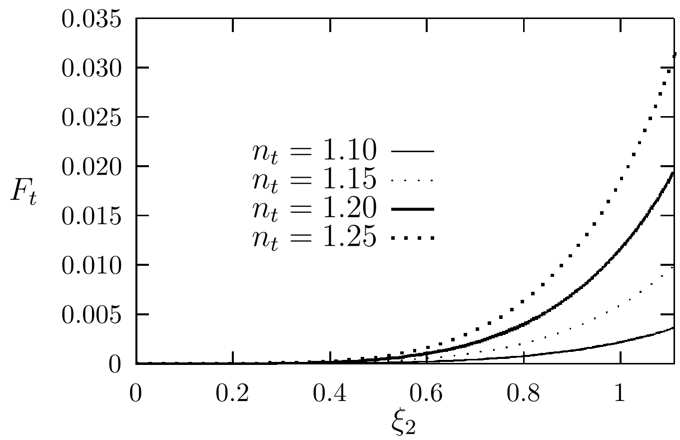

Figure 1: Plot of F_t vs. ξ_2 for various values of n_t, assuming a constant index of refraction and $\xi_1 = 0$. Although we have plotted the function over a common range of values $0 \leq \xi_2 \leq 1.11$, it should be noted that, for a given value of n_t, the allowed range of values of ξ_2 is limited by Eq. (34).

and, for values of ξ_2 around 1 (which implies photon energies of the order of the incident neutrino energy), the function increases rapidly as the index of refraction increases.

3.2 Longitudinal photons

For longitudinal photons, the formulas are analogous to those for transverse ones, with some obvious substitutions like replacing n_t by n_l. There is no polarization sum now, and we use Eq. (22). Then, using instead of Eq. (36) the relation appropriate for the longitudinal photons,

$$\pi_L = -\xi^2\omega^2\left(1 - \frac{1}{n_l^2}\right), \tag{40}$$

we obtain a formula for the rate which is similar to that in Eq. (37), but with F_t replaced by

$$F_l \equiv \int_{\xi_1}^{\xi_2} d\xi\; \xi^4 n_l N_l \left(1 - \frac{1}{n_l^2}\right)^3 \left[\left(2 - \frac{\xi}{n_l}\right)^2 - \xi^2\right]. \tag{41}$$

While the expressions for the longitudinal and transverse photons look similar, the longitudinal photons behave very different from the transverse ones. The dependence of the frequency on the wave vector is given by Eq. (10). It turns out that for values of the momentum of the order, or larger than, the inverse Debye screening length λ_D, the real and imaginary parts of Ω_l are comparable[14]. Thus, above those photon momenta, the longitudinal photon modes do not exist. Since λ_D is of the order of the Bohr radius, then for neutrino energies of the order of an MeV we actually have, for longitudinal photons, $\xi_{\max} \sim 10^{-4}$. On the scale of Fig. 1, this gives a negligible value of F_l, of order $\xi_{\max}^5$.

4 Numerical estimates

We now estimate the number of Cherenkov photons that will result from the formulas above. For a flux I of neutrinos, the number of Cherenkov events occurring in a time T in a detector of volume V is given by

$$\mathcal{N} = \frac{VTIR}{v}, \tag{42}$$

where R is the rate calculated in the last section and $v = c$ is the velocity of the neutrino. This gives

$$\mathcal{N}_{t,l} = 7.6 \times 10^9 \mathcal{A}^2 F_{t,l} \times \left(\frac{\omega}{1\,\mathrm{MeV}}\right)^5 \left(\frac{V}{1\,\mathrm{m}^3}\right) \left(\frac{T}{1\,\mathrm{day}}\right) \left(\frac{I}{I_\odot}\right), \tag{43}$$

where $I_\odot$ is the solar neutrino flux, $6 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$. While it may seem from Eq. (43) that the number of Cherenkov events increase simply as ω^5, this is not really true since $F_{t,l}$ are also functions of ω implicitly through ξ.

For optical photons, the formula above predicts a very small rate since in this range, $Q_2 \approx 3\,\mathrm{eV}$. Thus, for example, for a neutrino energy $\omega \approx 1\,\mathrm{MeV}$, we have $\xi_2 \approx 3 \times 10^{-6}$ so that ξ remains very small over the range of integration. Assuming that the index of refraction is of order one, $F_t \sim \xi_2^5 \sim 10^{-28}$. In general, if the upper range of Q is much smaller than ω, then $F_{t,l} \sim (Q_2/\omega)^5$, so that the rate in Eq. (43) scales as $(Q_2/1\,\mathrm{MeV})^5$.

Thus, for optical photons, the effects considered by Grimus and Neufeld [1] seem to be much larger, at least if the neutrino has a magnetic moment anywhere near the present experimental limit. However, this should not be discouraging because there are two important points to be remembered.

First, the effect considered in Ref. [1] hinges on the assumption that the neutrinos have an intrinsic magnetic moment of order $10^{-10}\mu_B$. On the contrary, the effect we have discussed in the present paper does not depend on any assumption about the neutrino properties and/or interactions beyond those specified by the Standard Model.

Secondly, the rate calculated by us for transverse photons has a very different dependence on neutrino energy than the rate calculated by Grimus and Neufeld [1]. In fact, using Eq. (18) of their paper, we can easily deduce the ratio of the two effects assuming, for illustration, that the refractive index in roughly constant and that $\xi_2 \gg \xi_1$:

$$\frac{\mathcal{N}_t}{\mathcal{N}_{\rm mag}} = 1.2 \left(\frac{n_t}{n_t^2 - 1} \right)^2 \mathcal{A}^2 F_t \left(\frac{\omega}{1\,{\rm MeV}} \right)^2 \left(\frac{\mu}{10^{-10}\mu_B} \right)^{-2} . \tag{44}$$

If we do not restrict ourselves to look only for photons in the optical range, but consider instead photons with energies that span all the kinematically allowed range, then it is easily seen that even at neutrino energies around 1 MeV, the effect we described in this paper is as important as the magnetic moment effects even if the magnetic moment is close to its present upper limit. If the magnetic moment is much lower, as indicated by considerations on the neutrino flux from the supernova SN1987A [15], then of course the effect we described is much stronger. In any case, as we emphasized before, the present effect is predicted from the physics of the Standard Model only, and therefore must be there. In this sense, the results of our calculations represent a firm prediction if an experiment can be set up. In fact, any deviation from this prediction can be taken as a serious indication that some of the neutrino properties and/or interactions are not the ones given by the Standard Model.

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