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PROBING THE EVOLUTION OF THE GAS MASS FRACTION WITH THE SUNYAEV-ZELDOVICH EFFECT

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ABSTRACT

Study of the primary anisotropies of the cosmic microwave background (CMB) can be used to determine the cosmological parameters to a very high precision. The power spectrum of the secondary CMB anisotropies due to the thermal Sunyaev-Zeldovich effect (SZE) by clusters of galaxies can then be studied, to constrain more cluster-specific properties (like gas mass). We show the SZE power spectrum from clusters to be a sensitive probe of any possible evolution (or constancy) of the gas mass fraction. The position of the peak of the SZE power spectrum is a strong discriminatory signature of different gas mass fraction evolution models. For example, for a flat universe, there can be a difference in the l-values (of the peak) of *as much as 2500* between a constant gas mass fraction model and an evolutionary one. Moreover, observational determination of the power spectrum, from blank-sky surveys, is devoid of any selection effects that can possibly affect targeted X-ray or radio studies of gas mass fractions in galaxy clusters.

Subject headings: cosmic microwave background — cosmology: observations — cosmology: theory — galaxies: clusters: general

1. INTRODUCTION

Clusters of galaxies are expected to contain a significant amount of baryons in the universe. From observational estimates of their total mass M_T and the gas mass M_g, the gas mass fraction ($f_g = M_g/M_T$) is obtained. These estimates can be used as probes of the underlying cosmological models. For example, the cluster f_g would give a lower limit to the universal baryon fraction Ω_b/Ω_m. Determination of f_g has been done by numerous people (Mohr, Mathiesen, & Evrard 1999; Sadat & Blanchard 2001), and the values are in agreement within the observational scatter. A point to be noted here is that the estimated f_g depends on the distance to the cluster (i.e., $f_g \propto d_{\rm ang}^{3/2}$). Hence, if f_g is assumed to be constant, then, in principle, one can use the "apparent" evolution of f_g over a large redshift range to constrain cosmological models (Sasaki 1996).

The question as to whether there is any evolution of the gas mass fraction, however, is still debatable, with claims made either way. For example, Schindler (1999) has investigated a sample of distant clusters with redshifts between 0.3 and 1 and conclude that there is no evolution of the gas mass fraction. A similar conclusion has been drawn by Grego et al. (2001). On the contrary, Ettori & Fabian (1999) have looked at 36 high-luminosity clusters and find evolution in their gas mass fractions (see also David, Jones, & Forman 1995; Tsuru et al. 1997; Allen & Fabian 1998; Mohr et al. 1999). Observations suggest that, although the f_g of massive clusters ($T_e \gtrsim 5$ KeV) appears to be constant, low-mass clusters may have lost gas as a result of preheating and/or post-collapse energy input (David et al. 1990, 1995; Ponman et al. 1996; Bialek, Evrard, & Mohr 2000). It is also well known that the intracluster medium (ICM) is not entirely primordial and there is probably continuous in fall of gas, thereby increasing f_g with time.

The ICM has been probed mainly through X-ray observations but also through the so-called Sunyaev-Zeldovich effect (SZE; Zeldovich & Sunyaev 1969) in the last decade (see Birkinshaw 1999). The SZE from clusters is a spectral distortion of the cosmic microwave background (CMB) photons due to inverse Compton scattering by the hot ICM electrons, with its magnitude proportional to the Compton y-parameter, given by $y = (k_B\sigma_T/m_e c^2)\int n_e T_e\, dl$. Here k_B is the Boltzmann constant, σ_T is the Thomson scattering cross section, m_e is the electron mass, and n_e and T_e are the ICM electron density and temperature. Using SZE measurements, f_g has been obtained for a number of clusters (Grego et al. 2001). Since SZE does not suffer from the $(1+z)^{-4}$ "cosmological dimming," one can use SZE as an useful probe of the evolution of the cluster gas mass fraction.

Other than the targeted SZE observations, nontargeted "blank-sky" surveys of SZE are one of the main aims of future satellite and ground-based small angular scale observations (Aghanim et al. 1997; Holder & Carlstrom 1999). Once the power spectrum is extracted from observations, a comparison can be made with theory, to constrain cosmological parameters and relevant cluster scale physics. The SZE power spectrum as a cosmological probe has been well studied (Atrio-Barandela & Mücket 1999; Refregier et al. 2000; da Silva et al. 2000), although its use as a probe of the ICM has seldom been looked at.

Keeping such surveys in mind, in this Letter, we look at the SZE power spectrum as a probe of the ICM. We show it to be a very sensitive probe of the evolution of f_g. Measurements of the primary anisotropy would give us "precise" values of cosmological parameters (like h, Ω_m, Ω_Λ, Ω_b). Hence, for our calculations, we assume that we know these values and do not worry about their effect on the SZE power spectrum. Any feature of the SZE power spectrum is, then, attributed to specific cluster physics (like gas content).

Current observations of primary CMB anisotropies suggest a flat universe with a cosmological constant (Padmanabhan & Sethi 2000). For our calculations, we take a flat universe with $\Omega_m = 0.35$, $\Omega_b = 0.05$, and $h = 0.65$ as our fiducial model.

This Letter is structured as follows. In § 2, we discuss the distribution of clusters and model the cluster parameters. In § 3, we compute the Poisson and clustering power spectrum from the SZE, and finally, we discuss our results and conclude in § 4.

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2. THE SUNYAEV-ZELDOVICH EFFECT POWER SPECTRUM

2.1. *Distributing the Galaxy Clusters*

We set up an ensemble of galaxy clusters with masses of $10^{13}\ M_\odot \le M \le 10^{16}\ M_\odot$, using the abundance of collapsed objects as predicted by a modified version of the Press-Schechter mass function (Press & Schechter 1974) given by Sheth & Tormen (1999). The lower mass cutoff signifies the mass for which one expects a well-developed ICM. The gas mass is supposed to sit in the halo potential and is distributed in the same manner. We probe up to redshifts of 5. Most of the power, however, comes from objects distributed at $z \lesssim 1$.

We use the transfer function of Bardeen et al. (1986), with the shape parameter given by Sugiyama (1995) and the Harrison-Zeldovich primordial spectrum, to calculate the matter power spectrum $P_m(k)$. The resulting *COBE* Differential Microwave Radiometer normalized (Bunn & White 1997) mass variance (σ_8) is 0.9 for our fiducial model.

2.2. *Modeling the Cluster Gas*

We closely follow Colafrancesco & Vittorio (1994) in our modeling. We have, for the gas density, $n_e(r) = n_{e,0}[1 + (r^2/r_c^2)]^{-3\beta/2}$, with $\beta = \frac{2}{3}$. We take the gas to be extended up to $R_v = pr_c$, with $p = 10$. The central gas density, $n_{e,0}$ is given by $n_{e,0} = f_g[2\rho_0/m_p(1 + X)]$, where $X = 0.76$ is the average proton mass fraction and ρ_0 is the central gas mass density. To account for the fact that there is a final cutoff in the gas distribution, we introduce a Gaussian filter at the cluster edge R_v given by $n_e(r) \to n_e(r)e^{-r^2/\xi R_v^2}$, where $\xi = 4/\pi$ is the fudge factor.

We parametrize the gas mass fraction as

$$f_g = f_{g0}(1 + z)^{-s}\left(\frac{M}{10^{15}\ h^{-1}\ M_\odot}\right)^k, \qquad (1)$$

where the normalization is taken to be $f_{g0} = 0.15$, based on local rich clusters. We look at combinations of both mass and redshift dependence for a range of evolutionary models (see § 4), inspired from the literature.

For the core radius r_c and the temperature, we use

$$r_c(\Omega_0, M, z) = \frac{1.69\ h^{-1}\ \mathrm{Mpc}}{p}\frac{1}{1 + z}\left(\frac{M}{10^{15}\ h^{-1}\ M_\odot}\frac{178}{\Omega_0\Delta_c}\right)^{1/3}, \qquad (2)$$

$$k_B T_e = 7.76\beta^{-1}\left(\frac{M}{10^{15}\ h^{-1}\ M_\odot}\right)^{2/3}(1 + z)\ \mathrm{keV}. \qquad (3)$$

Here $\Delta_c(z)$ is the cluster overdensity relative to the background and $\beta = 0.67$.

Putting everything in, we have the change in spectral intensity to be $\Delta I = i_0 g(x)y(\theta)$, with $i_0 = 2(k_B T_{\mathrm{cmb}})^3/(hc)^2$ and

$$y(\theta) = \frac{\sigma_T n_{e0} r_c k_B T_e}{m_e c^2}\frac{\pi e^{1/\xi p^2}}{\sqrt{1 + (\theta/\theta_c)^2}}\,\mathrm{erfc}\sqrt{\frac{1 + (\theta/\theta_c)^2}{\xi p^2}}. \qquad (4)$$

The angular core radius $\theta_c = r_c/d_{\mathrm{ang}}$. The spectral dependence of temperature anisotropy from the thermal SZE is given by

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2}[x \coth(x/2) - 4], \qquad (5)$$

where $x = h\nu/k_B T_{\mathrm{cmb}}$. This specific spectral dependence can be used to separate it out from other CMB anisotropies (Cooray, Hu, & Tegmark 2000).

3. COMPUTING THE POWER SPECTRUM

The fluctuations of the CMB temperature produced by the SZE can be quantified by their spherical harmonic coefficients a_{lm}, which can be defined as $\Delta T(\boldsymbol{n}) = T_0^{-1}\sum_{lm} a_{lm}Y_{lm}(\boldsymbol{n})$. The angular power spectrum of the SZE is then given by $C_l = \langle |a_{lm}|^2\rangle$, the brackets denoting an ensemble average. The power spectrum for the Poisson distribution of objects can then be written as (Cole & Kaiser 1988; Peebles 1980)

$$C_l^{\mathrm{Poisson}} = \int_0^{z_{\max}} dz\ \frac{dV(z)}{dz} \times \int_{M_{\min}}^{M_{\max}} dM\ \frac{dn(M, z)}{dM}\,|y_l(M, z)|^2, \qquad (6)$$

where $V(z)$ is the comoving volume and dn/dM is the number density of objects and y_l is the angular Fourier transform of $y(\theta)$ (see Molnar & Birkinshaw 2000).

In addition to Poisson power spectra, one would expect contribution to a "correlation power spectrum" from the clustering of the galaxy clusters. Following Komatsu & Kitayama (1999), we estimate the clustering angular power spectrum as

$$C_l^{\mathrm{Clustering}} = \int_0^{z_{\max}} dz\ \frac{dV(z)}{dz}P_m \times \left[\int_{M_{\min}}^{M_{\max}} dM\ \frac{dn(M, z)}{dM}b(M, z)y_l(M, z)\right]^2, \qquad (7)$$

where $b(M, z)$ is the time-dependent linear bias factor. The matter power spectrum, $P_m(k, z)$, is related to the power spectrum of cluster correlation function $P_c(k, M1, M2, z)$ through the bias, i.e., $P_c(k, M1, M2, z) = b(M1, z)b(M2, z)D^2(z)P_m(k, z = 0)$, where we adopt $b(M, z) = (1 + 0.5/\nu^4)^{0.06-0.02n}[1 + (\nu^2 - 1)/\delta_c]$(see Jing 1999 for details). In the above equation, $D(z)$ is the linear growth factor of density fluctuation, $\delta_c = 1.68$ and $\nu = \delta_c/\sigma(M)$.

4. RESULTS AND DISCUSSION

In our study, we assume two things: (1) a "precise" and "a priori" knowledge of the cosmological parameters and (2) that in the l-range of relevance, other secondary anisotropies are either smaller in strength or have different spectral dependence (Aghanim, Balland, & Silk 2000; Majumdar, Nath, & Chiba 2001) than the thermal SZE.

We have plotted the Poisson SZE power spectrum in Figure 1 (*left*). Clearly, the primary feature distinguishing a nonevolutionary constant f_g model from an evolutionary one is the position of the peak. The model with a constant f_g peaks at a higher l-value and also has greater power. The constant f_g model peaks at $l \sim 4000$. This result is in agreement with that of Komatsu & Kitayama (1999). If one assumes that there is no

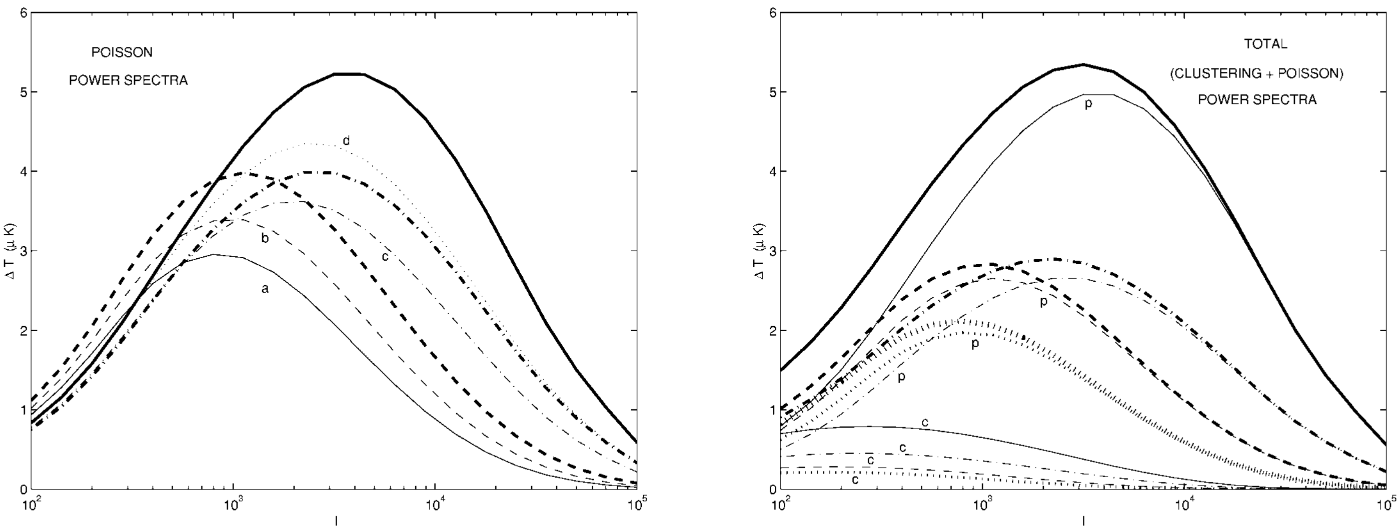

FIG. 1.—*Left*: Poisson power spectra due to SZE from galaxy clusters for different f_g-models. The thick solid line corresponds to a constant f_g model, the thick dashed line has no evolution with redshift, and the thick dash-dotted line has no evolution with total mass. The thin lines are for the cases (a) $k = 0.5$, $s = 1$; (b) $k = 0.5$, $s = 0.5$; (c) $k = 0.1$, $s = 0.5$; and (d) $k = 0.1$, $s = 0.1$. *Right*: Total power spectra (*thick lines*) and the corresponding Poisson (labeled "p") and clustering spectra (labeled "c"). The solid lines correspond to a constant f_g, the dashed lines to no evolution with redshift, the dash-dotted lines to no evolution with mass, and the dotted lines are for the case $k = 0.5$, $s = 1$.

evolution of f_g with redshift (i.e., $s = 0$, $k = 0.5$), the peak is at $l \sim 1100$, whereas in the case of no dependence on mass ($k = 0$, $s = 0.5$), the peak is at $l \sim 2500$. Based on Extended Medium-Sensitivity Survey data (David et al. 1990), Colafrancesco & Vittorio (1994; see also Molnar & Birkinshaw 2000) model f_g with $k = 0.5$ and $s = 1$. For this case, we see that the turnover is at a very low $l \sim 900$. Assuming a mild evolution ($k = 0.1$, $s = 0.1$), we get the peak at $l \sim 2100$. We also show results for ($k = 0.5$, $s = 0.5$) and ($k = 0.1$, $s = 0.5$). The last parametrization is based on the recent analysis of *ROSAT* data by Ettori & Fabian (1999). It is evident that the difference, in the l-value of the peak of the constant f_g scenario from an evolutionary one, can range between $l \sim 1500$ and 3200. Thus, the position of the peak strongly discriminates any evolution of f_g.

It is easy to understand the shift in the peak of the SZE power spectrum. Let us consider the case $s = 0$, i.e., f_g depends only on total mass. From equation (1), this means an enhanced reduction of f_g of smaller mass clusters relative to the larger masses and so a reduction of power at larger l (or smaller θ). Hence, the peak shifts to a lower l. For the case $k = 0$ (i.e., only redshift dependence), we now have structures at high z contributing less to the power (than without a redshift dependence). Since less massive structures are more abundant at high z, this negative dependence of f_g on redshift cuts off their contribution. Hence, once again there is less power at high l and the peak shifts to a lower l-value. The parametrization of equation (1) affects the larger masses less, as evident from almost equal power seen at $l \lesssim 600$ for all models.

We note that these results are irrespective of the arguments given (see Rines et al. 1999) to explain any possible evolution of f_g that *assume* f_g to be constant and relies on the cosmology to change the angular diameter distance, so that there is an "apparent" change in f_g. In their case, if there is actually even a slight evolution of f_g, then one can still account for it with a nonevolutionary model by simply changing the cosmological parameters. Our method *does not assume a priori* any constancy (or evolution) of f_g and tries to look for it. This also has the added advantage of being devoid of uncertainties that can creep in through "selection biases" in estimating the f_g using pointed studies of X-ray–selected galaxy clusters.

It may be possible to measure the power spectrum of SZE with the ongoing and future high angular resolution CMB observations. In principle, observations with the Sunyaev-Zeldovich Infrared Experiment, the Owens Valley Radio Observatory, the Berkeley-Illinois-Maryland Association array, and the Australia Telescope Compact Array can probe the

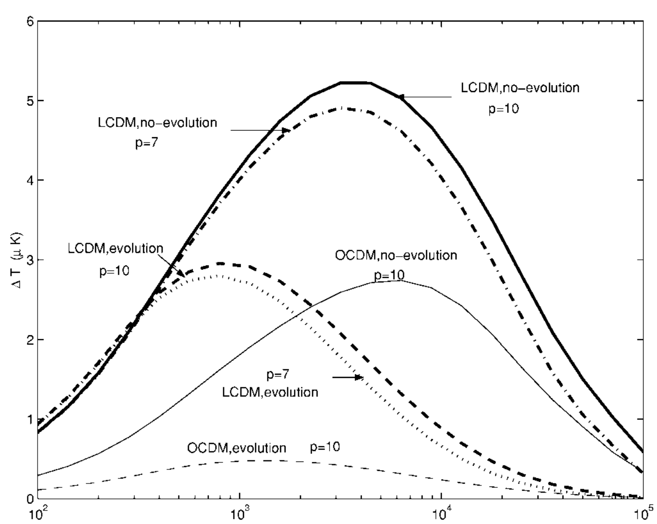

FIG. 2.—Poisson SZE power spectra plotted for different cosmologies and with different extensions of the gas mass. The solid lines are for a ΛCDM cosmology, with $\Omega_m = 0.35$, $\Omega_\Lambda = 0.65$, and $h = 0.65$, and the thin lines are for an open cold dark matter (OCDM) cosmology with $\Omega_m = 0.35$, $h = 0.65$. The OCDM lines have been multiplied by a factor of 10 in the plot. The solid and dashed lines are for gas mass extending up to $10r_c$, whereas the dash-dotted and the dotted lines are for extension up to $7r_c$.

range in l from ≈1000 to 7000 and a frequency range of ≈2–350 GHz. The SZE power spectrum would also be measured with increased precision by the proposed Atacama Large Millimeter Array and the Array for Microwave Background Anisotropy (which is geared for blank-sky surveys).

The observations will, however, measure the total power spectrum given by the sum of the Poisson and the clustering power spectrum. In Figure 1 (*right*), we show the total power spectrum for four parameter sets along with their Poisson as well as clustering power spectrum. Since for clustering, the peak depends mainly on the average intercluster separation, which is fixed once the cosmology is fixed, there is no appreciable spread of the clustering peaks in l-space. However, as is evident from the plot, the clustering power spectra are far smaller than the corresponding Poisson spectra, and hence addition of the clustering spectrum to the Poisson spectrum does not shift the peaks significantly. Thus, for the total power spectra, the difference in the l-value of the peak of the constant f_g scenario from an evolutionary one can lie in the range $l \sim 1100$–2500. The peak position thus remains a strong discriminatory signature of any evolution in f_g.

Finally, let us comment on the validity and robustness of our results. In Figure 2, we show results for an open universe ($\Omega_0 = 0.35$, $h = 0.65$). It is clearly seen that the difference in the peak position of constant f_g and evolutionary models remains far apart (in fact, for the same set of parameters the difference increases). It is seen that the turnover of the SZE power spectrum is insensitive to the mass cutoff, since the main contribution to the anisotropy comes from clusters with $10^{14}\ M_\odot < M < 10^{15}\ M_\odot$. We also indicate the effect of having a more compact gas distribution with $p = 7$. We see that shifts in the peaks are negligible (although the height is reduced a little). The use of a single β to model the full gas distribution introduces little error. This is because the major contribution to the anisotropy comes from around the core region, and increasing β slightly decreases the overall distortion, without affecting the peak. Also, a modified *M-T* relation (more suitable for ΛCDM) does not change the conclusions of this Letter (although amplitude of distortion slightly changes). For a more detailed analysis, however, one should take better observationally supported gas density and temperature profiles (see Yoshikawa & Suto 1999). These points will be discussed in greater detail in a future publication.

In conclusion, we have computed the angular power spectrum of SZE from clusters of galaxies. We have shown the position of the peak of the power spectrum to bear a *strong discriminatory signature* of different f_g evolution models. One of the goals of arcminute-scale observations of the CMB anisotropy is to measure the SZE power spectrum from blank-sky surveys. Such observational results can be used to constrain f_g-models. Our method, thus, provides a powerful probe of evolution (or constancy) of the gas mass fraction and can potentially resolve the decade-long debate.

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