Dispersion Velocity of Galactic Dark Matter Particles

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The self-consistent spatial distribution of particles of galactic dark matter is derived including their own gravitational potential, as also of that of the visible matter of the Galaxy. In order to reproduce the observed rotation curve of the Galaxy the value of the dispersion velocity of the dark matter particles, $\langle v^2 \rangle_{\rm DM}^{1/2}$ should be $\sim$600 km s^{-1} or larger. [S0031-9007(96)00231-1]

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More than 20 years ago it was suggested [1] that weakly interacting particles of nonzero rest mass which decouple from radiation and matter early after the big bang would form an invisible gravitating background of dark matter (DM) around galactic systems. Even though at that time the only available candidates for these particles were the neutrinos of the muon and electron flavors, the idea itself became the paradigm under which the newly discovered particles like the tau-neutrino and newly hypothesized particles within the context of possible physics beyond the standard model of particle physics and be incorporated. Also, during the latter half of the intervening 20 years, we have witnessed a tremendous growth in the experimental effort towards direct detection of these particles in the laboratory. The experiments are aimed at observing the effects of the impact of mainly the more massive candidate particles of DM with targets maintained at cryogenic temperatures which facilitate the observation of the tiny amount of energy deposited in the process against the background generated by internal and external radioactivity and by the cosmic rays. These developments are reviewed in detail by Trimble [2], Primack, Seckel, and Sadoulet [3], Caldwell [4], and Price [5].

The interpretation of these experiments to derive constraints on the properties of the unknown particles constituting a halo of dark matter in and around the Galaxy requires assumptions about the density and spectrum of velocities of the DM particles in the solar neighborhood. These parameters have been obtained thus far by describing the DM halo as a single component isothermal sphere which is truncated at a particular radius [6]. The normalization for the density of DM particles comes from an analysis originally suggested by Oort [7] in which the observed spatial and velocity distribution of stars near the solar system indicate a DM density of $\sim$0.3 GeV cm^{-3} in the solar neighborhood; Bahcall [8] gives a detailed account of this procedure. The three-dimensional dispersion velocity of the DM particles, $\langle v^2 \rangle_{\rm DM}^{1/2}$, has not been determined, however. It is customary to take recourse to the virial result pertaining to an isotropic isothermal sphere [9] and set $\langle v^2 \rangle_{\rm DM}^{1/2} = \sqrt{\frac{3}{2}}\,\Theta_\infty$, where Θ_∞ is the asymptotic value of the circular rotation speed. Since Θ_∞ for the Galaxy is not known, the usual practice is to *assume* that the rotation curve of the Galaxy [10–12], $\Theta(R)$, is flat from $R \sim 5$ kpc out to $R \gg R_0 \approx 8.5$ kpc (here and below R denotes the galactocentric distance in the plane of the Galaxy, R_0 being the Sun's position), and set $\Theta_\infty \approx \Theta(R_0) \approx 220$ km s^{-1}, the rotation speed near the solar system. This yields $\langle v^2 \rangle_{\rm DM}^{1/2} \approx 270$ km s^{-1}, which is the value usually assumed in most studies of issues related to galactic DM. However, as noted in the recent review by Fich and Tremaine [12], "Much of the data indicates that the rotation curve continues to rise beyond R_0." Thus the estimate $\langle v^2 \rangle_{\rm DM}^{1/2} \sim 270$ km s^{-1} derived by assuming $\Theta_\infty = \Theta(R_0)$ is uncertain. Moreover, the assumption of a pure isothermal sphere for the description of the dark matter halo neglects the possible deviation from spherical symmetry induced by the dislike distribution of the visible matter.

Keeping these points in mind, we focus attention on the observed rotation curve of the Galaxy, and develop a theoretical framework, the salient features of which are as follows: (a) A model for the Galaxy comprised of visible matter and particles of DM with a *self-consistent* inclusion of their gravitational interactions, and (b) departure from spherical symmetry due to the dislike distribution of the visible matter which will be treated as axially symmetric. The quantity $\langle v^2 \rangle_{\rm DM}^{1/2}$ appears as a free parameter in our framework and is determined by comparing the theoretical rotation curve with the observed data.

We adopt well-established models to describe the density distribution of the normal visible matter and the resulting gravitational potential. In this Letter we present our results for a two-component model of the visible matter consisting of a spheroidal bulge [9,13,14] with density $\rho_s(r)$, and an axisymmetric disk [14] with density $\rho_d(R, z)$,

$$\rho_s(r) = \frac{\rho_0}{(1 + r^2/a^2)^{3/2}}, \tag{1}$$

$$\rho_d(R,z) = \frac{\Sigma_0}{2h}\, e^{-(R-R_0)/R_d} e^{-|z|/h}, \qquad (2)$$

where $r = (R^2 + z^2)^{1/2}$, and $\Sigma_0 \equiv \int_{-\infty}^{\infty} \rho_d(R_0,z)dz$ is the disk surface density at the solar position, z being the vertical distance from the plane of the disk. The values of the parameters are given by [13,14] $a = 0.103$ kpc, $R_d = 3.5$ kpc, $h = 0.3$ kpc, and $\rho_s(R_0) = 7 \times 10^{-4} M_\odot\, pc^{-3}$. (Note that he rotation curve in the outer regions of the Galaxy is relatively insensitive to the spheroid parameters There are conflicting reports on the value of Σ_0: Whereas Kuijken and Gilmore (KG) [14] suggest $\Sigma_0 \sim 40 M_\odot\, pc^{-2}$ on the basis of data on ~512 K dwarf stars, Bahcall *et al.* [15,16] in their reanalysis of essentially the same data suggest a number for Σ_0 which is about twice as large. In our calculations we consider values of Σ_0 in the range (40–80)$M_\odot\, pc^{-2}$. The estimate of the local surface density of the galactic disk due to the identified matter such as visible stars is $\sim (48 \pm 8) M_\odot\, pc^{-2}$. Thus Bahcall *et al.'s* kinematical estimate of Σ_0 seems to indicate the presence of a substantial amount of unseen matter in the galactic disk, whereas KG's estimate is consistent with no disk dark matter. (Note that analyses of Refs. [14–16] are all based on one-dimensional solutions to the Boltzmann equation, which, in the given situation, are strictly valid for an infinite disk only). In any case, the dark matter associated with the disk is likely to be dissipational in contrast to that constituting the extended halo which would be collisionless and nondissipative. We are concerned with this latter type of dark matter in this paper. We use the conventional nomenclature "visible" to describe effectively the *total* matter associated with the disk and write the total visible matter density, ρ_v, as $\rho_v = \rho_s + \rho_d$, the corresponding potential being $\Phi_v = \Phi_s + \Phi_d$. The expressions for the potentials Φ_s and Φ_d corresponding to the chosen forms of ρ_s and ρ_d are given in Refs. [19,13,14].

Now, for the DM component, the exercise is to calculate the distribution of the DM particles by self-consistently including the effects of the self-gravitation of the DM particles themselves *and* the potential due to the total visible component specified above. The procedure we follow is analogous to the one developed earlier [17] with this difference that we now have to contend with the axial symmetry of the potentials. Since the DM particles obey the steady-state collisionless Boltzmann equation, the assumption of Maxwellian phase-space density allows us to write the spatial density, $\rho_{DM}(R,z)$, of DM as

$$\rho_{DM}(R,z) = \rho_{DM}(0,0)\exp\Big[-\frac{3}{\langle v^2\rangle_{DM}} \times \{[\Phi_{DM}(R,z) - \Phi_{DM}(0,0)] + [\Phi_v(R,z) - \Phi_v(0,0)]\}\Big], \qquad (3)$$

where the DM potential, $\Phi_{DM}(R,z)$, satisfies the Poisson equation

$$\nabla^2 \Phi_{DM}(R,z) = 4\pi G \rho_{DM}(R,z). \qquad (4)$$

The solution of the coupled equations (3) and (4) for Φ_{DM} is effected through the iterative scheme ($n = 1,2,3,\ldots$)

$$\nabla^2 \phi_n(R,z) = 4\pi G \rho_{n-1}(R,z), \qquad (5)$$

where $\rho_{n-1}(R,z)$ is equal to the right-hand side (rhs) of Eq. (3) with Φ_{DM} replaced by $\phi_{n-1}(R,z)$, and $\{\phi_0(R,z) - \phi_0(0,0)\} = 0$ is the initial choice for the iteration process. The quantities $\rho_{DM}(0,0)$ and $\langle v^2\rangle_{DM}^{1/2}$ are taken as free parameters.

Details of the iterative scheme and the numerical procedure are described elsewhere. After a few iterations (typically, $n \le 10$) the potentials ϕ_n converge towards the desired potential Φ_{DM}. We checked our numerical code against test equations whose exact solutions are known. We also check our numerical results for the actual equations (3) and (4) against analytical results for small and large values of R and z.

Once Φ_{DM} has been calculated, the rotation curve, $\Theta(R)$, is obtained through the relation

$$\Theta^2(R) = \left(R\frac{\partial}{\partial R}[\Phi_{DM}(R,z) + \Phi_v(R,z)]\right)_{z=0}. \qquad (6)$$

Note that the contribution of the visible disk to $\Theta^2(R)$ is proportional to its surface density [see Eq. (4-159) of Ref. [9]], while that of a perfect isothermal sphere is proportional to the square of the velocity dispersion of its particles [see Eq. (4-127b) of Ref. [9]].

The theoretical rotation curves thus obtained for various values of the parameters $\rho_{DM}(0,0)$ and $\langle v^2\rangle_{DM}^{1/2}$ are to

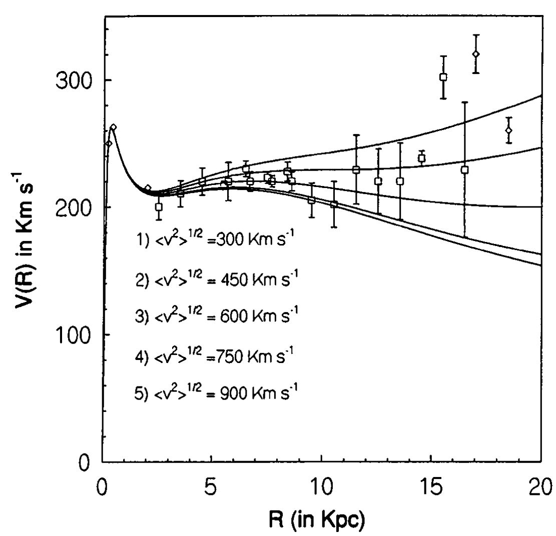

FIG. 1. The theoretically calculated rotation curve of the Galaxy for various values of $\langle v^2\rangle_{DM}^{1/2}$ compared with the available observational data [10–12]. All curves are for $\rho_{DM}(0,0) = 1$ GeV cm^{-3} and $\Sigma_0 = 80 M_\odot\, pc^{-2}$ (see text). The data and error bars for R in the range ~2–17 kpc are from Fig. 3 of Ref. [11], and those for $R > 17$ kpc are from Fig. 2 of Ref. [12]. The data for R below ~2 kpc are from Ref. [10].

be compared with observations [10–12] to ascertain the domain of the parameter space which is acceptable. This comparison is shown in Fig. 1 for $\Sigma_0 = 80 M_\odot \mathrm{pc}^{-2}$ and $\rho_{\rm DM}(0,0) = 1\ \mathrm{GeV\,cm^{-3}}$. The value of $80 M_\odot \mathrm{pc}^{-2}$ for Σ_0, it being the upper limit on the allowed value of Σ_0 in our calculation, gives us a conservative estimate of (i.e., a *lower* limit on) $\langle v^2\rangle_{\rm DM}^{1/2}$. This is because, for a given value of Θ at a given value of R, a lower value of the disk surface mass density (Σ_0) requires a higher value of $\langle v^2\rangle_{\rm DM}^{1/2}$ [for a fixed value of $\rho_{\rm DM}(0,0)$]. Our choice of $\rho_{\rm DM}(0,0) \approx 1\ \mathrm{GeV\,cm^{-3}}$ is dictated by the constraint [7,8] that $\rho_{\rm DM}(R_0,0) \sim 0.3\ \mathrm{GeV\,cm^{-3}}$ and the need to fit the rotation curve. A slightly lower value of $\rho_{\rm DM}(0,0)$ generally requires higher values of $\langle v^2\rangle_{\rm DM}^{1/2}$ in order to satisfy the above constraint and to fit the rotation curve. In this sense, our choice of $\rho_{\rm DM}(0,0) \approx 1\ \mathrm{GeV\,cm^{-3}}$ yields, again, a lower limit to $\langle v^2\rangle_{\rm DM}^{1/2}$. A higher value of $\rho_{\rm DM}(0,0)$, on the other hand, can be consistent with the constraint $\rho_{\rm DM}(R_0,0) \sim 0.3\ \mathrm{GeV\,cm^{-3}}$ for sufficiently low values of $\langle v^2\rangle_{\rm DM}^{1/2}$; however, in this case, the rotation curve falls steeply beyond the solar circle and thus provides a poor fit to the rotation curve.

In order to determine (a lower limit to) the best-fit value of $\langle v^2\rangle_{\rm DM}^{1/2}$ we have calculated $\chi^2 \equiv (1/N)\sum_{i=1}^{N}\{[\Theta_i(R_i) - \Theta_{i,0}(R_i)]/\sigma_i\}^2$ as a function of $\langle v^2\rangle_{\rm DM}^{1/2}$ [for $\Sigma_0 = 80$ and $40 M_\odot \mathrm{pc}^{-2}$ and $\rho_{\rm DM}(0,0) = 1\ \mathrm{GeV\,cm^{-3}}$], where N is the number of observational data points, $\Theta_i(R_i)$ and $\Theta_{i,0}(R_i)$ are the theoretical and observational values of the rotation speed, respectively, for the ith data point for which $R = R_i$, and σ_i is the 1σ uncertainty in the measured value of $\Theta_{i,0}(R_i)$. We calculate the above χ^2 for the entire data set for R in the range $\sim$2–20 kpc as well as for the restricted data set for R in the range $\sim$10–20 kpc in which the observed rotation curve data show a conspicuous rising trend. For $\Sigma_0 = 80 M_\odot \mathrm{pc}^{-2}$, both data sets give a minimum χ^2 at $\langle v^2\rangle_{\rm DM}^{1/2} \sim 600\ \mathrm{km\,s^{-1}}$. For $\Sigma_0 = 40 M_\odot \mathrm{pc}^{-2}$, the minimum of the χ^2 lies at $\langle v^2\rangle_{\rm DM}^{1/2} \sim 750\ \mathrm{km\,s^{-1}}$ for the restricted data set while the minimum is beyond $900\ \mathrm{km\,s^{-1}}$ for the full data set. From the above analysis we conclude that the lower limit on $\langle v^2\rangle_{\rm DM}^{1/2} \sim 600\ \mathrm{km\,s^{-1}}$.

Notice from Fig. 1 that for $\langle v^2\rangle_{\rm DM}^{1/2} \sim 300\ \mathrm{km\,s^{-1}}$ the potential of the visible component concentrates the distribution of DM towards the center, causing the rotation curve to fall below the observational data at large galactocentric distances. As the kinetic energy of the DM particles increases with increased value of $\langle v^2\rangle_{\rm DM}^{1/2}$ the particles are affected progressively less by the potentials and spread out farther. This causes the rotation curves to be elevated.

We thus see that the rms velocity of particles of DM needed to generate the observed rotation curve is higher then that adopted in a variety of discussions of DM [18]. Indeed, we had an inkling that this might be so, based on our analytic estimate made earlier in this context [19]. The implications of this result are as follows.

(1) Since the typical velocity of individual DM particles is higher by at least a factor of $\sim$2 on the average, the energies they would deposit in the detectors would be higher by at least a factor of $\sim$4. This would make these events stand out against the background.

(2) The higher velocities imply higher fluxes, and the event rates would be increased by at least a factor of $\sim$2.

(3) When the observed pulse height spectrum in the detectors are reanalyzed taking the above two points into account, the existing bounds on the masses and other properties of dark matter particles would become substantially more stringent.

(4) The higher velocities would also mean lower rates of capture by the Sun by accretion; consequently, the flux of high energy neutrinos arising from their annihilations in the central region of the Sun [20] is expected to be correspondingly smaller.

(5) The large velocities would also imply an extended halo (with an estimated mass of $\sim 1.5 \times 10^{12} M_\odot$ up to $\sim$100 kpc) whose influence on the dynamical motions within our Galaxy and on the local group, as also the tidal effects on the dwarf spheroidals would become important. For example, the high temperature halo will impart stability to the disk according to the criterion derived by Peebles and Ostriker [21].

These issues are under study and will be reported elsewhere.

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[1] R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972); Astrophys. J. **180**, 7 (1973).
[2] V. Trimble, Annu. Rev. Astron. Astrophys. **25**, 425 (1989).
[3] J.R. Primack, D. Seckel, and B. Sadoulet, Annu. Rev. Nucl. Part. Sci. **38**, 751 (1988).
[4] D.O. Caldwell, in *Proceedings of the XXVIII Recontre de Moriond,* edited by J. Tran Tranh Van *et al.* (Editions Frontiers, Gif-sur-Yvette, France, 1993), p. 187.
[5] P.B. Price, in *Proceedings of the International Conference on Non-Accelerator Particle Physics, Bangalore, India, 1994*, edited by R. Cowsik (World Scientific, Singapore, 1995), p. 239.
[6] R.A. Flores, Phys. Lett. B **215**, 73 (1988); C.S. Frenk and S.D.M. White, Mon. Not. R. Astron. Soc. **193**, 295 (1980).
[7] J.H. Oort, Bull. Astron. Inst. Neth. **6**, 249 (1937); **15**, 45 (1960).
[8] J.N. Bahcall, Astrophys. J. **276**, 169 (1984).
[9] J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton University Press, Princeton, 1987).
[10] W.B. Burton and M.A. Gordon, Astron. Astrophys. **63**, 7 (1978).
[11] M. Fich, L. Blitz, and A. Stark, Astrophys. J. **342**, 272 (1989).

[12] M. Fich and S. Tremaine, Annu. Rev. Astron. Astrophys. **29**, 409 (1991).
[13] J. A. R. Caldwell and J. P. Ostriker, Astrophys. J. **251**, 61 (1981).
[14] K. Kuijken and G. Gilmore, Mon. Not. R. Astron. Soc. **239**, 571 (1989); **239**, 651 (1989); Astrophys. J. **367**, L9 (1991).
[15] J. N. Bahcall, M. Schmidt, and R. M. Soneira, Astrophys. J. **265**, 730 (1983).
[16] J. N. Bahcall, C. Flynn, and A. Gould, Astrophys. J. **389**, 234 (1992).
[17] R. Cowsik and P. Ghosh, Astrophys. J. **317**, 26 (1987).
[18] We have repeated the calculations with King's model [9] which ensures that the density of dark matter vanishes at a finite distance. The bound on $\langle v^2\rangle_{\rm DM}^{1/2}$ quoted in this paper remain unaffected by this change. A manuscript incorporating the density contours, mass distribution, etc., is under preparation.
[19] R. Cowsik, Curr. Sci. **61**, 759 (1991).
[20] G. Steigman, C. L. Sarazin, H. Quintana, and J. Faulkner, Astron. J. **83**, 1050 (1978); W. H. Press and D. N. Spergel, Astrophys. J. **296**, 679 (1985); A. Gould, Astrophys. J. **321**, 571 (1987); **368**, 610 (1991); J. Silk, K. Olive, and M. Srednicki, Phys. Rev. Lett. **55**, 257 (1985); T. Gaisser, G. Steigman, and S. Tilav, Phys. Rev. D **34**, 2206 (1986); M. Kamionkowski, Phys. Rev. D **44**, 3021 (1991); N. Sato *et al.*, Phys. Rev. D **44**, 2220 (1991).
[21] P. J. E. Peebles and J. P. Ostriker, Astrophys. J **186**, 467 (1973).