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Is the Hubble flow a result of inverse cascade?*

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A few general characteristics of nonlinear open systems are described. A turbulent fluid is one such system which exhibits order by supporting structures in an otherwise random medium through the transfer of energy from small spatial scales to large spatial scales. The spatial distribution of energy so derived is found to account well for two disparate situations like the solar granulation and the rotation curves of galaxies. Encouraged by these successes, one wonders if the spatial distribution of energy at the largest scales, i.e. $V(L) \propto L$ has anything to do with the Hubble flow.

COHERENT structures, correlated motions and well-defined patterns are observed on a variety of spatial as well as temporal scales. Organized states of matter and motion can be seen in a convection cell, cloud complexes, a tornado, a cyclone, zonal flows on planetary surfaces, the Red Spot of Jupiter, convective flows on stellar surfaces, spiral patterns in galaxies, clusters of galaxies and perhaps ourselves. Figure 1 *a–c* represents distribution of clouds in the earth's atmosphere, of convective motions on the solar surface and of galaxies in clusters of galaxies. Could you tell one from the other? Figure 2 *a, b* represents the velocity vectors in a cluster of galaxies and on the solar photosphere. Both show converging and diverging flows. The usual interpretation for clusters of galaxies is infall of matter in the strong gravitational field of the unseen dark matter, whereas the solar photosphere acquires the same pattern due to the formation of fluid vortices. Can the vortices account for the flow patterns in clusters of galaxies? Is the invisible matter indispensable? In other words, do these organized states of matter and motion arise under equilibrium or non-equilibrium conditions? Is it substance and or style? Are these dissipative structures?

Our proposal[1–6] is that apparently disparate phenomena of (i) non-equilibrium motions on stellar surfaces, (ii) the large scale organization of matter, motion and magnetic field or in general the large scale structure of the universe have their origin in the inverse cascade of energy leading to self-organization in an otherwise nonlinear turbulent medium.

Novelties of non-equilibrium systems

Near equilibrium, a system, when perturbed, comes back

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to its initial state and is said to be stable. Under far-from equilibrium conditions, a system, when driven sufficiently far, may never return to its initial state and is said to be unstable. The system undergoes a bifurcation, where the initial solution becomes unstable and the new state behaves in a manner completely different from that of the initial state. The new state that emerges after a bifurcation is known as a dissipative structure. A spectacular example of this phenomenon is provided by chemical clocks. A bifurcation is associated with the breaking of a symmetry. One starts with a system describable by completely symmetric equations and finds their solutions which are not symmetric, i.e. a bifurcation splits into a so-called right handed and a left handed solution. The physical reality may be embedded in only one of these solutions. For example, sea shells usually have a preferential chirality. It is seen that in a highly unstable dynamical system, the trajectories starting from infinitesimally close points diverge exponentially in time and such a system is said to be in a chaotic state. Thus a completely deterministic description supports a chaotic solution, which lends it a probabilistic behaviour, i.e. the system is intrinsically random and irreversible. For example, a collection of particles moving parallel to each other, develops randomness with the passage of time, i.e. they try to reach equilibrium. Now time inversion symmetry requires that the reverse must happen, i.e. randomly directed velocity vectors must align themselves with the passage of time. But this is never observed (Figure 3). This means that we need representations of dynamics which are not invariant with respect to time inversion and such representations do exist for highly unstable systems. This reduces the predictability of a system. Tomorrow is not already present in today! Instead, it emerges in an unpredictable way, the way improvization is achieved in Indian music.

a

b

c

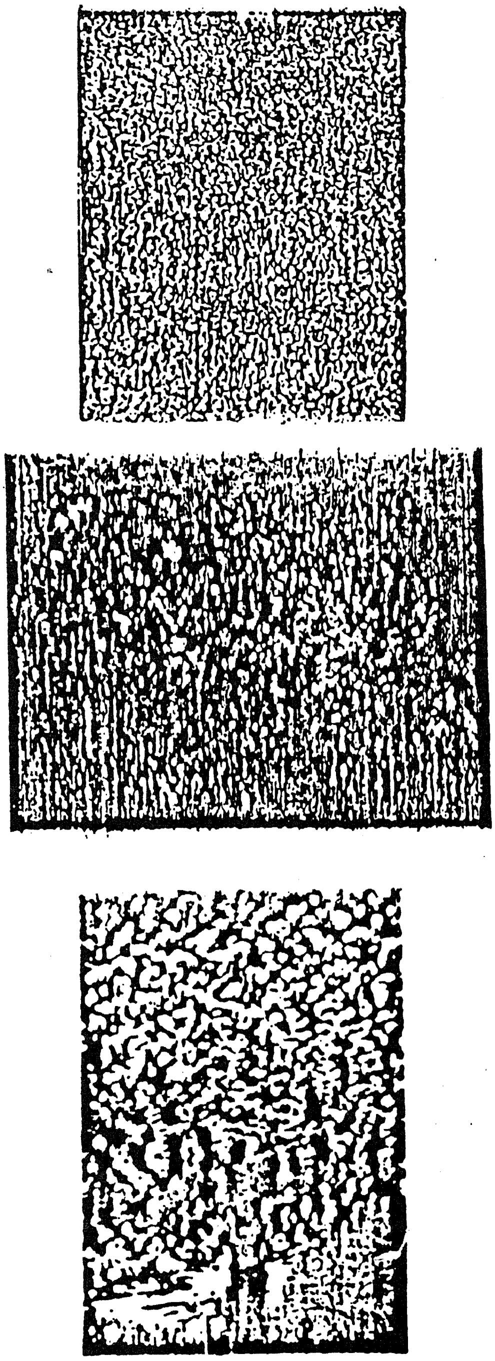

Figure 1 *a–c*. Distribution of clouds in the earth's atmosphere of convective motions on the solar surface and of galaxies in clusters of galaxies.

a

b

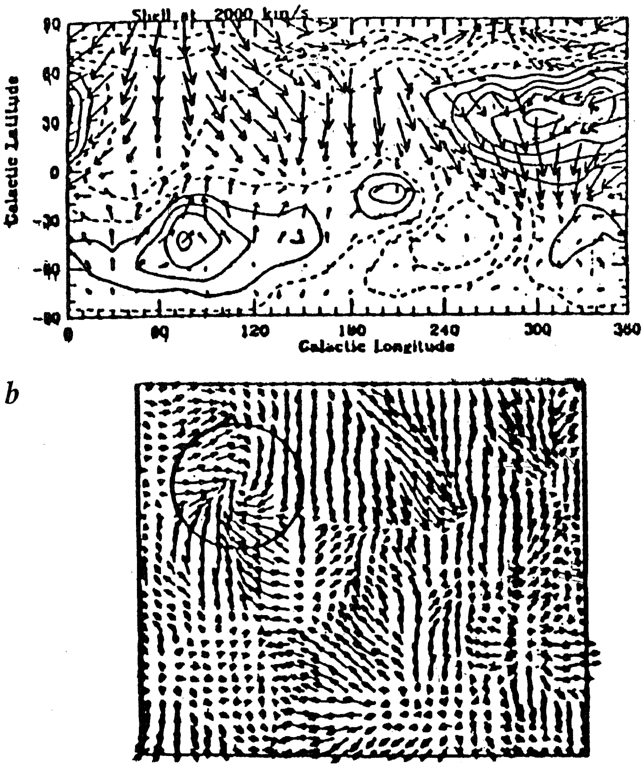

Figure 2. Velocity vectors in a cluster of galaxies and on the solar surface.

It is in intrinsically random, irreversible, unsymmetric, non-equilibrium systems that all phenomena of material existence are embedded. The conditions for formation of dissipative structures can therefore be summarized as[7,8]: (i) a macroscopic system far from equilibrium, so that it can develop correlations among its different parts, (ii) the system is open towards its environment, i.e. it can draw from its surroundings, and (iii) it should be able to self-reinforce itself autocatalytically by a sustained level of fluctuations deriving from its neighbourhood.

Fluid turbulence

Far from equilibrium conditions exist everywhere in the universe, including its origin, the big bang. Many structures like galaxies and clusters of galaxies are suggested to form and exist in non-equilibrium states. The belief in equilibrium has led us to darkness, since, equilibrium commands that most of the matter in the universe remain invisible. To investigate the role of non-equilibrium nonlinear processes in astrophysical objects, we need to go to the fluid description since several aspects of stellar, galactic and the universe as a whole can be understood by treating them as fluids, always in a state of turbulence. The fluid turbulence is a tricky subject. Even Heisenberg found the problem too difficult. He is reputed to have said that he hoped, before he dies, some one would explain quantum mechanics to him, but after he died, he hoped, God would explain turbulence to him[9]. Nevertheless, a few characteristics of fluid turbulence can be summarized here. It is a random state of fluid motion that supports several interacting length and time scales through the excitation of instabilities. The scales are constrained by boundaries, buoyancy and dissipation. Turbulence affects all transport and diffusion processes, generally leading to an enhanced efficiency.

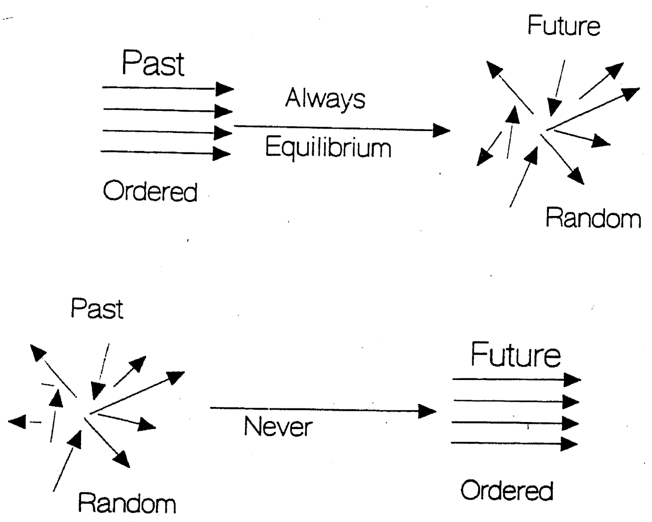

Figure 3. Irreversible processes.

Quantification of turbulence

Mean values of the products of field variables (like velocity U, magnetic field, etc.) and their derivatives form the fabric of a turbulent medium. Out of these, the two-point correlation function $R_{ij}(\mathbf{r})$ defined as

$$R_{ij}(\mathbf{r}) = \langle U_i(\mathbf{X})\, U_j(\mathbf{X}+\mathbf{r}) \rangle \qquad (1)$$

is the most important. Here, the angular brackets represent the space average. For homogeneous and stationary turbulence $R_{ij}(\mathbf{r})$ depends only on the configuration and not on its location. The Fourier transform $\phi_{ij}(\mathbf{K})$ of $R_{ij}(\mathbf{r})$ is defined as:

$$\phi_{ij}(\mathbf{K}) = \frac{1}{(2\pi)^3} \int R_{ij}(\mathbf{r})\, e^{-i\,\mathbf{kr}}\, d^3r\,. \qquad (2)$$

The total kinetic energy W per unit mass of the fluid is given by

$$W = \frac{1}{2} \langle U_i(X) U_i(X) \rangle = \frac{1}{2} \int \phi_{ii}(\mathbf{K})\, d^3K = \int E(K)\, dK\,, \qquad (3)$$

where $E(K)$ is the omnidirectional energy spectrum. From the Kolmogorov law, that in a quasi-steady state, there should be a stationary flow of energy in K space from the source to the sink, i.e. the energy density flow rate should be a constant and equal to the dissipation rate E of the energy density at the sink, one finds the energy spectrum

$$E(K) \sim \varepsilon^{2/3} K^{-5/3} \qquad (4)$$

describing the flow of energy from large spatial scales to small spatial scales. But what if the assumptions of homogeneity and isotropy are dropped?

Inverse cascade in 2D turbulence

In a 2D incompressible and ideal system, there are two invariants, the total energy W and the enstrophy U_E defined as:

$$U_E = \int \frac{(\nabla \times \mathbf{U})^2}{2}\, d^3r\,. \qquad (5)$$

Therefore one derives two types of inertial ranges, one for the energy and the other for the enstrophy. The two energy spectra are:

$$E(K) \sim \varepsilon'^{2/3} K^{-3} \quad \text{for enstrophy} \qquad (6)$$

and $E(K) \sim \varepsilon^{2/3} K^{-5/3}$ for energy and it has been argued and demonstrated that energy cascades from small spatial scales to large spatial scales according to $K^{-5/3}$ law, thus setting up an inverse cascade[10].

Inverse cascade in 3D turbulence

The appearance of large scale structures in the 3D atmospheres of planets compels us to look for the possibility of inverse cascade in 3D systems. Specifically, the observations of helical type of flow structures in circumstances varying from oceans to cloud complexes, brought to the fore, the importance of helicity in turbulent fluids. The fundamental idea that needed to be appreciated was that the large helicity fluctuations always exist in a turbulent medium, even if the average helicity vanishes. It was shown that the fluctuating topology of the vorticity field in turbulent flows can be characterized by the statistical helicity invariant I represented by conserved mean square helicity[11–13]:

$$I = \int \langle h(\mathbf{X})\, h(\mathbf{X}+\mathbf{r}) \rangle \, d^3 r \,, \tag{7}$$

where

$$h = \mathbf{U} \cdot \nabla \times \mathbf{U} \,.$$

Thus E and I are the two invariants of a helically turbulent 3D system. Again using Kolmogorovic arguments for the I invariant, one finds the spectrum in the inertial range as

$$E(K) \propto K^{-1}$$

and the total energy density

$$W = \int E(K)\, dK \propto \log\,[L(t)/l] \,. \tag{8}$$

Here $L(t)$ is the transient length scale excited at time t. One observes that the energy grows very slowly as the spatial scale $L(t)$ grows. So, practically, there is little transfer of energy towards large scales. But, what happens is that as the correlation length of helicity fluctuations increases, the velocity and vorticity become more and more aligned and as a consequence the nonlinear term $(V \cdot \nabla)V$ of the Navier–Stokes equation decreases and flow of energy towards small scales is retarded. On the other hand, the growth of correlation length cannot go on indefinitely. Especially if the medium is restricted in the vertical direction by gravity or buoyancy as is true of atmospheres of any celestial object, may it be a planet or a star. Under such circumstances, the correlation length continues to grow in the horizontal plane and the system becomes more and more anisotropic. In addition, $I(K)$ is found to be dominant at small K while $E(K)$ is larger at large K and the inverse cascade of I follows.

What is achieved by the growth of correlation length of helicity fluctuations is the anisotropy in the system which can now be approximated to a quasi 2D system. Here, the horizontal scale is much larger than the vertical scale and the vertical velocity is smaller than the horizontal velocity. Under these conditions, the energy and the I spectrum become identical and go as $K^{-5/3}$, as in the 2D case. One expects that an increasing fraction of energy is transferred to large spatial scales as the anisotropy in the system increases. This can go on until coriolis force begins to be effective. The length scale L_c where the nonlinear term of the Navier–Stokes equation becomes comparable to the coriolis force is given by $L_c = U/\Omega$, where Ω is the angular velocity. At these large scales, the system simulates 2D behaviour and enstrophy conservation begins to play its role. One may consider scales $L \geq L_c$ as a source of vorticity injection into the system. The enstrophy then cascades towards small scales with a power law spectrum given by:

$$E(K) \propto K^{-3}$$

$$W \propto L^2 \,. \tag{9}$$

Thus the complete energy spectrum of a hydrodynamic turbulent medium is given in Figure 4. What is the evidence for this spectrum?

Solar granulation

The cellular velocity patterns observed on the solar surface are believed to be manifestations of convective phenomena occurring in the sub-photospheric layers. The quality observations obtained at Pic-du-Midi indicate the existence of a continuum of sizes instead of one or two

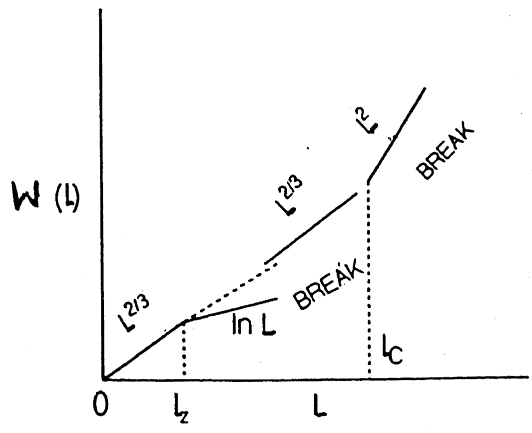

Figure 4. Turbulent energy spectrum, L_z, normalizing length; L_c, break due to coriolis force (Krishan[1,2]).

dominant scales in the solar granulation. We have proposed an inverse cascade model for the entire granulation phenomenon. It is encouraging to find that the energy spectrum of granular motions deduced from observations does show a branch with $K^{-5/3}$ which turns into a $K^{-0.7}$ law towards small K (Figure 5). This agrees fairly well with the predictions of the inverse cascade model where the Kolmogorov branch $K^{-5/3}$ develops into a K^{-1} behaviour towards large spatial scales. Thus the solar granulation provides one example where the flat branch K^{-1} seems to account well for an observed phenomenon. Is there another example?

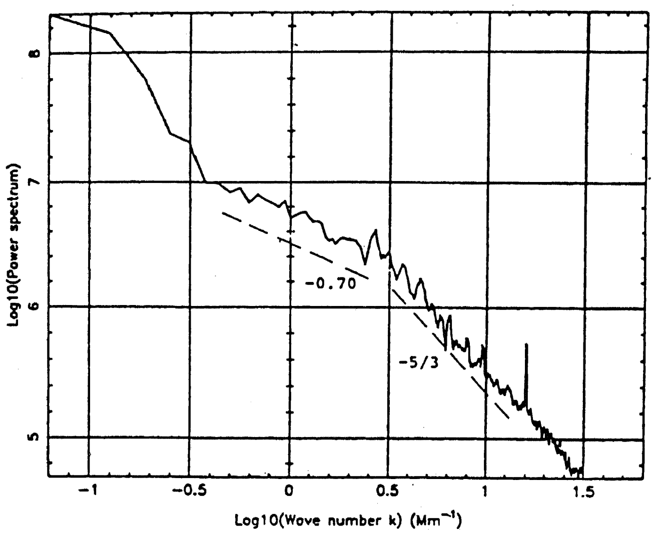

Figure 5. Power spectrum of the solar photospheric motions (Zahn[15]).

Rotation curves of galaxies

The issue of the flat rotation curves of galaxies and the need for dark matter is described precisely in Figure 6. The flat nature of the orbital motion in galaxies is accounted through the relationship $\frac{1}{2}mV^2 = (GMm/R)$ by assuming that $M \propto R$ and therefore, the velocity $V = \text{constant}$. Since the mass $M \propto R$ has no luminosity associated with it, it is known as dark matter. We present an alternative explanation for the flat nature of the velocity by applying the ideas of inverse cascade in a turbulent atmosphere of a galaxy. We propose a law of velocities of the following type:

$$V(L) = AL + BL^{1/3}$$

in the inner region, i.e. for $L < L_z$ and (10)

$$V(L) = CL^{-1/2} + D[\ln L/L_z]^{1/2}$$

in the outer region, i.e. for $L > L_z$ of a galaxy. A, B, C and D are determined from the fits with the observed velocity curves. We have successfully accounted for the flat nature of the rotation curves for nearly hundred galaxies. An example is given in Figure 7.

We have discussed two cases, from completely disparate phenomena, where the Kolmogorov branch $K^{-5/3}$ and the flat branch K^{-1} together have explained the observed behaviour very well. What about the rest of the spectrum given in Figure 4? Particularly, is there any evidence for the spectrum at the largest spatial scales, i.e. for $E(K) \propto K^{-3}$ or $W(L) \propto L^2$?

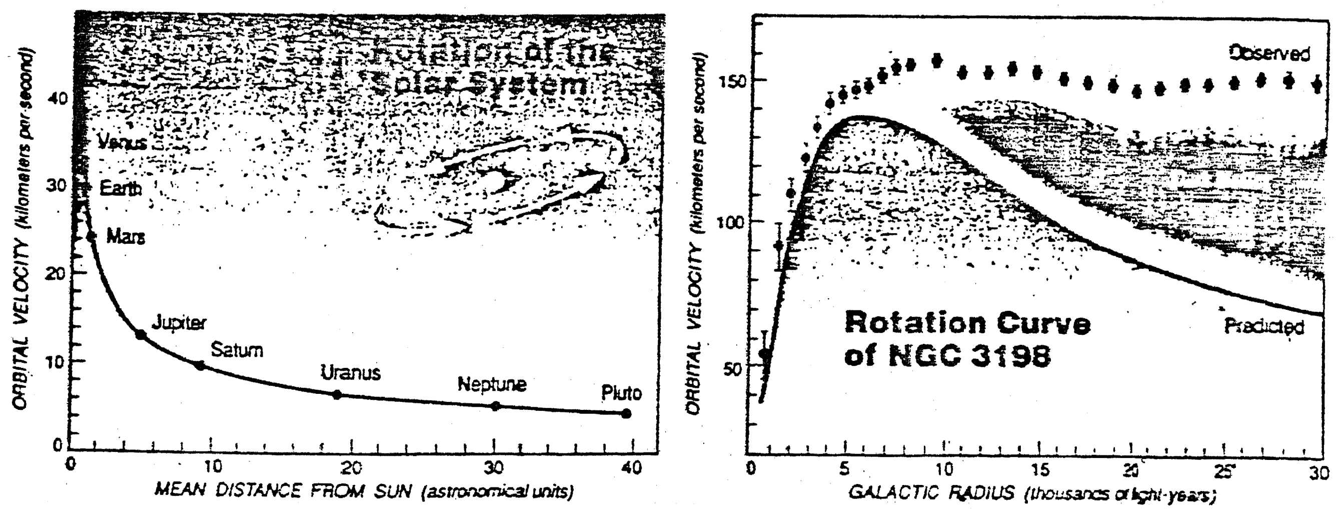

Figure 6. The case for dark matter in spiral galaxies. *Left*: The orbital velocities of the planets (dots) decrease with distance from the Sun exactly as predicted by Newtonian gravitation (line), assuming a system dominated by one solar mass at its center. *Right*: The cosmos is not as well behaved on galactic scales. Here a graph of orbital velocity versus radius has been computed for NGC 3198, a spiral galaxy in Ursa Major, assuming that the distribution of light serves as a good indicator of the distribution of mass. The failure of the observed velocities (dots) to match the predicted ones is striking and points to an unseen component of dark matter in the galaxy.

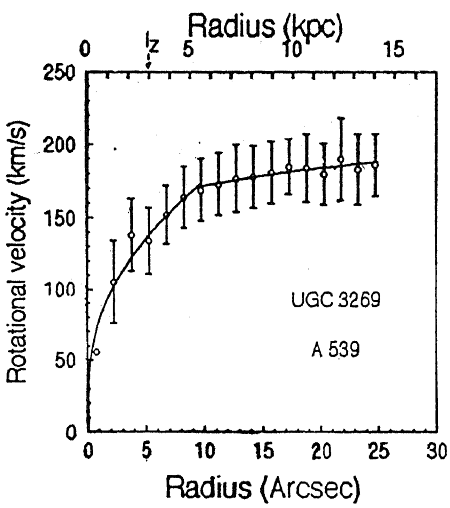

Figure 7. Flat rotation curve of a galaxy fitted by equation (10).

The Hubble flow

The energy spectrum at the largest spatial scales is governed by enstrophy transfer and goes as

$$E(K) \propto K^{-3}. \tag{11}$$

Putting in the proportionality constants, the total energy density per unit gram varies as

$$W(L) = V^2(L) \simeq (\varepsilon/L_c^2)^{2/3} L^2$$

or

$$V(L) \simeq (\varepsilon/L_c^2)^{1/3} L. \tag{12}$$

$\varepsilon = V^2/\tau$ is the energy injection rate and (ε/L_c^2) is the enstrophy injection rate. Substituting for L_c, the scale at which the coriolis force becomes comparable to nonlinear inertial force, one finds

$$V(L) \simeq (\Omega^2/\tau)^{1/3} L \tag{13}$$

which, when compared with the Hubble flow expressed as

$$V = HL$$

gives the value of the Hubble constant H to be

$$H = (\Omega^2/\tau)^{1/3}. \tag{14}$$

Does this mean that the velocities with which the galaxies are receding from each other like dots on an expanding balloon are given by the largest spatial scale end of the same energy spectrum whose small spatial scale part accounts so very well for the flat rotation curves? If one assumes that the angular speed at scales $\sim L_c$ is such that $\Omega^{-1} = \tau$, then one gets H^{-1} = age of the universe $= \tau$, the duration for the injection of energy. If stars are the main contributors to turbulence, the age of the universe τ is nothing but the age of the oldest stars. Of course, the big bang itself is a source of turbulence in the universe.

Conclusion

The need of the hour is to think non-equilibrium. This may alleviate some of the caveats that arise from the belief that equilibrium rules the universe. Hydrodynamics in addition to gravitational processes must be included in the scheme of things since a self-consistent distribution of matter and motion must be determined. One must realize that converging flows commonly occur in a turbulent fluid without the necessity of invoking great masses. The whole question of mass distribution or the large scale structure of the universe can only be settled by including the physics of hydrodynamical processes in the making of structures. The angular momentum must be treated with reverence!

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