ELLIPSOMETRY OF COELOSTAT COATINGS USING A BABINET COMPENSATOR: SIMULATION OF THE EXPERIMENTAL ACCURACY

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Abstract. The precise measurement of solar magnetic fields requires an accurate measurement of the Muller matrix of the optical components in the path of the light beam, which again requires a careful measurement of the optical constants of the reflecting surfaces in the case of a 3-mirror coelostat system. Here we present a method to measure the optical constants (the real and imaginary part of the refractive index) to an accuracy of the order of 1% for bulk aluminium. This work is directed towards the measurement of instrumental polarisation at the Kodaikanal solar tower telescope, although it can be used for any metallic coated optics.

1. Introduction

The Muller matrix for any optical system is usually determined by measuring the Stokes vector at the output (Gerard and Burch, 1975) for various input Stokes vectors. At least four linearly independent input Stokes vectors are needed to completely specify the 4×4 Muller matrix. This requirement poses serious difficulties for large coelostats. Ellipsometry offers certain simplifications since the Muller matrix of a single reflecting optics depends on only two terms, namely the ratio of the reflection coefficients of linearly polarised light parallel and perpendicular to the plane of incidence (say X) and the phase difference between the reflected light polarised parallel and perpendicular to the plane of incidence (say τ) (Kawakami, 1983; Stenflo, 1993).

There are several different methods to find the values of X and τ (Heavens, 1965; Abeles, 1971; Hass, Heany, and Hunter, 1982; Hjortsberg, 1980) for thick as well as thin metallic films. These two terms X and τ are functions of the complex refractive index and the angle of incidence. Since the angle of incidence in the case of solar observations can be obtained analytically for the particular day, time and particular place (Balasubramaniam, Venkatakrishnan, and Bhattacharrya, 1985), the problem reduces to one of measuring the complex refractive index.

This paper aims to develop a procedure to accurately measure the complex refractive index of the mirror surfaces of the Kodaikanal solar tower telescope. The chief consideration here is the fact that the mirrors should remain in their respective cells during measurement, and also that the measurements are done without physically touching the mirrors to avoid damage to the surfaces.

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2. The Telescope

The Kodaikanal tower telescope (Bappu, 1967) consists of a 3-mirror coelostat system, each of 61-cm diameter. The primary mirror is an equatorial mount (latitude 10° 14′ N). The sunlight from the primary mirror is reflected to the secondary which in turn reflects the light vertically down to a third mirror which is kept fixed at 45° inclination. Hence the reflected light from the third mirror will be parallel to the ground. The light then passes through a 38-cm lens which images the Sun on to the slit of the spectrograph.

For polarisation measurement one should try to avoid oblique reflections and hence ideally this kind of telescope should not be used. However, one cannot prevent oblique reflections in existing installations not originally meant for polarisation measurements. Hence the study of the instrumental polarisation is needed, especially to determine the limits of accuracy that are achievable in such installations.

3. The Method Adopted

One way to evaluate the optical constants η (the real part) and κ (the imaginary part) requires the measurement of the transmittance T at the normal incidence and the reflectance at near-normal incidence (Nilsson, 1968). This can only be applicable for transparent objects. For reflecting surfaces one can determine η and κ by measuring the reflection coefficient at two different angles of incidence and by using the graphical method (Tousey, 1939). However, the accuracy achieved in the calculation of η and κ is of the order of 4% (Avery, 1952). In this paper we will examine the accuracy that can be achieved from a minimization technique.

Basically, the method adopted here is ellipsometry. For a coating with thickness greater than $(\lambda/2\pi\kappa)$, the 'skin depth', the layer can be treated as a bulk material (Makita and Nishi, 1970). From the Fresnel formulae for reflection at a conducting surface, we can write the phase retardation, τ, of the electric field vibrations along the plane of incidence with respect to the vibration normal to the plane of incidence,

$$\tan(\tau) = \frac{2b\sin(i)\tan(i)}{\sin^2(i)\tan^2(i) - (a^2+b^2)}\,, \tag{1}$$

while the ratio of reflectivities of these two components is given by

$$X^2 = \frac{a^2+b^2-2a\sin(i)\tan(i)+\sin^2(i)\tan^2(i)}{a^2+b^2+2a\sin(i)\tan(i)+\sin^2(i)\tan^2(i)}\,, \tag{2}$$

where

$$a^2 = \tfrac{1}{2}[\eta^2-\kappa^2-\sin^2(i)+\sqrt{(\eta^2-\kappa^2-\sin^2(i))^2+4\eta^2\kappa^2}]\,,$$

$$b^2 = \frac{1}{2}[-\eta^2 + \kappa^2 + \sin^2(i) + \sqrt{(\eta^2 - \kappa^2 - \sin^2(i))^2 + 4\eta^2\kappa^2}] \, .$$

Equations (1) and (2) are valid for bulk material. The phase shift and the reflection coefficient are functions of the angle of incidence and the complex refractive index.

In the proposed method, light polarized linearly at 45° to the plane of incidence will be intercepted, after reflection, by a Babinet compensator (whose axes are at 45° to the input polarisation) followed by a polaroid crossed to the input polarisation. The values of τ can be obtained by initially inserting a reference polaroid in the reflected beam to produce identical polarisation as the incident beam. This will produce a set of reference fringes. The shift in fringes seen after removing the reference polaroid will give the value of τ. Because of unequal reflection of the two polarisation components in and orthogonal to the plane of incidence, the position angle of the resulting elliptical polarisation will rotate producing a decrease in the fringe contrast, to that of the reference fringes. The contrast can be maximized once again by rotating the polaroid behind the BC. If θ is the angle of rotation, then

$$X^2 = \tan(45 + \theta) \, . \tag{3}$$

For the simulation, we first assume a particular value for the complex refractive index. The simulated measurements of τ and X^2 are then obtained from Equations (1) and (2). While generating the value of τ, we add the maximum error in the fringe shift measurement (which can be as low as 1μ for CCD detection of fringes using a BC with a wedge angle of 0.5°) as well as the systematic error of the angle of incidence measurement (which is kept as a free parameter in the problem). The determination of κ is best done separately from the τ measurement and the η can then be determined using the previously determined value of κ. While generating the experimental value of X^2, we add the systematic error of the fringe contrast measurement which is limited by the photometric accuracy of the detector. For a well exposed CCD frame with 8 bits of digitization, this turns out to be $\approx 0.4\%$. Having generated such 'experimental' values, $\tau_{\rm exp}$ and $X^2_{\rm exp}$ for several angles of incidence, we compute the following minimization functions, viz.,

$$\sigma^2_\tau = \sum_i (\tau_{\rm the} - \tau_{\rm exp})^2 \, , \tag{4}$$

$$\sigma^2_X = \sum_i (X^2_{\rm the} - X^2_{\rm exp})^2 \, , \tag{5}$$

where the summation is over the angle of incidence i and the theoretical values $\tau_{\rm the}$ and $X^2_{\rm the}$ are obtained by substituting arbitrary values of κ and η into Equations (1) and (2). Thus, $\sigma^2_{\tau,X}$ are surfaces in κ, η space. The minimum of these surfaces yield the value of κ and η. The σ^2_τ function has a sharper minimum in the κ direction than in the η direction. It is for this reason that we prefer to determine κ alone from the τ measurement and then use this value of κ in the X^2 measurement to determine η.

4. Results

The deviations of κ and η, determined using the minima of σ_τ^2 and σ_X^2, from the input values of κ and η can be considered as estimates of the errors of the method. We have noticed that the errors of the κ determination chiefly depends on the errors in the measurement of the angle of incidence. We have plotted in Figure 1 the κ errors as a function of the error in i, the angle of incidence. Each curve is for a specific wavelength as described in the caption. We see that the angle of incidence must be measurable with an accuracy better than 0.04° to obtain a κ determination to better than 1% accuracy. This is an important consideration while designing the experiment. In Figure 2, we plot the error in η determination as a function of error in κ measurement for different λ assuming eight-bit digitization of the intensity data. Figure 2 shows that for a 1% accuracy in κ determination, the accuracy in η will be better than 1% for all wavelengths.

5. Discussions and Conclusions

The measurement of the change of polarisation by reflection for solar optical telescopes is needed if one wants to measure high resolution magnetic structure in the Sun (Lites, 1987; Lites, Scharmer, and Skumanich, 1990; November, 1990). Hence, the accurate measurement of the complex refractive indices of the reflecting mirrors, used in the solar telescopes, is necessary. One method which one could possibly use for accurate measurement has been described. It has been found that with a BC of 0.5° quartz wedge and with a CCD as a detector, the accuracy in the calculation of η and κ is high. It is seen that the angle of incidence measurement error is important in the case of τ measurement, which is the main factor that limits the accuracy of the experiment.

The model used above is valid for a film whose thickness is larger than the skin depth. The skin depth increases with wavelength and hence the material will act as a thin film. Hence, the Fresnel formulae have to be modified and would depend on the number 'n' of layers one would like to model. In this case also the transmission coefficient cannot be measured because the back side of the telescope will be closed. So, from the two sets of measurements of τ and X^2, $3n$ parameters (η, κ and the film thickness of each layer) have to be determined. (See, however, Bosch and Monzonis (1995) for some recent attempts on ellipsometry of samples with arbitrary number of layers.)

From the η and κ calculated using the bulk material approximation, one can calculate the Muller matrix for a single reflecting optics as

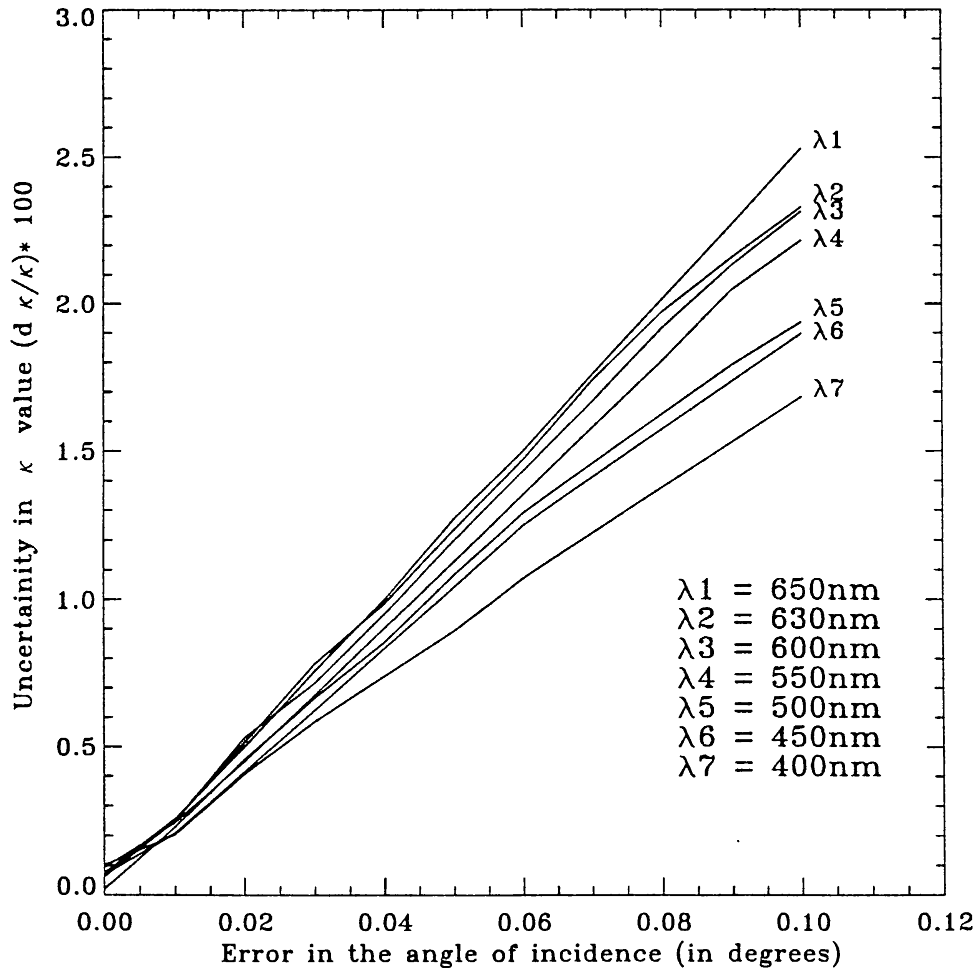

Figure 1. Variation of the percentage error in κ with the error in angle of incidence measurement for different wavelengths mentioned in the figure.

$$[M] = \frac{1}{2}\begin{pmatrix} 1+X^2 & 1-X^2 & 0 & 0 \\ 1-X^2 & 1+X^2 & 0 & 0 \\ 0 & 0 & 2X\cos(\tau) & 2X\sin(\tau) \\ 0 & 0 & -2X\sin(\tau) & 2X\cos(\tau) \end{pmatrix},$$

where X and τ is given by Equations (1) and (2), using the angle of incidence calculated from the time of observation as in Balasubramaniam, Venkatakrishnan, and Bhattacharrya (1985).

It can be seen that the errors in η and κ estimation would result in errors in $[M]$ and finally errors in the magnetic field value derived from the observed Stokes' parameters. An error of 1% in η and κ can grow into an error of a few percent in the corresponding Muller matrix for the Kodaikanal installation (Balasubramaniam, 1988). For example, a 10% crosstalk of V into Q or U would result in a spurious value of 10% for Q or U if V is 100%. If the error in compensation is a few percent, then this leaves a residual error of $\leq 1\%$ in Q or U after applying the

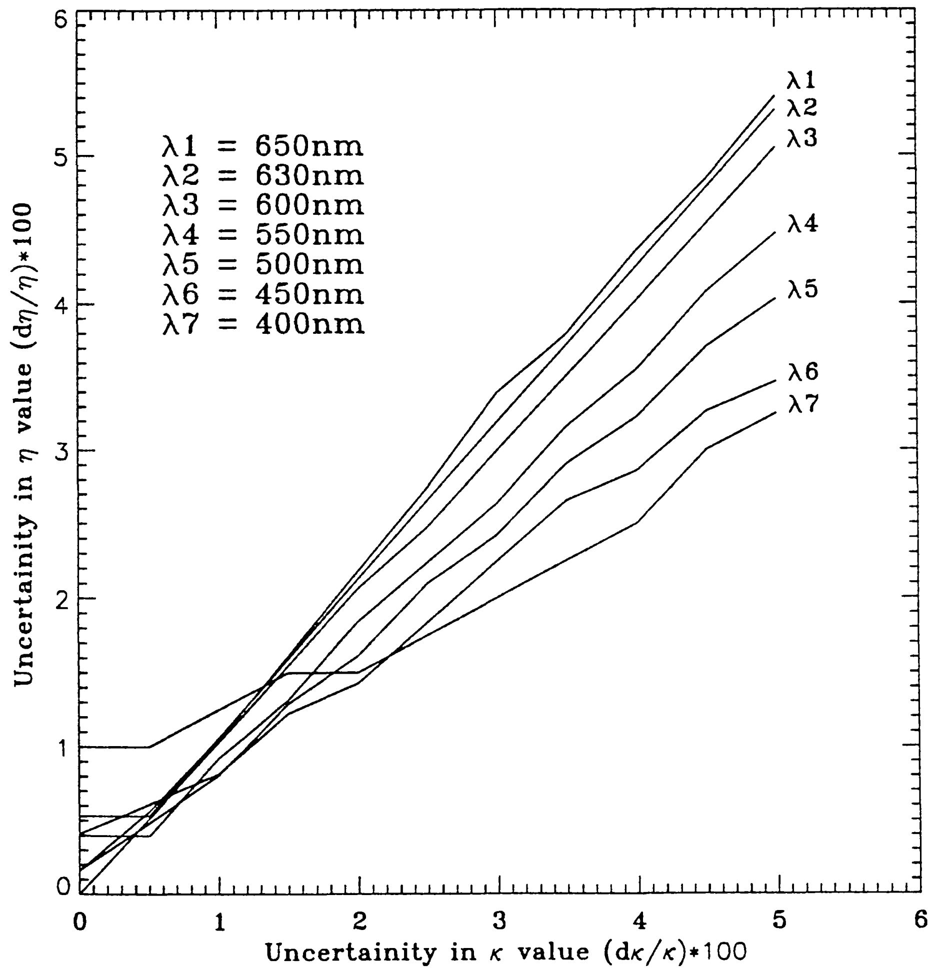

Figure 2. Variation of the percentage error in η with the percentage error in κ.

compensation. This limits the sensitivity of the transverse field measurements for weak fields ($\leq$ 100 G) but poses no serious difficulty for moderate fields ($\geq$ 500 G).

The technique suggested for measuring η and κ is also useful for monitoring the weathering of the coelostat mirrors and for determining the uniformity of the coatings over the mirror surface. One consequence of weathering would be the formation of an oxide layer on the aluminium layer. To simulate the oxide layer one needs to know about the thickness as well as the refractive index of the layer which cannot be known a priori in the case of solar telescopes. The thickness of the layer will depend on the time of exposure, humidity, etc. Hence, it will be different for different observation sites and could also be a function of time. Modelling a reflecting surface as oxide free even in the presence of a thin oxide layer results in a drastic reduction of the equivalent η and κ values. However, the predicted X and τ values using these pseudo-values of η and κ compared well with the experimentally determined values (Burge and Bennett, 1964). For example, pseudo-value of the refractive index η and κ calculated with the bulk material approximation will be a systematic function of the angle of incidence. The variation in these pseudo-values turns out to be less than 3% for aluminium for angle of incidence greater than

20° (Burge and Bennett, 1964). Hence, the bulk material approximation is valid to within an error of 3% for large angles of incidence. In our coelostat system the most prominent effect will be from the third mirror kept at 45°. Hence, the bulk material approximation will be applicable at least for the third mirror with an error less than 3% which happens to contribute a large part of the instrumental polarisation. The variation of X and τ has to be measured experimentally and a model with the oxide layer has to be fitted with the experimental data in order to achieve greater accuracy. For the present, this study serves to highlight the areas in the experimental set up where special care must be maintained. We conclude that the τ measurement is ideal for estimating κ, but the chief source of error is the error in the angle of incidence measurement. The X measurement is ideal for estimating η and is limited only by the accuracy of the intensity measurement.

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