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Scattering of light by periodic structure with randomness

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Light scattering by a rough periodic surface is considered with the aim to extract the periodic part from the roughness. In the scattering geometry (figure 1), the elevation at any point is given by $\varsigma(x,y)=a\cos Qx + \delta\varsigma(x,y)$, where $\delta\varsigma(x,y)$ is a zero mean Gaussian stationary random variable with $<\delta\varsigma(0)\delta\varsigma(r)>=\sigma^2\exp((-r/l)^{\theta})$. The intensity for such a composite rough surface is given in reference (Chatterjee, 2000). Defining the wave vectors of scattering as: $v_x = k(\sin\theta_1 - \sin\theta_2\cos\theta_3)$, $v_y = -k\sin\theta_2\sin\theta_3$, $v_z = -k(\cos\theta_1 + \cos\theta_2)$, $v_{xy}^2 = v_x^2 + v_y^2$ where $k=2\pi/\lambda$, λ being the wavelength of light, following reference (Beckmann and Spizzichino, 1963, Beckmann, 1965, 1967) we define the ratio, $<\rho\rho^*>$ = Intensity of light scattered by the rough surface in the direction (θ_2,θ_3)/Intensity of light scattered by a smooth surface in the specular direction (i.e. $v_x=0=v_y$) and present below the signatures of periodicity in the $<\rho\rho^*>$.

We find (Chatterjee, 2000)$<\rho\rho^*>=J_0^2(av_z)f(v_x)+\Sigma J_n^2(av_z)\{f(v_x+nQ)+f(v_x-nQ)\}$ where the f's are determined by the randomness. For low randomness $<\rho\rho^*>$ shows distinct sharp peaks at $v_x = \pm nQ$. The peaks are smeared for large randomness since the f's are so wide that the peaks overlap and hence cannot be distinguished. We show that even in such cases the distinct peaks can be recovered, if one tries to show that the above $<\rho\rho^*>$ is similar to a train of pulses of identical shapes but shifted by $\pm nQ$ with Raman Nath type amplitudes $\propto J_n^2$. This method can thus be likened to a matched filtering in signal detection, involving joint fit to the f's as also to the J_n^2's. First $<\rho\rho^*>$ is differentiated (figure 2) with the derivatives indicating the hidden structures in the scattered intensity. We next find the "best fit" $f(v_x)$ to be the one for which $Z(v_x)\equiv[<\rho(v_x)\rho^*(v_x)>/<\rho(0)\rho^*(0)> - f(v_x)]$ has two peaks on both sides of the origin but of shape same as that of $f(v_x)$ (figure 3). The positions of these peaks are identified as $\pm Q$. By comparing the heights of these peaks one can extract the ratio of the amplitudes:$(J_1/J_0)^2$. The method is illustrated in figures 2-3 that follow. We have first computationally generated the data by choosing a=330 Å, $Q=1.0057\mu^{-1}$, σ=1600Å, l=5 μ, λ = 6328Å and θ = 1.0. We find that by our matched filter method, in the first step itself the best fit values give Q within 11%

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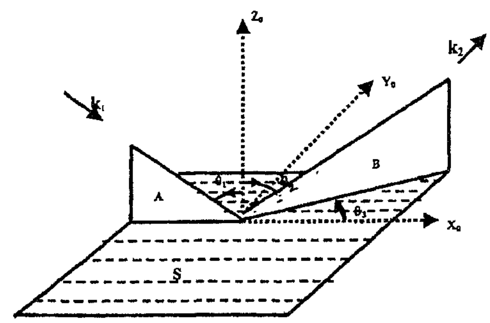

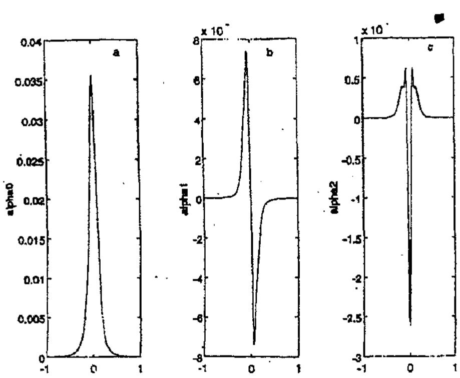

Figure 1. The scattering geometry.

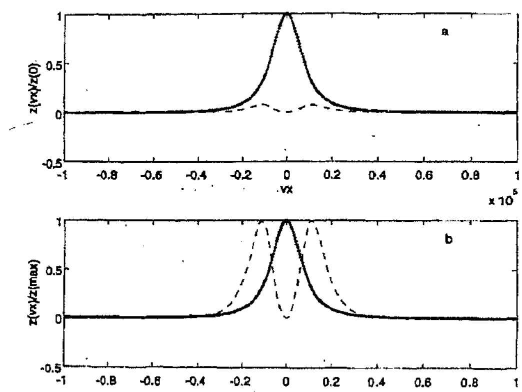

Figure 2. Plots of (a) $\alpha_0=\langle\rho\rho^*\rangle/\langle\langle\rho\rho^*\rangle_0$
(b) $\alpha_1=(\partial\langle\rho\rho^*\rangle/\partial v_x)\langle\rho\rho^*\rangle_0$
(c) $\alpha_2=(\partial^2\langle\rho\rho^*\rangle/\partial v_x^2)\langle\rho\rho^*\rangle_0$

Figure 3. (a) Plot of $Z(v_x)/Z(0)$
(b) Plot of $Z(v_x)/Z_m$, (side lobes) and comparison with $f(v_x)$ (centre)

and "a" within 12%. The details of the method will be published in a separate communication and improvement of the method promises to give higher accuracy (Chatterjee and Vani, 2002). For astronomical seeing, $\theta = 5/3$ and the peaks are sharper than for $\theta = 1.0$ case, making the detection easier. A comparison of this method with Bayesian analysis will be reported soon.

References

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