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NEW MOUNTING FOR X-RAY AND EXTREME ULTRA VIOLET CONCAVE HOLOGRAPHIC GRATING

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Nouveau montage de réseau holographique concave pour rayons X et ultraviolet lointain

SUMMARY : An entirely new mounting of concave holographic grating for X-ray and extreme ultra violet (EUV) has been proposed and its properties have been investigated theoretically. Design parameters for recording a holographic concave grating for minimum spectral image aberrations have been calculated. It is found that this mouting is very suitable.

RÉSUMÉ : On étudie théoriquement les propriétés d'un montage entièrement nouveau de réseau holographique concave pour l'ultraviolet lointain et les rayons X mous. Les paramètres du montage pour l'enregistrement du réseau ont été calculés pour minimiser les aberrations dans le spectre. Ce montage se révèle très bien adapté.

1. — *INTRODUCTION*

The diffraction gratings, prepared holographically, have been put on the market by now and have attracted considerable attention in the field of spectroscopy. These gratings have some advantages over the mechanically ruled gratings. Of these advantages, the design of a grating that meets specific requirements, such as elimination and/or reduction of certain types of aberrations, have been paid quite a lot of attention by the manufacturer, designer as well as user. All these holographically recorded gratings (HRDG) are being produced to satisfy Rowland circle conditions as well as correction of aberrations.

Not much attention has been paid to propose design parameters to record HRDG with different type of focal properties other than the conventional focal properties of the mechanically ruled concave diffraction gratings. Flamand *et al.* [1] has paid some attention on it. Our studies in detail have revealed that quite a new useful mountings can be proposed for HRDG and at the same time aberrations of the spectral lines can be corrected [2]. Pouey [3] has designed new holographic grating devices for hot plasma diagnostics, Singh *et al.* [4] have designed new holographic gratings for extreme ultra violet and X-ray region.

In the present paper, we describe an entirely new mounting of concave holographic grating for X-ray and EUV. The calculations have been performed for a 30 mm × 50 mm grating with a radius of curvature $R = 1\,000$ mm. The X-ray and EUV region of electromagnetic spectrum are very important for the study of comet, sun and other celestial objects.

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2. — THEORETICAL ANALYSIS

Let us take the centre of the grating rulings, 0, as the origin of the Cartesian coordinate system *(fig. 1)*, the z-axis being parallel to the rulings, the x-axis along the grating normal and the y-axis as shown *(fig. 1)*. Let $A(x, y, z)$, $B(x', y', z')$ and $P(u, w, l)$ be points on the source slit, spectrum line and grating ruling respectively. We further assume that the recording sources are represented by $C(x_C, y_C, O)$ and $D(x_D, y_D, O)$ and that the difference of the distances of these recording sources from 0 is an integer multiple of λ_0 the wavelength of the recording laser light, and that the zeroth groove passes through 0. Now let us take the cylindrical polar coordinates of the points $A(x, y, z)$, $B(x', y', z')$, $C(x_C, y_C, o)$ and $D(x_D, y_D, o)$ as (r, α, z), (r', β, z'), (r_C, γ, o) and (r_D, δ, o) respectively. All these angles are measured in the xy plane. The angles α and β are the angles of incidence and diffraction respectively. The signs of α and β are opposite if points A and B lie on different sides of the xz plane. The same kind of sign condition holds good for γ and δ. The signs of α and β should be consistent with the signs of γ and δ.

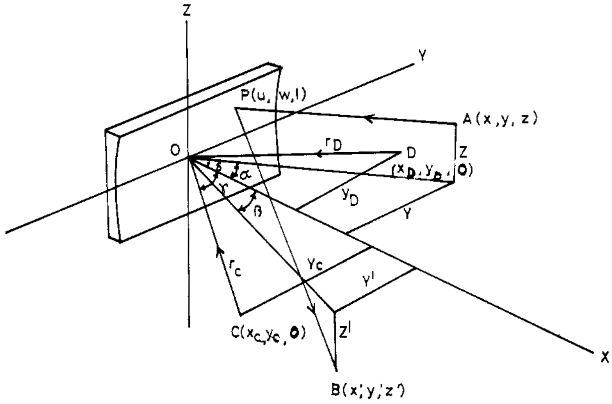

FIG 1. — *Schematic diagram of the optical system.*

Now following Noda *et al.* [5-6], the horizontal focal condition of the grating will be given by the relation

$$\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} - \frac{\sin\alpha}{(\sin\delta - \sin\gamma)}\left\{\left(\frac{\cos^2\delta}{r_D} - \frac{\cos\delta}{R}\right) - \left(\frac{\cos^2\gamma}{r_C} - \frac{\cos\gamma}{R}\right)\right\} + $$
$$+ \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} - \frac{\sin\beta}{(\sin\delta - \sin\gamma)}\left\{\left(\frac{\cos^2\delta}{r_D} - \frac{\cos\delta}{R}\right) - \left(\frac{\cos^2\gamma}{r_C} - \frac{\cos\gamma}{R}\right)\right\} = 0 \quad (1)$$

This relation is in fact the basis of all mountings of the HRDG. If in this equation we put some definite values of r and α we shall get a relation, involving r' and β, which will be the polar equation of the focal curve for the particular position (r, α) of the source. Now on an examination of Eq. (1), we see that the first three terms contain only the variables r and α while the remaining three terms contain only the variables r' and β. Thus, if we choose r and α in such a way that the first part of the equation, viz.

$$\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} - \frac{\sin\alpha}{(\sin\delta - \sin\gamma)}\left\{\left(\frac{\cos^2\delta}{r_D} - \frac{\cos\delta}{R}\right) - \left(\frac{\cos^2\gamma}{r_C} - \frac{\cos\gamma}{R}\right)\right\} = \frac{K}{R}$$

and the recording parameters $(r_C, \gamma, r_D, \delta)$ such that

$$\frac{R}{(\sin\delta - \sin\gamma)}\left\{\left(\frac{\cos^2\delta}{r_D} - \frac{\cos\delta}{R}\right) - \left(\frac{\cos^2\gamma}{r_C} - \frac{\cos\gamma}{R}\right)\right\} = 1 \quad (2)$$

we get for the source curve as

$$r = \frac{R\cos^2\alpha}{\cos\alpha + K + \sin\alpha} \quad (3)$$

and for the focal curve as

$$r' = \frac{R\cos^2\beta}{\cos\beta - K + \sin\beta} \quad (4)$$

a relation which is independent of the position of the source. In otherwords, we may say that if the source point be any where on the curve denoted by Eq. (3) which may be called the source curve, the spectra will always be formed on a fixed focal curve given by Eq. (4). Of course, as the position of the source changes on the source curve, different regions of the spectrum will come up at a particular place on the focal curve. In case $K = 0$, Eqs. (3) and (4) reduce respectively to

$$r = \frac{R \cos^2 \alpha}{\cos \alpha + \sin \alpha} \tag{5}$$

$$r' = \frac{R \cos^2 \beta}{\cos \beta + \sin \beta} \tag{6}$$

which represent the same curve for the source and the diffracted images. The values of K may be selected between $+1$ and -1. In *figure 2*, [2] the source curves and the corresponding focal curves have been shown for the values of $K = 0$, ± 0.1045, ± 0.1736, ± 0.2079, ± 0.5, ± 0.7071, ± 0.9397, ± 0.9848. In *figure 2*, we have plotted Rowland Circle i.e. $r' = R \cos \beta$, $r = R \cos \alpha$ along with the focal curve and source curves for different values of K. It can be noticed easily from this figure that the Rowland circle is cutting the source curve and the focal curve for the same value of K, in a well defined manner i.e. at an angle $\pm \theta$, such that $K = \sin \theta$. For example if we take $K = -0.5$, the Rowland circle cuts the source curve at an angle $\alpha = \sin^{-1} 0.5$ and the Rowland circle cuts the focal curve at an angle $\beta = \sin^{-1}(-0.5)$ i.e. $\alpha = \theta = 30°$, $\beta = -\theta = -30°$. The same fact holds good for other values of K.

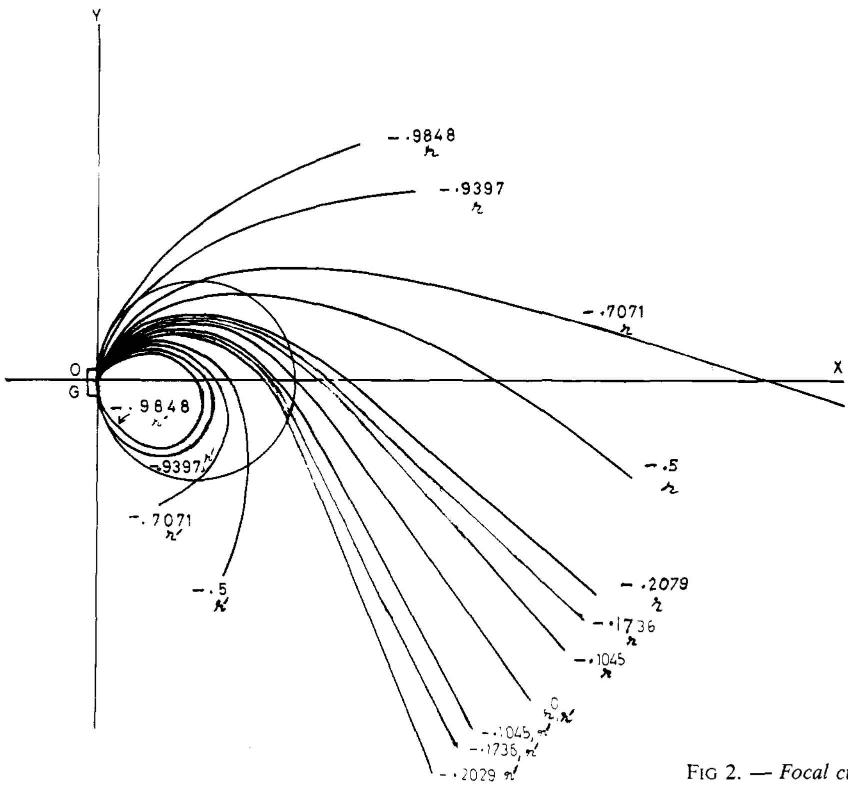

FIG 2. — *Focal curves and source curves at different values of K.*

3. — ESTIMATION OF THE ABERRATIONS AND CALCULATIONS OF DESIGN PARAMETERS

By substituting Eqs. (3) and (4) into the relation of optical path function and applying Fermat's principle $\frac{\partial F}{\partial l} = 0$ and $\frac{\partial F}{\partial \omega} = 0$, where $F(r, r', \alpha, \beta, \omega, l, z, z', \gamma, \delta, r_C, r_D)$ is optical path function [5], we get the following expressions for the length of the spectral images (due to astigmatism) and the spread of the spectral images (due to coma, and other aberrations) for a point source, respectively as.

$$\Delta z' = l \left[1 + \frac{\cos^2 \beta}{\cos \beta - K + \sin \beta} \left\{ (CF1) + \frac{\omega}{R} CF2 \right\} \right] \bigg/ \left(1 + \frac{\omega}{R} \frac{\sin \beta (\cos \beta - K + \sin \beta)}{\cos^2 \beta} \right) \tag{7}$$

where

$$CF1 = \frac{\cos \alpha + K + \sin \alpha}{\cos^2 \alpha} - (\cos \alpha + \cos \beta) - (\sin \alpha + \sin \beta)\, BF1$$

$$BF1 = \left(\frac{R}{r_D} - \frac{R}{r_C} + \cos\gamma - \cos\delta \right) \frac{1}{\sin\delta - \sin\gamma}$$

$$CF2 = \frac{\sin\alpha\,(\cos\alpha + K + \sin\alpha)\,(\cos\alpha\,\sin^2\alpha + K + \sin\alpha)}{\cos^4\alpha} + \frac{\sin\beta\,(\cos\beta - K + \sin\beta)\,(\cos\beta\,\sin^2\beta - K + \sin\beta)}{\cos^4\beta} + (\sin\alpha + \sin\beta)\,BF2$$

$$BF2 = \left(\frac{R^2}{r_C^2}\sin\gamma - \frac{R^2}{r_D^2}\sin\delta - \sin\gamma\cos\gamma\,\frac{R}{r_C} + \sin\delta\cos\delta\,\frac{R}{r_D} \right) / (\sin\delta - \sin\gamma)$$

$$\Delta\lambda = \frac{\sigma}{-2\,mR^2}\,((\Delta z')^2 - 2\,l\,\Delta z')\,CF - \frac{\sigma l^2}{2\,mR^2}\,CF2 - \frac{3\,\omega^2\sigma}{2\,mR^2}\,CF3 - \frac{\sigma\omega\,(l^2+\omega^2)}{2\,mR^3}\,CF4 - \frac{\sigma\omega^3}{mR^3}\,CF5 + O\left(\frac{\omega^4}{R^4}\right) \tag{8}$$

where m is the order of the spectrum :

$$CF = \frac{\sin\beta\,(\cos\beta - K + \sin\beta)^2}{\cos^2\beta}$$

$CF2$ = as defined earlier

$$CF3 = \frac{\sin\alpha\,(K + \sin\alpha)\,(\cos\alpha + K + \sin\alpha)}{\cos^2\alpha} + \frac{\sin\beta\,(-K + \sin\beta)\,(\cos\beta - K + \sin\beta)}{\cos^2\beta} + (\sin\alpha + \sin\beta)\,BF3$$

$$BF3 = \left(\frac{R^2}{r_C^2}\sin\gamma\cos^2\gamma - \frac{R^2}{r_D^2}\sin\delta\cos^2\delta - \frac{R}{r_C}\sin\gamma\cos\gamma + \frac{R}{r_D}\sin\delta\cos\delta \right) / (\sin\delta - \sin\gamma)$$

$$CF4 = \frac{\cos\alpha\,\sin^2\alpha + K + \sin\alpha}{\cos^2\alpha} + \frac{\cos\beta\,\sin^2\beta - K + \sin\beta}{\cos^2\beta} - (\sin\alpha + \sin\beta)\,BF4$$

$$BF4 = \left(\frac{R}{r_D} - \frac{R}{r_C} + \cos\gamma - \cos\delta \right) / (\sin\delta - \sin\gamma)$$

$$CF5 = \frac{2\sin^2\alpha\,(K + \sin\alpha)\,(\cos\alpha + K + \sin\alpha)^2}{\cos^4\alpha} + \frac{2\sin^2\beta\,(-K + \sin\beta)\,(\cos\beta - K + \sin\beta)^2}{\cos^4\beta} - \frac{(K + \sin\alpha)^2\,(\cos\alpha + K + \sin\alpha)}{2\cos^2\alpha} - \frac{(-K + \sin\beta)^2\,(\cos\beta - K + \sin\beta)}{2\cos^2\beta} + (\sin\alpha + \sin\beta)\,BF5$$

$$BF5 = \left(2\frac{R^3}{r_C^3}\sin^2\gamma\cos^2\gamma - 2\frac{R^2}{r_C^2}\sin^2\gamma\cos\gamma \right) - 2\frac{R^3}{r_D^3}\sin^2\delta\cos^2\delta + 2\frac{R^2}{r_D^2}\sin^2\delta\cos\delta - \frac{R}{2\,r_C}\left(\cos^2\gamma\,\frac{R}{r_C} - \cos\gamma\right)^2 + \frac{R}{2\,r_D}\left(\cos^2\delta\,\frac{R}{r_D} - \cos\delta\right)^2 / (\sin\delta - \sin\gamma)\,.$$

From Eqs. (7), we get the following expression, similar to those obtained in other cases [5], for the length of the astigmatic images $(Z')_{\text{ast}}$ formed by a point source as,

$$(Z')_{\text{ast}} = L\left(1 + \frac{\cos^2\beta}{\cos\beta - K + \sin\beta}\,CF1\right) \tag{9}$$

where L is the total length of the grooves.

The condition for zero astigmatism is given by

$$\left(\frac{R}{r_D} - \frac{R}{r_C} + \cos\gamma - \cos\delta \right) / (\sin\delta - \sin\gamma) = \left[\frac{\cos\beta - K + \sin\beta}{\cos^2\beta} + \frac{\cos\alpha + K + \sin\alpha}{\cos^2\alpha} - (\cos\alpha + \cos\beta) \right] / (\sin\alpha + \sin\beta) = f(\alpha, \beta, K)\,. \tag{10}$$

For $\alpha = 10°$, $K = -0.1736$, $f = 1.1765$, the values of $(Z')_{ast}/L$ are given in *figure 3* at different wavelengths. The zero astigmatism condition is chosen at $\beta = 0$. This condition gives very small astigmatism throughout the wavelength region considered. This gives at one more wavelength minimum astigmatism.

The equation for secondary focal curve in polar coordinates (r_h', β) is given by

$$r_h' = \frac{R}{(\cos\alpha + \cos\beta) - \dfrac{\cos\alpha + K + \sin\alpha}{\cos^2\alpha} + (\sin\alpha + \sin\beta)\, f(\alpha, \beta, K)}. \tag{11}$$

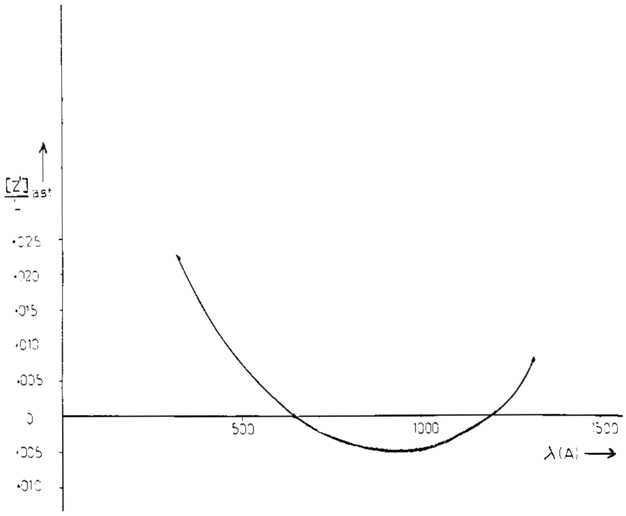

FIG 3. — *Astigmatism per unit grooves length at different wavelengths.*

If we plot both the primary focal curve given by Eq. (4) and secondary focal curve given by Eq. (11), on the same diagram, the point of intersection of these two curves, give the position for zero astigmatism and the separation between these curves at different points an insight into the astigmatic properties of the spectral images. *Figure 4* represents source curve and primary focal curve at $K = -0.1736$ and secondary focal curve at an angle of incidence 10°.

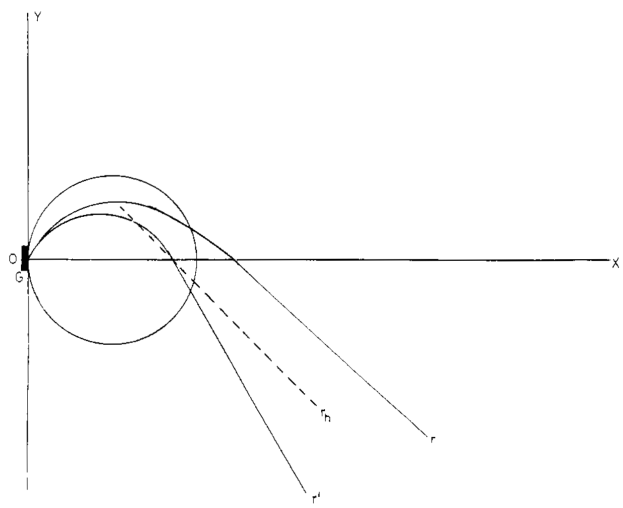

FIG 4. — *Secondary focal curve and primary focal curve.*

By using Eq.s (2) and (10) we get the relations for calculating the recording parameters r_C and r_D at some selected values of γ and δ ; $f(\alpha, \beta, K)$. These recording parameters will give spectral line without astigmatism at the wavelengths given by the values of (α, β) which have been used in Eq. (10) i.e. $f(\alpha, \beta, K)$, for these calculations. These relations are as given below :

$$\frac{R}{r_C} = \frac{\sin^2\delta}{\cos\delta + \cos\gamma} + \frac{(\sin\delta - \sin\gamma)}{(\cos^2\delta - \cos^2\gamma)} \times \left[1 - f(\alpha, \beta, K)\cos^2\delta\right] \tag{12}$$

$$\frac{R}{r_D} = \frac{\sin^2\gamma}{\cos\delta + \cos\gamma} + \frac{(\sin\delta - \sin\gamma)}{(\cos^2\delta - \cos^2\gamma)} \times \left[1 - f(\alpha, \beta, K)\cos^2\gamma\right]. \tag{13}$$

By using Eqs. (12) and (13) the following recording parameters for making the concave diffraction grating have been obtained.

$$\frac{r_C}{R} = 1.3758$$

$$\frac{r_D}{R} = 0.4337$$

$$\gamma = -45°$$
$$\delta = 30°.$$

By using a laser light of wavelength $\lambda_0 = 0.45793\ \mu m$, for making the grating, the grating constant σ is $0.37936\ \mu m$.

4. — RAY TRACING, SPOT DIAGRAMS

Using Eqs. (7) and (8) and the design parameters as decided in earlier paragraphs, the spot diagrams at λ (Å) = 328.15, 658.57, 989,37 and 1 357.52 are presented in *figure 5*. In *table I* we have presented the max values of ΔZ (mm) and $\Delta\lambda$ (mm) at different wavelengths. These figures are very much encouraging. The reciprocal linear dispersion at

328.15 Å, 658.57 Å, 989.37 Å and 1 317.52 Å are 2.67 Å/mm, 3.38 Å/mm, 3.56 Å/mm and 3.70 Å/mm respectively and the practical resolution is 0.0064 Å, 0.0074 Å, 0.0061 Å and 0.0037 Å respectively.

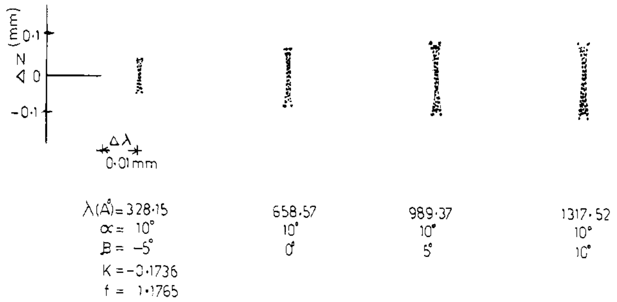

FIG 5. — *Spot diagrams at different wavelengths.*

Table 1

f = 1.1765	δ = 30°
r_C = 1.3758 m	α = 10°
r_D = 0.4337 m	λ_o = 173,6 nm
R = 1 m	γ = - 0.45°
σ = 0.37936 m	K = - 0.1736

λ(nm)	β	ΔZ (mm)	$\Delta\lambda$(mm)	Practical resolution nm
32.815	-5°	0.2117	0.0024	0.00064
65.857	0	0.1960	0.0022	0.00074
98.937	5°	0.1535	0.0017	0.00061
131.752	10°	0.0885	0.0010	0.00037

5. — *CONCLUSION*

The proposed mounting is very much suitable for X-ray and EUV region. It can be exploited in various ways for designing new types of monochromators and spectrographs, necessary for the study of comet, sun and other celestial objects. The resolution limit of this design is far better than the instruments used in recent ultra violet explorer and other space craft experiments [7-10].

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