Alfvén vortices in nonuniform dusty magnetoplasmas

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The Alfvén waves are shown to be linearly coupled with finite-frequency convective cell modes in a nonuniform cold dusty magnetoplasma. The dynamics of weakly interacting finite-amplitude electromagnetic oscillations is governed by a pair of nonlinear equations. The latter admit dipolar vortices as possible stationary solutions. The present results can have relevance to the nonlinear electromagnetic wave propagation in a space environment such as cometary tails or interstellar clouds.

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Dusty plasmas are low-temperature ionized gases that contain dust grains of micrometer or submicrometer size. The dust grains are very massive, negatively (or positively) charged particulates in the background plasma. The electrostatic charging of the dust grains results from various processes, such as electron and ion collection from the ambient plasma, photoelectric emission, secondary electron and ion emission, field emission, etc. The dust grains can acquire an electric charge, which can be thousands of units of charge [1–5]. Charged particulate matter (dust) appears to be an almost ubiquitous component of space environment, such as asteroid zones, planetary rings, cometary tails, interstellar clouds, as well as Earth's noctilucent clouds. Charged particulates can also be confined for hours in laboratory discharges [6–8].

Recently, there has been a great deal of interest in studying the collective behavior of dusty plasmas in which, besides the electrons and the ions, a charged dust component is also present. It has been found that charged dust grains can modify and even dominate wave propagation [9–13], wave scattering [14,15], gradient and velocity-space-driven instabilities [16,17], self-similar plasma expansion [18], velocity modulation [19], transport processes [20], and ion trapping [21].

Linear and nonlinear properties of low-frequency electrostatic waves in a nonuniform dusty magnetoplasma have been the subject of many recent investigations [22, 23]. In this Brief Report, we study linear as well as nonlinear properties of low-frequency (in comparison with the ion gyrofrequency) coupled Alfvén and finite-frequency convective cell modes in a nonuniform cold magnetized plasma whose constituents are the electrons, ions, and static charged dust grains.

The quasineutrality condition at equilibrium is now given by

$$\sum_j eZ_j n_{j0}=0 \ , \tag{1}$$

where e is the magnitude of the electron charge, Z_j is the charge number on the jth species ($j=e$ for the electrons, i for the ions, and d for the dust grains), and n_{j0} is the equilibrium number density, which is assumed to have a gradient along the x axis.

Consider now the perturbation of the equilibrium in the presence of three-dimensional electromagnetic oscillations, the electric and magnetic fields of which are, respectively, $\mathbf{E}=-\nabla\phi-(1/c)\hat{\mathbf{z}}\partial_t A_z$ and $\mathbf{B}_\perp=\nabla A_z\times\hat{\mathbf{z}}$, where ϕ is the electrostatic potential, A_z is the z component of the vector potential, $\hat{\mathbf{z}}$ is the unit vector along the constant external magnetic field $\mathbf{B}_0$, and c is the speed of light. The compressional magnetic-field perturbation has been neglected in view of the low-β approximation. In the electromagnetic fields, the perpendicular (to the constant external field $\mathbf{B}_0$) components of the charged-particle fluid velocities in the drift approximation ($|\partial_t|\ll\omega_{ci}=eB_0/m_ic$, where B_0 is the strength of the external magnetic field and m_i is the ion mass) are

$$\mathbf{v}_{e\perp}\approx\mathbf{v}_{EB}+v_{ez}\mathbf{B}_\perp/B_0 \tag{2}$$

and

$$\mathbf{v}_{i\perp}\approx\mathbf{v}_{EB}+\mathbf{v}_{ip} \ , \tag{3}$$

where $\mathbf{v}_{EB}=c\mathbf{E}\times\hat{\mathbf{z}}/B_0$ and $\mathbf{v}_{ip}=-(c/B_0\omega_{ci})(\partial_t+\mathbf{v}_{EB}\cdot\nabla)\nabla_\perp\phi$ are the $\mathbf{E}\times\mathbf{B}_0$ and the ion polarization drifts, respectively. The ion motion is assumed to be two dimensional.

The parallel component of the electron fluid velocity is determined from the z component of Ampère's law, yielding

$$v_{ez}=(c/4\pi n_{e0}e)\nabla_\perp^2 A_z \ , \tag{4}$$

where $\nabla_\perp^2=\partial_x^2+\partial_y^2$. In the presence of static charged dust grains, the relevant governing equations for our purposes are the conservation of the charge density

$$\partial_t(n_e-n_i)+\nabla\cdot(n_e\mathbf{v}_e-n_i\mathbf{v}_i)=0 \ , \tag{5}$$

Poisson's equation

$$\nabla^2\phi=4\pi e(n_e-n_i) \ , \tag{6}$$

and the z component of the inertialess electron momentum equation

$$(\mathbf{E}+\mathbf{v}_e\times\mathbf{B}_\perp/c)\cdot\hat{\mathbf{z}}=0 \ , \tag{7}$$

where the ions are assumed to be singly charged and the static dust grains carry the charges with number Z_d, which can take both the positive and negative values, in general. For simplicity, Z_d has been assumed constant.

Letting $n_j = n_{j0}(x) + n_{j1}$, where $n_{j1} \ll n_{j0}$, we obtain from Eqs. (1)–(7) a pair of nonlinear equations

$$d_t\left[\nabla^2 + \frac{\omega_{pi}^2}{\omega_{ci}^2}\nabla_\perp^2\right]\phi - \frac{4\pi ecZ_d}{B_0}\hat{\mathbf{z}}\times\nabla n_{d0}\cdot\nabla\phi + cd_z\nabla_\perp^2 A_z = 0 , \quad (8)$$

and

$$d_t A_z + c\partial_z\phi = 0 , \quad (9)$$

where $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the ion plasma frequency. We have denoted $d_t = \partial_t + \mathbf{v}_{EB}\cdot\nabla_\perp$ and $d_z = \partial_z + \nabla A_z \times \hat{\mathbf{z}}/B_0$.

The origin of the various terms in (8) is obvious. The first two terms come, respectively, from the deviation from the quasineutrality and the linear and nonlinear ion polarization drifts, the third term originates from the $\mathbf{E}\times\mathbf{B}_0$ convection of the equilibrium dust number density since $n_{e0} \neq n_{i0}$, whereas the last term arises from the z variation of the parallel electron flow velocity and the divergence of the nonlinear electron drift associated with the coupling of the parallel electron flow velocity with the perturbed magnetic field $\mathbf{B}_\perp$. In the absence of the charged dust grains, the third term vanishes by virtue of the equilibrium quasineutrality condition.

In the linear limit and local approximation, Fourier transformation of (8) and (9) and their combination yields the wave frequency

$$\omega = \tfrac{1}{2}\omega_{sv} \pm \tfrac{1}{2}(\omega_{sv}^2 + 4k_z^2 v_A^2)^{1/2} , \quad (10)$$

where $\omega_{sv} = 4\pi Z_d eck_y d_x n_{d0}/B_0(k^2 + k_\perp^2\omega_{pi}^2/\omega_{ci}^2)$ is the dust convective cell frequency, and $v_A^2 = k_\perp^2 c^2/(k^2 + k_\perp^2\omega_{pi}^2/\omega_{ci}^2)$. Here $k^2 = k_\perp^2 + k_z^2$, $k_\perp^2 = k_x^2 + k_y^2$, $k_x(k_z)$ is the wave number along the density gradient (external magnetic field), and k_y is the wave number along the y axis, which is transverse to the equilibrium density gradient and the external magnetic field. In the local approximation, the wavelength $(2\pi/k)$ is much smaller than the scale length of the density inhomogeneity.

For $k_\perp\omega_{pi}/\omega_{ci} \gg k$, we have

$$\omega_{sv} = 4\pi\omega_{ci}^2 Z_d eck_y d_x n_{d0}/B_0 k_\perp^2\omega_{pi}^2$$

and

$$v_A = B_0/(4\pi n_{i0}m_i)^{1/2} .$$

Thus, the convective cell modes (with the frequency ω_{sv} being directly proportional to $d_x n_{d0}$) and Alfvén waves (with the frequency $k_z v_A$) are linearly coupled. Note that the presence of ω_{sv} is attributed to the presence of the static charged dust grains, and would not arise in a pure electron-ion plasma without the dust grains.

In the following, we show that the presence of the dust density-gradient term in (8) provides the possibility of a two-dimensional coherent dipolar vortex structure [24]. Accordingly, we look for the stationary solution of the nonlinear equations (8) and (9) in the stationary frame $\xi = y + \alpha z - ut$, where α is a constant and u is the translational speed of the vortex. Thus, assuming $\nabla^2 \ll \omega_{pi}^2\nabla_\perp^2/\omega_{ci}^2$, we can write (8) and (9) in the form

$$-u\partial_\xi\nabla_\perp^2\phi + u_d\partial_\xi\phi + (c/B_0)J_1(\phi,\nabla_\perp^2\phi) + (v_A^2/c)\alpha[\partial_\xi - (1/B_0)J_2(A_z,\nabla_\perp^2 A_z)] = 0 , \quad (11)$$

and

$$-u\partial_\xi A_z + (c/B_0)J_1(\phi, A_z) + \alpha\partial_\xi\phi = 0 , \quad (12)$$

where $u_d = -4\pi ec\omega_{ci}^2 Z_d d_x n_{d0}/B_0\omega_{pi}^2$, $J_1(\phi,\nabla_\perp^2\phi) = (\partial_x\phi\partial_\xi - \partial_\xi\phi\partial_x)(\partial_x^2 + \partial_\xi^2)\phi$, and $J_2(A_z,\nabla_\perp^2 A_z) = (\partial_x A_z\partial_\xi - \partial_\xi A_z\partial_x)(\partial_x^2 + \partial_\xi^2)A_z$.

Equation (12) is exactly satisfied by

$$A_z = (c\alpha/u)\phi . \quad (13)$$

Substituting for A_z from (13) into (11), we obtain

$$(1 - \alpha^2 v_A^2/u^2)[\partial_\xi U - (c/uB_0)J_1(\phi,U)] - (u_d/u)\partial_\xi\phi = 0 , \quad (14)$$

where $U = (\partial_x^2 + \partial_\xi^2)\phi$.

The dipolar vortex solution [24] of (14) in the outer region $(r > R)$ is

$$\phi(r,\theta) = \frac{CR}{\lambda_1^2 + \lambda_2^2}\frac{K_1(\lambda_1 r)}{K_0(\lambda_1 R)}\cos\theta , \quad (15)$$

whereas the solution in the inner region $(r < R)$ is given by

$$\phi(r,\theta) = \frac{CR}{\lambda_2^2}\left[\frac{r}{R} - \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\frac{J_1(\lambda_2 r)}{J_0(\lambda_2 R)}\right]\cos\theta , \quad (16)$$

where $C = uB_0(\lambda_1^2 + \lambda_2^2)/c$, $\lambda_1^2 = uu_d/(u^2 - \alpha^2 v_A^2)$, $r = (x^2 + \xi^2)^{1/2}$, $\theta = \tan^{-1}(\xi/x)$, and R is the vortex radius. Furthermore J_1 (K_1) is the Bessel function (modified Bessel function) of the first order. Note that the outer solution is well behaved provided that $u_d/(u^2 - \alpha^2 v_A^2) > 0$. It emerges that for $u_d > 0$ (<0), we must have $u > \alpha v_A$ $(<\alpha v_A)$. For a given value of $\lambda_1 > 0$, the value of λ_2 is determined from

$$\frac{J_2(\lambda_2 R)}{J_1(\lambda_2 R)} = -\frac{\lambda_2 K_2(\lambda_1 R)}{\lambda_1 K_1(\lambda_1 R)} , \quad (17)$$

where J_2 and K_2 are the Bessel and modified Bessel functions of the second order.

To summarize, we have investigated the linear as well as nonlinear properties of coupled Alfvén and convective cell modes in nonuniform multicomponent dusty plasmas embedded in a homogeneous magnetic field. It is found that the presence of static charged dust grains provides a linear coupling between the Alfvén waves and the finite-frequency convective cell modes, whose frequency is directly proportional to the gradient of the equilibrium dust number density. The nonlinear coupling of weakly interacting electromagnetic modes in nonuniform magnetized dusty plasmas is governed by a pair of nonlinear equations. The stationary solution of the latter is represented as a coherent dipolar vortex, whose translational speed is well determined but whose size is arbitrary. We stress that, in the absence of charged dust grains, the formation of a dipolar Alfvén vortex is forbidden in the absence of parallel electron inertia in a cold plasma [25].

The present results should be useful for understanding the linear and nonlinear properties of electromagnetic turbulence in dusty plasmas that often exist in cometary tails and interstellar clouds.

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