RESEARCH ACCOUNT

On the black hole trail . . .: A personal journey*

C. V. Vishveshwara

Indian Institute of Astrophysics, Bangalore 560 034, India

Beginning of the trail

It is a joy to give this talk as a tribute to Professors Vaidya and Raichaudhuri, the two father-figures of general relativity in India. If my talk is rather autobiographical in nature, the responsibility rests with Naresh Dadhich and Bala Iyer, respectively the President and the Secretary of the Indian Association for General Relativity and Gravitation, who persuaded me to make it so.

My personal journey along the black hole trail started in the sixties when I was a graduate student of Charles Misner at the University of Maryland. I had transferred from Columbia University in New York specifically to work with him. I had been encouraged to follow this course by Robert Fuller, my mentor at Columbia University and like Misner a former student of John Wheeler. Fuller had remarked, 'If you want to work in general relativity, why not go to one of the best in the field!' Also, that was when I first came to know about Vaidya and Raychaudhuri. Leepo Cheng was doing her master's thesis with Misner on the Vaidya metric. She was appalled by my ignorance when I told her that I did not know who Vaidya was. Later on, we were told that Raychaudhuri was coming as a Visiting Professor. Again, my colleagues were suitably impressed by my ignorance when I confessed that I did not know who this Raychaudhuri was either. I went on not only to take a course on cosmology from him but also to pass it with a little bit of honest cheating. Never did I dream that some day I would be delivering a lecture in honour of these two gentlemen.

Let me come back to black holes. That is not what they were called at that time. Schwarzschild singularity, which is a misnomer. Schwarzschild surface, which is better. The term 'Black Hole' was to be coined later on by John Wheeler. Perhaps, it was because of such an intriguing name that so many people were enticed into working on the physics of the black hole. This is known as the Schicklgruber Effect. Scholars have speculated on how human history might have been different if Schicklgruber had not changed his name. But, he did change his name to Hitler. Misner proposed the following problem for my Ph D thesis. Take two of these entities that are now called black holes. Revolving around each other, they come closer as energy is radiated away in the form of gravitational waves. They coalesce into an ellipsoidal 'Schwarzschild surface' still rotating and radiating. Study the whole process, computing all the characteristics of the emitted gravitational radiation. Fine,

P. C. Vaidya
The Radiant Rider
Discoverer of the well-known Vaidya metric which represents the spacetime of a radiating star. Now in his youthful seventies, Vaidya rides his bike with such a star for the lamp.

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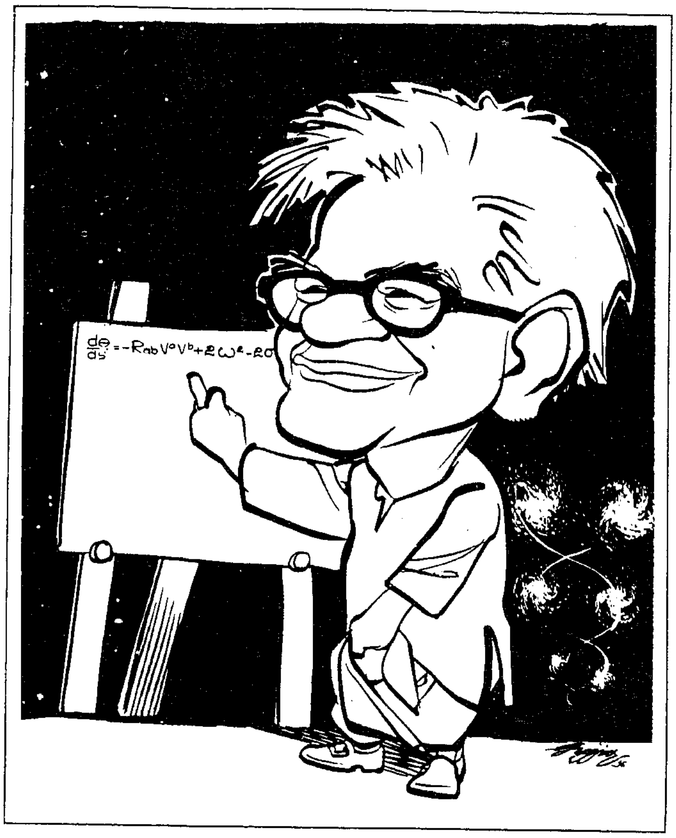


A. K. Raychaudhuri
The Cosmic Converger
Deriver of the extensively used Raychaudhuri equation which describes the motion of galaxies and shows how they must have emerged from an initial singularity in the past.

I said, thy will be done! No one at the time could have realized the magnitude of this problem. Had I pursued it, I might have entered the Guinness book of records as the oldest graduate student alive that too without financial support. Anyway, this proposed problem required the understanding of two aspects of black holes: the geometrical structure of a black hole and the perturbations of its spacetime.

Geometry of black holes

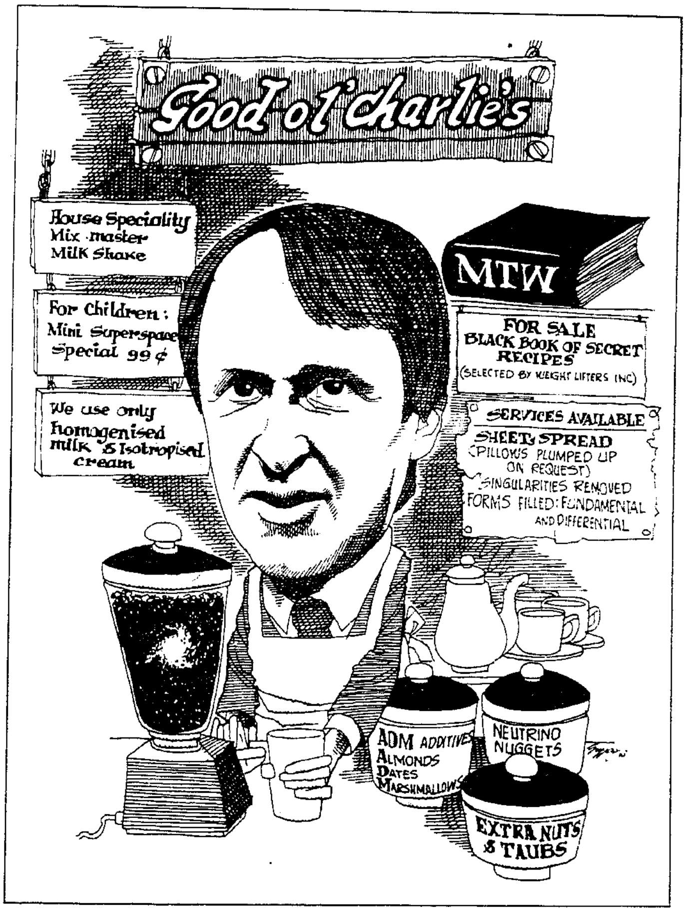
Those were the early days when very little was known about black holes. Wheeler was going around giving his talk 'Gravitational Collapse: To What?' with missionary zeal. There was some vague notion of the metric component g_{00} of static spacetimes tending to zero on some surface. I distinctly remember the cold morning when, on the way to grab a sandwich at the little store run by the school of dairy research, Misner suggested that I look into this shady business. Fine, I said, thy will be done! There were false starts. I had this excruciating experience of translating to myself a lengthy paper in German by Ehlers and Sachs – or was it Ehlers, Kundt and Sachs, I forget – on ray optics and optical scalars in the hope that it would throw some light on the matter. It did not. Nevertheless, it was the article by Ehlers and Kundt[1] that gave the clue to the secret of the black hole structure. The covariant approach to the unravelling of the black hole geometry was via the spacetime symmetries or Killing vector fields. This way the fundamental properties of both Schwarzschild and Kerr black holes could be analysed, compared and contrasted. For instance, given a Killing vector field ξ one could derive the equation[2],

$$n^a n_a = \tfrac{1}{2}\,[\xi^a \xi_a (\xi_{b;c}\xi^{b;c}) - \omega^r \omega_r]\,, \qquad (1)$$

where ω^a is the vorticity of the Killing congruence and n^a is the normal to surfaces of constant Killing norm, i.e.

$$\Sigma : \xi^a \xi_a = \text{constant}. \qquad (2)$$

This shows that the surface on which ξ^a becomes null



C. W. Misner
The Master Mixer
Known for, among other things, his mix-master universe, cosmic isotropization by neutrinos, ADM formulation of general relativity, elucidation of the Taub-NUT spacetime, co-authorship of the big black book *Gravitation*, and physics through spread sheets.

($\xi^a\xi_a = 0$), is itself a null surface, equivalently a one-way membrane or an event horizon ($n^a n_a = 0$), provided the vorticity also becomes null on the surface. The first condition implies that static observers cannot exist at and beyond this surface on which ξ^a is null. On the other hand, a null surface, satisfying the second condition, acts as a one-way membrane through which one can fall but cannot re-emerge from. This is the event horizon or the black hole. In the case of the Schwarzschild spacetime the surface on which the global timelike vector field ξ^a becomes null is a null surface since the vorticity of ξ^a is identically zero. In the case of Kerr spacetime, this is achieved for a suitable combination of the global timelike Killing vector ξ^a and the rotational Killing vector η^a. Consequently, the Schwarzschild black hole is both a one-way membrane and a static limit whereas in the Kerr spacetime, these two surfaces are distinct, thereby possessing the ergosphere between them. And this is where interesting phenomena like the Penrose process and the consequent energy extraction can occur.

Although the global timelike Killing vector field ξ^a of the Kerr spacetime possesses non-zero vorticity or rotation, Kerr spacetime admits an irrotational vector field

$$\chi^a = \xi^a - \frac{(\xi^b\eta_b)}{(\eta^c\eta_c)}\,\eta^a\,, \tag{3}$$

which is timelike down to the black hole. This vector field defines the Locally Non-Rotating Frames (LNRF)[3], or the Zero Angular Momentum Observers (ZAMO)[4]. But this vector field exhibits much more interesting properties. These were investigated by Richard Greene, Englebert Schücking and myself[5] around 1970. I had by then joined Schücking at New York University after a stint at the Institute for Space Studies of NASA in New York and a short period of unemployment. Perhaps it was too much to expect that black holes would be a source of income, since they were not sources of anything in the first place. To continue, we were able to generalize the irrotational vector field to arbitrary stationary, axisymmetric spacetimes with orthogonal transitivity. It was shown to be globally hypersurface orthogonal normal to $t =$ constant surfaces. These are maximal surfaces. The vector field could become null on an event horizon. Some features of this study, such as the physical interpretation of the mathematical conditions necessary for these properties, are still open problems. Incidentally, Iyer and I[6] have recently renamed LNRF or ZAMO as GHOST – Globally Hypersurface Orthogonal Stationary Trajectories!

The geometry of Killing trajectories, i.e. the integral curves of Killing vector fields, that play such a basic role in elucidating the black hole structure, sneaked into our investigations in an indirect manner. Eli Honig was studying the motion of charged particles in homogeneous electromagnetic fields using the Frenet–Serret (FS) formalism[7]. This formalism offers a geometric description of an arbitrary curve characterizing it by certain scalars and an orthonormal frame of reference at each point. In three dimensions these scalars are κ, the curvature and τ_1, the torsion. In four-dimensional general relativity, we have an additional torsion τ_2. Furthermore, the derivatives are with respect to proper time and, as a consequence, κ turns out to be the magnitude of four-acceleration. Similarly, the precession rate of a gyroscope carried along the curve has components τ_1 and τ_2 with respect to two members of the Frenet–Serret tetrad at each point of the curve. Now the worldliness of charges moving in a constant electromagnetic field F_{ab} bears striking resemblance to Killing trajectories. In both cases, each member of the FS tetrad satisfies the Lorentz equation. For Killing trajectories

$$F_{ab} = e^{\psi}(\xi_{a;b})\,, \tag{4}$$

where the normalization factor $e^{\psi} = (\xi^a\xi_a)^{-1/2}$. In both cases, one can show κ, τ_1 and τ_2 are constants along the worldline and

$$\kappa^2 - \tau_1^2 - \tau_2^2 = \tfrac{1}{2}\,F_{ab}F^{ab}\,. \tag{5}$$

In the case of Killing trajectories, τ_1 and τ_2 turn out to be the components of voriticity. Further, acceleration is given by n_a, the gradient of equipotentials $\xi^a\xi_a =$ constant, so that κ^2 is proportional to $n^a n_a$. With these substitutions, equation (5) reduces to equation (1). So, we have indirectly rederived the original black hole equation.

We shall return later to gyroscopic precession which we have mentioned here in passing.

Stability of the Schwarzschild black hole

The ultimate problem for my Ph D thesis, as mentioned earlier, was supposed to be the coalescence of two black holes. In order, at least, to make a beginning on this problem, I had to study perturbations superposed on the Schwarzschild spacetime as the background. The canonical paper in this area was, of course, the one by Regge and Wheeler[8]. To me this was a completely unknown territory. I remembered vaguely, a remark in Wheeler's book *Geometrodynamics*[9] to the effect that the stability of the Schwarzschild spacetime was a problem far from having been solved satisfactorily. In fact, it was this book, totally incomprehensible to me when I was a first year graduate student in Columbia University and hence highly intriguing, that had drawn me towards general relativity. Another student of Misner, Lester Edelstein and I rederived the perturbation equations and published them[10] since these equations, as they appeared in the paper by Regge and Wheeler as well as at other places, contained errors. Lester was the first one to work out the radiation emitted by a particle falling into a black hole. He could not track down a factor of two that was missing when he compared his formula with the one of Landau and Lifshitz in the weak field limit. Unfortunately, as a result, he never completed his thesis and eventually switched from general relativity to actuaries. It was around this time that S. Chandrasekhar visited Maryland. It was a big event. Wheeler and his group, that included Uri Gerlach, Bob Geroch and Kip Thorne, drove down from Princeton. Chandra was getting interested in general relativity and in particular, black hole perturbations. We gave him our newly derived, yet unpublished equations. I had met Chandra a couple of years earlier at the Boulder Summer School in Colorado when I was aspiring to be a particle physicist in Columbia University. Other participating Indian students and I discussed with him our research as well as a bit of Indian science. In later years, I was to have the great privilege of having many discussions with Chandra on black hole physics, which became his chosen territory, and Indian science, in which he was keenly interested.

Stability analysis, as Ed Salpeter once put it, consists in finding out whether a system breaks apart if an ant sneezed in its vicinity. In the case of the black hole, the ant's sneeze is represented by metric perturbation which is a product of Fourier time mode $\exp(i\omega t)$, angular function which is a suitable tensor spherical harmonic and a radial function. Assuming the radial function to be well behaved, one had to show that imaginary frequencies that would make the perturbations grow exponentially in time were not admitted. Good behaviour had to be tested in reference to Kruskal coordinates that are singularity free at the black hole. Moreover, the radial functions corresponding to real frequencies had to be shown to form a complete set so that wave packets could be built that did not blow up in time. All this could be done for odd parity perturbations for which the radial function was governed by a Schrödinger-type equation with an equivalent potential. Frank Zerilli[11] would later derive a similar equation in the case of even parity perturbations. But, at the time, the even parity equation was a mess with frequency appearing all over the place. Stability analysis did not seem to go through. I was stuck, hopelessly stuck. Misner, who was going away to Cambridge for a year, suggested that I find a few simple, solvable problems and string them together into a thesis. My heart jumped into my mouth and my other organs rearranged themselves accordingly. I decided to devote another two weeks to the problem – body, mind and soul – and then quit if I did not make any progress. Those were the days when Joe Weber was setting up his gravitational wave detector. Weber and his group were observing a rather peculiar phenomenon. Regularly around midnight the detector would record a sharp, beautiful peak. And then again another peak after an interval of a few minutes. Joe Sinsky, a graduate student, stayed on in the laboratory one night to investigate this puzzling phenomenon. Around midnight the door opened, a security guard came in and banged the door shut – the first peak. After making sure everything was secure in the laboratory he went out banging the door shut again – the second peak. Probably it was the same security guard who used to visit me around one in the morning. My working hours used to be from nine in the night to two in the morning. He would remove his shoes and his belt heavy with holstered gun, put up his feet on the desk and rest for a while. He would tell me what was going on in the world, including the parking lot, appreciate my working hard all alone through the night, sympathize with my non-existent wife waiting for me and then move on. On the eighth day from my decision to give stability a last try, my friend found me in a state of absolute euphoria. I had solved the problem. It had taken quite a bit of complicated analysis of the messy equation. Misner did not believe at first that the stability problem had been solved. But, after being convinced, he pronounced that my thesis was in the bag[12] and went away to Cambridge. And I, on my part, goofed off for one whole year.

Apart from establishing the stability of the Schwarzschild black holes[13], the perturbation analysis had shown that the spacetime did not admit static perturbations that were regular at both the black hole and infinity. This was an indication that distorted static black holes could not exist in isolation. Nevertheless, it was startling to learn that Werner Israel had discovered the uniqueness of the Schwarzschild black hole[14]. There would be no potato-shaped black holes for instance. Nature had been robbed of her infinite variety. On the other hand, this clearly exhibited nature's simplicity. A static black hole could have only the shape of a sphere – the most perfect figure. After all, the philosopher Xenophanes, as early as in the sixth century BC, had declared that even God, being perfect, had to be spherical in shape!

Quasinormal modes

Halfway through the defense of my Ph D thesis, the examiner from the mathematics department asked the question, probably in a rhetorical vein, why one should bother to prove the stability of an object that was impossible to observe and was of doubtful existence in the first place. My thesis advisor did not like the question in the least especially coming from a mathematician. The rest of the examination ended up as a verbal battle between the two which I watched with great satisfaction. But, the question remained: how do you observe a solitary black hole? To me the answer seemed obvious. It had to be through scattering of radiation, provided the black hole left its fingerprint on the scattered wave. I remembered from my first-year graduate course in quantum mechanics, how the reflection coefficient displayed maxima and minima in a wave scattered from a square barrier. In the case of the black hole also, the scattering was from a barrier, although of a different shape. So I thought, I might discover maxima in reflection coefficient characteristic of the black hole. In order to carry out this calculation you needed a computer, since the radial equation had to be numerically integrated. The days of the PCs were far in the future.

I was working at the Institute for Space Studies in New York where we did enjoy some luxuries. One of them was chilled beer that was sold at a quarter a can during seminars. So much so, the listeners soaked up more alcohol than astrophysics. The other luxury was computer time which was quite dear and scarce at other places. In addition, we had the help of a numerical analyst and a computer programmer. The reflection coefficient did show maxima albeit extremely faint. I became highly excited. But, when the range of integration was increased, the maxima shifted to some other frequency region. After quite a bit of computer experimentation, I decided that these were spurious maxima produced by the abrupt cut off of the effective potential. My conjecture was that a completely smooth potential would not give rise to maxima in the scattering cross section. I consulted Regge and Wheeler when I gave a talk at Princeton in 1969 with the alliterative title 'Schwarzschild Surface as a Stable Scattering Centre'. It was just before my seminar that I heard for the first time the term 'black hole' newly coined by Wheeler, which he illustrated with a picture of automobile junkyard he drew on the blackboard. Regge and Wheeler both agreed that there was no theorem connecting the smoothness of the potential to the non-existence of maxima in the scattering cross section. I still do not know the answer.

Although the scattering of monochromatic waves did not show obvious characteristics of the black hole, I felt that scattering of wave packets might reveal the imprint of the black hole. So, I started pelting the black hole with Gaussian wave packets. If the wave packet was spatially wide, the scattered one was affected very little. It was like a big wave washing over a small pebble. But when the Gaussian became sharper, maxima and minima started emerging, finally levelling off to a set pattern when the width of the Gaussian became comparable to or less than the size of the black hole. The final outcome was a very characteristic decaying mode, to be christened later as the quasinormal mode. The whole experiment was extraordinarily exciting.

By the time the above work was published in *Nature*[15], I had moved to New York University. Chandra made a visit and gave a talk on ellipsoidal figures of rotating fluids. He was very much interested in my work on scattering and in the phenomenon of decaying modes. Later on he was to compute the quasinormal mode frequencies with Detweiler[16]. Many calculations in this direction would follow finally culminating in the accurate determination of the frequencies by Nils Andersson[17].

Quasinormal modes are generated in astrophysical scenarios such as gravitational collapse and coalescence of black holes. Ed Seidel has shown how well the fundamental mode matches the outcoming wave during the coalescence of binary black holes[18]. Recently Aguirregabiria and I have studied the sensitivity of the quasinormal modes to the scattering potential[19]. The motivation is to understand how any perturbing influence, such as another gravitating source, that might alter the effective potential would thereby affect the quasinormal modes. Interestingly, we find that the fundamental mode is, in general, insensitive to small changes in the potential, whereas the higher modes could alter drastically. The fundamental mode would therefore carry the imprint of the black hole, while higher modes might indicate the nature of the perturbing source.

Quasinormal modes are perhaps the rebuttal to the criticism of my thesis examiner regarding the nonobservability of black holes.

Ultracompact objects

One of the indirect offshoots of black hole research was the study of ultracompact objects or UCOs. While investigating the scattering of gravitational waves from the Schwarzschild black hole, I had noticed a peculiar phenomenon which I did not publish. Although at the time, the radial equation for even parity perturbations was quite complicated and was not in the Schrödinger form, it yielded exactly the same reflection coefficient as the odd parity perturbation for a given angular parameter l. One day I got a very excited telephone call from Chandra enquiring whether I knew this fact. I answered, yes, I did. Did I know why this happened? No, I did not. He had found the reason, he said triumphantly. He asked me for the numbers I had computed which I sent him. He went on to publish his interesting conditions under which two potentials lead to identical scattering cross sections mentioning my foreknowledge of the fact but not the reason.

It is the same story with neutrinos as well. The two equivalent potentials corresponding to the two helicities are quite different from each other[20], but lead to identical reflection coefficients. One of them has the peculiar feature in that it has a potential well in the region $r<3m$ attached to the usual potential barrier. This was terribly intriguing. Could there be a bound state in the potential well giving rise to some sort of neutrino trapping by the black hole? One can estimate the maximum number of bound states by integrating the potential over its spatial range. The well depth increases with the angular momentum quantum number and in the limit of its tending to infinity you get the answer one for the maximum number of possible bound states. In other words, there are no bound states.

I did not publish any of the above results. But out of it all another interesting question arose. Suppose you replaced the black hole by a spherical star of radius $r<3m$. Then the potential well would not only exist, but would also be deepened by the enhanced gravitation of the matter. Could there then be bound states trapping neutrinos within the star? Ajit Kembhavi and I worked on this problem, found and computed the complex frequencies corresponding to the bound states of the neutrinos[21]. In a way, these neutrino bound states with complex frequencies were forerunners of the quasinormal modes of ultracompact stars worked out by Chandrasekhar and Ferrari[22] as has been pointed out by Andersson[23]. It is a very happy feeling that some of the problems I had worked on interested Chandra also.

Ultracompact objects with radius $r<3m$ are in fact quite interesting entities. In principle, trapping of mass less particles in their potential well is possible. Or the object can oscillate in its quasinormal modes. Van Paradijs[24] has pointed out the peculiar behaviour of redshift for $r<3m$. Recently, Abramowicz and Prasanna[25] have discussed the reversal of centrifugal force at $r=3m$ for which you need a black hole or a UCO.

But, do such highly compact objects or stars with radius $r<3m$ exist in nature? This question was considered by Dhurandhar, Iyer and myself[26,27] and we also coined the name 'Ultra-Compact Objects' or 'UCOs'. By studying very carefully the general relativistic stellar models with different equations of state we established that, as a matter of fact, stable ultracompact objects can exist in nature.

Gyroscopic precession and inertial forces

We discussed earlier how the black hole structure can change dramatically when going from static to stationary spacetimes on account of the rotation inherent to the latter. The study of Killing trajectories in these spacetimes led to a covariant description of gyroscopic precession via the Frenet–Serret formalism. Precession is an important phenomenon. For instance, the Earth precesses. Ancient astronomers knew this. Astrologers did not, thereby making predictions that were doubly wrong. Or, can two wrongs add upto one right? In atomic physics, Thomas precession, a manifestation of special theory of relativity, played a crucial role. Spacetime curvature gives rise to Fokker–De Sitter precession in the Schwarzschild spacetime. The spin of Kerr black hole contributes additional precessional effects. All these can be studied elegantly using the Frenet–Serret description[6].

Another related area concerns the general relativistic analogues of inertial forces as developed by Abromowicz and coworkers[28]. A particle at rest in a static spacetime experiences only the gravitational force, but is acted upon by the centrifugal force as well if it is moving uniformly in a circular orbit. In a stationary spacetime, there is an additional force, the Coriolis–Lense–Thirring force, which arises as a consequence of the metric components mixing space and time. In static spacetimes, such as the Schwarzschild spacetime, centrifugal force reverses at the circular photon orbit[25]. So does gyroscopic precession. The situation is far more complicated in stationary spacetimes[29,30]. My young colleague Rajesh Nayak and I have studied these effects and established covariant connections between gyroscopic precession on the one hand and inertial forces on the other[31–33]. These considerations should be of interest in black hole physics from a conceptual point of view as well as for astrophysical applications.

The trail goes on . . .

I have tried to offer a glimpse, just a fleeting one at that, of my personal journey along the black hole trail. It has been a long journey spanning some three decades. There have been all sorts of ups and downs along the way. For instance, I have had my share of tussle with journals and referees. My very first paper[10], the one with Edelstein, was unceremoniously rejected as nothing more than a bunch of formulae. Misner had to write a strong letter pointing out that the same journal that had previously published the wrong equations was now rejecting the correct ones. The paper on the structure of black holes[2] was also rejected as it was considered to be just mathematics and had to be published in the *Journal of Mathematical Physics*. The stability paper[13] too had to cross some hurdles before seeing the light of the day. As with any important field, black hole physics has had its sociological factors sometimes leading to, among other things, inadequate recognition of significant contributions. All this becomes trivial in comparison to the exhilarating experience of exploration. It is a rare good fortune to have been trekking along the track right from the beginning. To have watched the seed germinate, the sapling sprout and the tree grow. It is also a good fortune to have had the company of congenial co-travellers on the journey – marvellous friends to work with and keen minds to lead the way. If sometimes you stray away from the road, you keep coming back. Even now my colleagues and I are working on different aspects of black hole physics, such as quasinormal modes, rotational effects and black holes in cosmological backgrounds.

What is the most important lesson I have learnt having traversed the trail for so long? Let me answer that question by quoting the Spanish poet Antonio Machado, who wrote:

Caminante, no hay camino
Se hace camino al andar.

Traveller, there is no path,
Paths are made by walking.

It is gratifying, to feel that you have made a path however short, however narrow that has helped build a trail that was planned and paved by so many. It has been a joy to follow that trail. And I hope the trail will never end.

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