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Why Does the Sun Have Kilogauss Magnetic Fields?

S.S. Hasan[1] and A.A. van Ballegooijen

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

Abstract:

Magnetic fields in the solar photosphere are concentrated in flux tubes with kilogauss field strength surrounded by nearly field-free plasma. Observations show that the flux tubes are located in convective downdrafts where the temperature is lower than average. We assume that the convective downdrafts extend to large depths in the convection zone, and that flux tubes follow the downdrafts to these depths. We develop a model for the magnetic field strength $B(z)$ in the flux tubes as a function of depth z below the surface. Our calculations reveal that ϵ, the ratio of magnetic pressure to gas pressure, has a large depth variation: at the base of the convection zone where $\epsilon \sim 10^{-5}$ ($B \sim 10^5$ G), while at the top $\epsilon \sim 1$, in broad agreement with solar observations. Thus the model can explain why the field strength at the photosphere is around 1 kG.

1. Introduction

It is generally accepted that the small-scale magnetic field at the solar surface is structured in the form of vertical magnetic flux tubes with field strengths around 1500 G and diameters of about 100 km (see the reviews by Stenflo 1989 and Solanki 1993). Various observations have determined that flux tubes occur preferentially at the intergranule boundaries, which are also the sites of downflows (e.g., Muller & Roudier 1992; Berger & Title 1996). This suggests that flux tubes are surrounded by plumes of down flowing material, in which the temperature is lower than the average surrounding temperature. We find that this has interesting consequences for the depth variation of the magnetic field.

In the present investigation we develop a theoretical model for the equilibrium structure of a vertical flux tube by including the presence of downflowing plumes just outside the flux tube. Physically, the Reynolds stresses associated with the downflowing gas, effectively increase the external pressure, leading to a larger field strength (on account of horizontal pressure balance). This model is based on earlier work by van Ballegooijen (1984) where the stresses for turbulent flows in the convection zone were estimated using mixing length theory. It was found that the field strength in the photosphere could be determined by specifying its value at the base of the convection zone. The calculations showed that the surface value was insensitive to the choice of strength at the base of the convection zone. However, the surface value calculated using the above model

[1]Permanent address: Indian Institute of Astrophysics, Koramangala, Bangalore–560034, India

turned out to be smaller by at least a factor of 2 than that suggested by observations. In the present investigation, we re-examine this question by using numerical simulations to compute the Reynold's stresses just below the photosphere. This refinement leads to a much better fit of the computed field strength at the surface with observations.

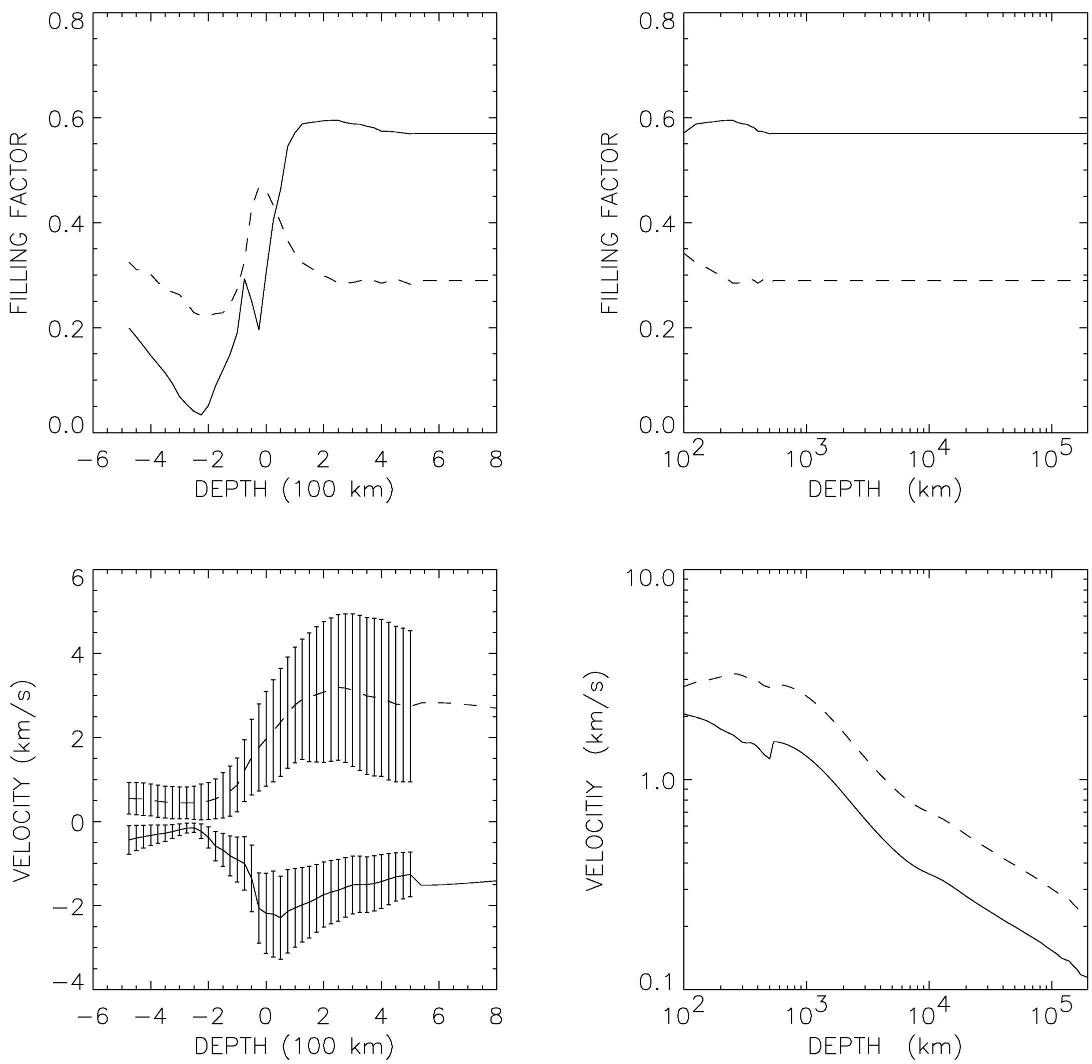

Figure 1. Average filling factors and vertical velocities of upflows and downflows in the solar convection zone, as derived from a 3-D simulation of solar granulation by Stein & Nordlund (1989) and a simple two-component model for the deeper layers. *Full curves*: upflows, *dashed curves*: downflows. The top panels show the filling factor for the surface layers (left) and for the deeper layer (right). The bottom panels show the vertical velocity (positive for downflows). The error bars denote the 1σ variation of vertical velocity as derived from the simulation.

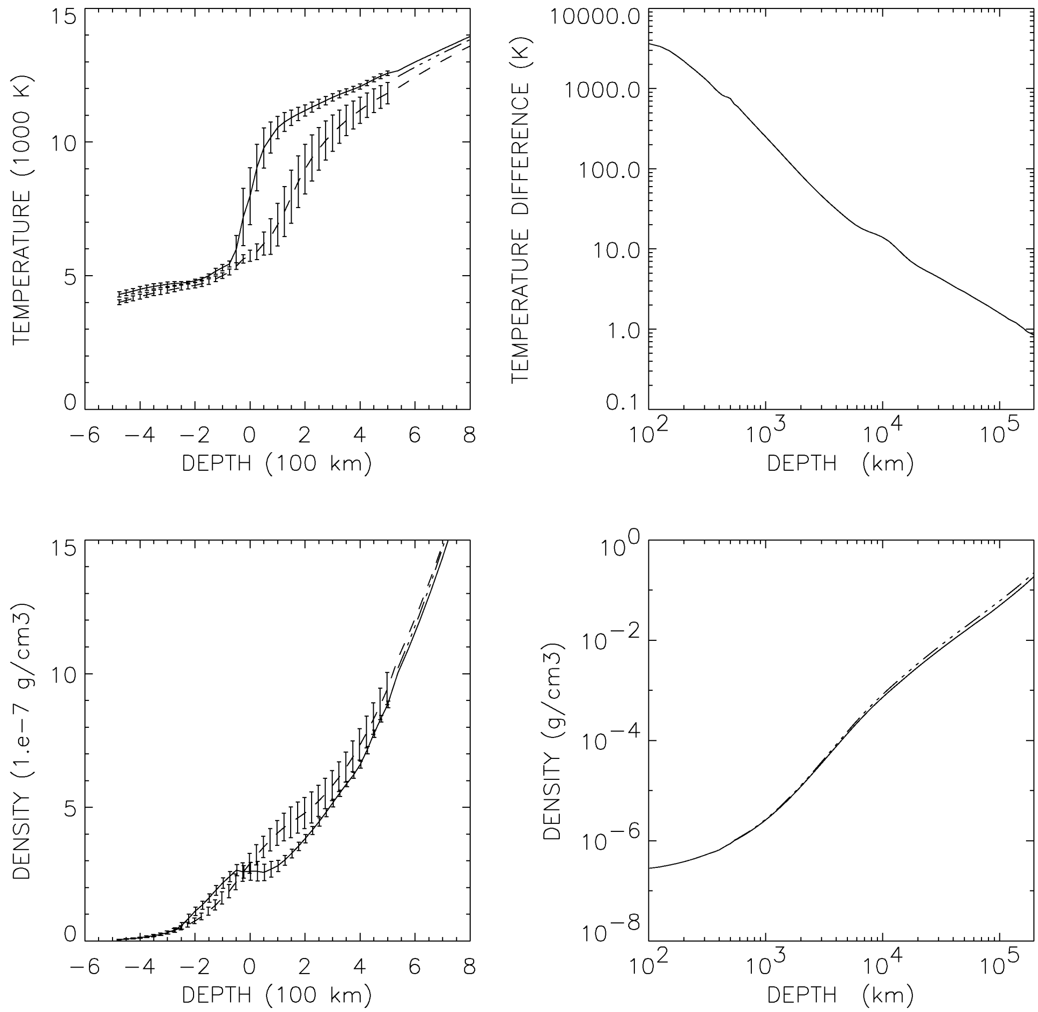

Figure 2. Temperatures and densities of upflows and downflows in the solar convection zone. *Full curves*: upflows, *dashed curves*: downflows. The error bars denote the 1σ variation of temperature and density as derived from the simulation by Stein & Nordlund (1989). The upper right panel shows the temperature difference between upflow and downflow at larger depth. The dash-dotted curves show the mean temperature and density from Spruit's (1977) mixing-length model.

2. Convection Zone Model

We use the results from a 3-D simulation of solar granulation by Stein & Nordlund (1989) to extract mean temperatures, densities, gas pressures and vertical velocities as functions of depth up to a depth of 500 km below the photosphere. These quantities are computed separately for *upflow* and *downflow* regions. We also derive area filling factors $f_u(z)$ and $f_d(z)$. Here upflows are defined as regions where the vertical velocity is upward and the temperature is larger than average; downflows are regions with downward velocity and lower-than-average temperatures. In general the filling factors do not add up to unity. The results of this analysis are shown in Figures 1 and 2. In order to improve the match at z=500 km we increased the pressures and densities from the simulation by 10%.

For the deeper layers ($z > 500$ km) we use a two component model of convection consisting of upflows and downflows. The average temperature $T(z)$, mean molecular weight $\mu(z)$, and specific heat $c_p(z)$ are taken from the mixing length model of Spruit (1977). The parameters of the upflows and downflows are computed from the following equations for conservation of mass, momentum and energy:

$$f_u \rho_u v_u + f_d \rho_d v_d = 0, \tag{1}$$

$$\frac{d}{dz}\left(p_u + \rho_u v_u^2\right) = \rho_u g, \tag{2}$$

$$\frac{d}{dz}\left(p_d + \rho_d v_d^2\right) = \rho_d g, \tag{3}$$

$$F_{conv} = f_u \rho_u v_u \left(c_p \Delta T_u + {}^1\!/_2 v_u^2\right) + f_d \rho_d v_d \left(c_p \Delta T_d + {}^1\!/_2 v_d^2\right), \tag{4}$$

where f_u and f_d are the area filling factors of the upflows and downflows, $\rho_u(z)$ and $\rho_d(z)$ are the densities, $v_u(z)$ and $v_d(z)$ are the vertical velocities, $p_u(z)$ and $p_d(z)$ are the gas pressures, $\Delta T_u(z)$ and $\Delta T_d(z)$ are the temperature differences relative to the mean temperature $T(z)$, and $g(z)$ is the acceleration of gravity. The terms $\rho_u v_u^2$ and $\rho_d v_d^2$ describe the deviations from hydrostatic equilibrium. For simplicity we assume that the upflows and downflows have the same gas pressure, so that $p_u(z) = p_d(z)$, and furthermore we use the ideal gas law. The above equations are solved numerically by integrating from $z = 500$ km to larger depth. The results are shown in Figures 1 and 2.

3. Flux Tube Model

We work within the framework of the thin flux tube model, since the diameter of the tube is typically less than the local pressure scale height. Let us consider a vertical flux tube surrounded by a plume of downflowing plasma. We neglect the presence of flows within the flux tube and assume that the tube interior is in hydrostatic equilibrium, so that

$$\frac{dp_i}{dz} = \rho_i g, \tag{5}$$

where p_i and ρ_i are the internal gas pressure and density. Horizontal pressure balance leads to,

$$p_i + \frac{B^2}{8\pi} = p_d. \tag{6}$$

Furthermore, let us assume that the flux tube is in thermal equilibrium with its surroundings, so that

$$T_i(z) = T_d(z). \tag{7}$$

We define the ratio of magnetic pressure to gas pressure:

$$\epsilon(z) \equiv \frac{B^2}{8\pi p_i}. \tag{8}$$

Subtracting (5) from (3), we obtain the following differential equation:

$$\frac{d}{dz}\left[\ln\left(1+\epsilon\right)\right] = -\frac{1}{p_d}\frac{d}{dz}\left(\rho_d v_d^2\right). \tag{9}$$

We solve this equation for $\epsilon(z)$ by integrating upward from the base of the

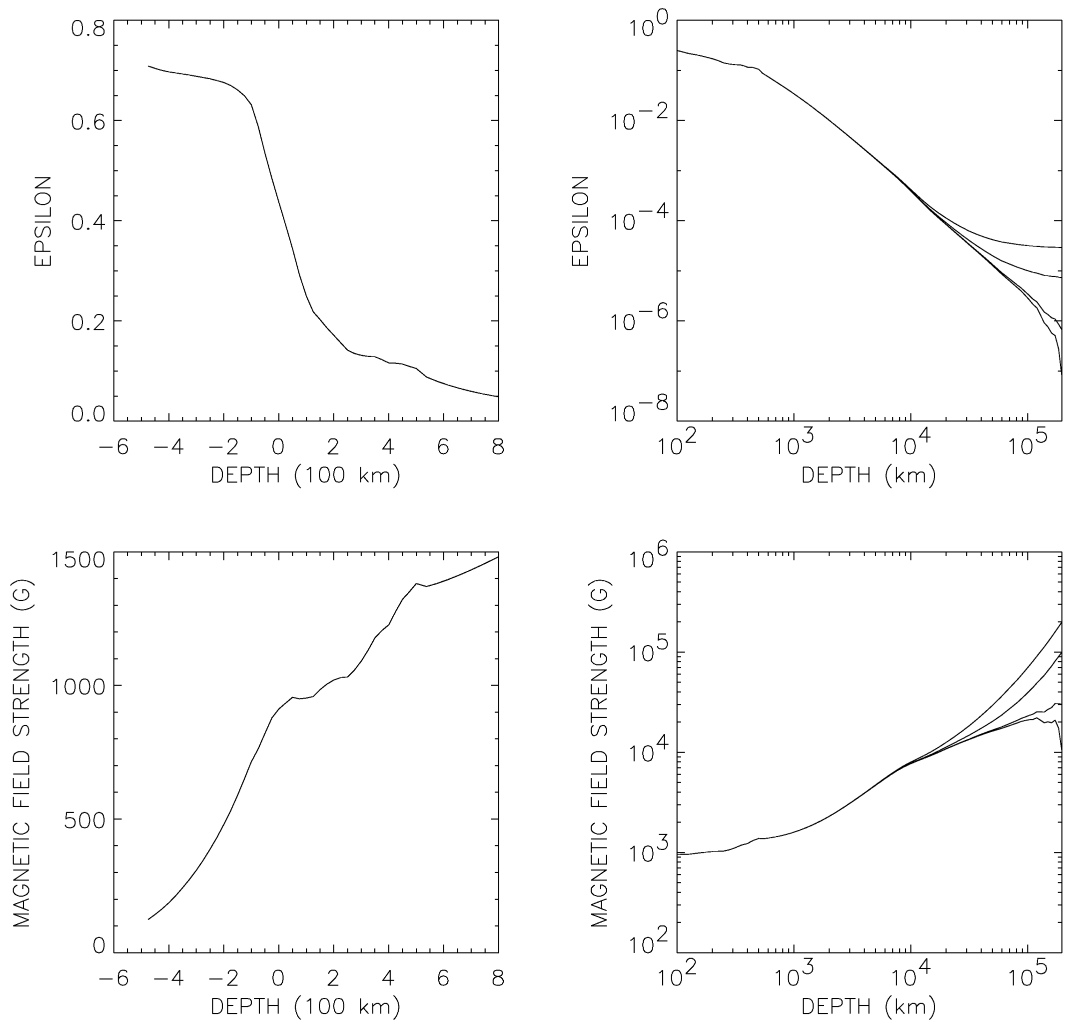

Figure 3. Magnetic field strength $B(z)$, and ratio of magnetic pressure and gas pressure, $\epsilon(z)$, as functions of depth z in the solar convection zone. The different curves represent different values of magnetic field strength B_0 at the base of the convection zone ($B_0 = 10^4$, 3×10^4, 10^5 and 2×10^5 G). Note that for $z < 10^4$ km the field strength is independent of B_0.

convection zone (also see van Ballegooijen 1984). The results are shown in Figure 3 for different values of the field strength B_0 at the base. Following each curve upward in height, for large B_0 the ratio ϵ is nearly constant with height in the deeper layers, until $\rho_d v_d^2/p_d$ becomes of order ϵ. Thereafter, $\epsilon(z)$ approximately equals $\rho_d v_d^2/p_d$, which increases with height. Consequently, the value of ϵ at the solar surface is nearly independent of B_0. At the solar surface we find $\epsilon \approx 0.45$, which corresponds to a field strength $B \approx 1000$ G. This should be compared with the observed field strengths of 1300 to 1500 Gauss (e.g. Solanki 1993).

4. Discussion and Conclusions

The model shows that the downflows in the immediate surroundings of flux tubes play an important role in determining the magnetic field strength as a function of depth in the convection zone. Using a simple model of the convection, we are able to reconcile the presence of relatively weak fields in the deeper layers ($\epsilon \sim 10^{-5}$) with the existence of kilogauss fields at the solar surface ($\epsilon \sim 1$). The surface field strength is virtually independent of the field strength at the base. The predicted surface field strength of 1000 G is somewhat smaller than the observed values. This may be due to the fact that the flux tubes are actually located in the coolest parts of the downflows (i.e., cooler than the average downflow). Alternatively, the flux tubes may be cooler than their local surrounding in the first few 100 km below the surface due to radiative transfer effects. Further refinements to the model are being worked out.

In summary, we have shown that the inclusion of Reynold's stresses associated with downflowing plumes in the immediate periphery of flux tubes can lead to kilogauss field strengths at the surface and produce a depth variation of the field in the deep convection zone that is compatible with dynamo theories. Observational tests for the model could come from time distance helioseismology.

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