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Time delay in gravitational lens systems and cosmological parameters

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Abstract. The time delay between the multiple images in gravitational lens systems is affected by large scale mass distribution in the lens, even though it might not have observable effects on the macro images. Inhomogeneity at very small scale will not directly affect the time delay but will influence the models of mass distribution in the lens galaxy. How this degeneracy can be eliminated is described. A method to determine time delay based on smoothed cubic splines is explained. The value of the Hubble Constant, inferred from the time delay in 5 well–constrained lens systems is 56 ± 8 km s^{-1} Mpc^{-1}.

Keywords : Cosmology: cosmological parameters; Hubble constant; gravitational lensing - time delay

1. Introduction

Gravitational lensing is caused when the beam of photons from a background source is bent and distorted due to the gravitational pull of an intervenous massive object like a galaxy or galaxy–cluster. In the strong lens systems, multiple images of the background source can appear, the maximum separation between them being an indication of the mass of the lens and the *effective distance* to the system,

$$D_{eff} = \frac{D_{lens} D_{source}}{D_{LS}} \tag{1}$$

where the distances in the right side are respectively angular diameter distances from the lens and the source to the observer and from the source to the lens. Intrinsic variability

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in the source will appear in the images at different times. This time interval is called the *time delay*, and is in the range of days to months for normal galaxy–lenses. It is caused by the different path length of photons forming the multiple images (*geometrical time delay*) as well as the difference in the gravitational potential along the path (*potential time delay*). The time delay is proportional to the effective distance to the lens system, and varies as the square of the image separation, but it is also sensitive to the image configuration, through the details of the mass density distribution in the lens. Hence, if the geometry of the lens configuration is determined from the observed image features or the lens characteristics, D_{eff} can be determined from the time delay.

The point lens, where the mass of the lens is assumed to be concentrated at a point, can be used to illustrate the principle. A point lens produces two images, (say, at angular distances θ_1 and θ_2 from the lens) which appear at the two sides of the lens and all the three will be collinear. If the source is exactly behind the lens, the Einstein ring of radius R_E will appear, where,

$$\mu = R_E^2 = \frac{4GM}{D_{eff}c^2}. \qquad (2)$$

μ is a measure of the strength of the lens, and the image separation, Δ, varies as the square root of μ. If A is the magnification ratio of the images, μ is obtained from the observables as

$$\mu = \sqrt{A}\left[\frac{\Delta}{(1+\sqrt{A})}\right]^2 \qquad (3)$$

It will be noted that $\theta_1 = \frac{\Delta}{(1+\sqrt{A})}$. The time–delay between the images, τ is given by,

$$\tau = \frac{D_{eff}}{2c}\left[\theta_1^2 - \theta_2^2 + \mu * \ell n\frac{\theta_2}{\theta_1}\right] \qquad (4)$$

Thus, from the image separation, the magnification ratio and the time–delay between the images produced by a point lens, the distance combination to the lens system, D_{eff} can be determined. The distance can be used to obtain the Hubble constant for a specified cosmological model, from the redshift to the lens and to the source. If an accurate value of the Hubble constant is measured for a number of lens systems having a combination of redshifts, even cosmological parameters can be constrained. For example, a system with very low lens redshift and a low source redshift will measure Hubble constant independent of the other cosmological parameters, while a high lens redshift and a comparable source redshift will give very different D_{eff} for a model of the Universe with or without a cosmological constant. Currently, there are more than half a dozen AGNs lensed by galaxies at moderate to high redshift for which time delay measurements as well as redshifts are available. Among them, well–constrained systems like $PG1115+080$ and $B1422+231$ have low lens redshift and the time delay can be translated into Hubble Constant fairly independent of other cosmological parameters. Some other systems for which vast observational data are available, like $B0218+357$ have source and lens redshift moderately high and comparable; consequently, for this system the relation between D_{eff} and the Hubble Constant depends to some extent on the cosmological model. Lenses like $PKS1830-211$ having high lens redshift in addition to high redshift of the source with respect to the lens can provide good constraint on the density parameter. On the other hand some less constrained systems like $HE1104-1805$ or $Q0957+561$ could probe possible dark halo mass in the lens, if a value for the Hubble Constant is assumed. Evidently, the range of image separation, image multiplicity and redshift combination for potentially useful arcsecond scale lenses can probe cosmological models, cosmological parameters as well as provide powerful constraints on the location of dark matter. However, it must be remembered that ***the presently available lens systems with measured time delays are not capable of distinguishing between an open universe having 40% of critical density and a flat Universe with cosmological constant contributing to 60% of the energy.*** But, with slightly improved time–delay measurements, we should be able to distinguish between a flat Universe with and without appreciable amount of vacuum energy.

2. Problems in getting the distance estimate from time–delay

The realistic lenses are multi–component. The time delay depends on the gravitational potential integrated along the photon path, while separation between the images, on its gradient and magnification, on the second derivatives of the potential. Our failure to determine the gradients of the potential everywhere results in degeneracies in the estimation of the potential. Specifically, multiple components of the lens affect the image separation and time delay differently. A small scale inhomogeneity like a microlens does not change the time delay directly, but the magnification of the images is modified, affecting the lens models. Also, the microlens light curve acts as a noise to the intrinsic variability of the source, leading to faulty values of the time delay.

A large scale mass distribution in the lens increases the image separation though the relative magnifications are not noticeably altered. Consequently, a nearly constant mass sheet having surface mass density $\kappa\times$ the ***critical density for strong lens*** reduces time delay to a fraction $1-\kappa$. Here the ***critical mass density*** of the lens projected on the lens plane is given by $\Sigma_{cr} = \frac{c^2 D_{source}}{4\pi G D_{lens} D_{LS}}$. It signifies the required amount of gravitating mass in the lens for noticeable lens signature, were a source located just behind along the line of sight to the lens.

The effects of the multiple lens components can, in principle, be modeled with additional observations. Some of the possible constraints from the image plane or direct observation of the lens galaxy are: (a) the image multiplicity, relative orientation and lengths of the images of an arcsecond scale jet may be sensitive to the mass distribution in a dark halo at hundred kiloparsec scale but the the parsec scale features of VLBA scale jet or knot will probe the galaxy–scale mass distribution. More generally, weak lens constraints like distortion of background galaxy–images at ten arcsecond scale and information of image characteristics at multiple source redshifts can lift the degenaracy in the lens mass distribution. (b) Asymmetry of microlens light curve is directly dependent on the *shear of the macrolens* (Subramanian, Chitre, Narasimha 1985; Chitre & Narasimha 1997) $\frac{\gamma}{1-\kappa}$. (c)The rotation curve of a spiral lens galaxy like PKS1830-211 or B0218+357 was used to directly estimate the mass distribution in the inner parts of the lens (Chengalur, de Bruyn, Narasimha 1999; Narasimha 1999). To some extent, this could eliminate the mass degeneracy caused by observations restricted to a single plane.

Equally important is accurate measurement of the time delay. Experience has shown that the time delay estimated from the observations is invariably susceptible to noises. Here we outline a method to alleviate the problem to a good extent, if the intrinsic variability of the source is restricted to a narrow band.

3. Estimation of time delay using smoothed cubic splines

Conventional methods to cross correlate the fluxes from the images to determine the time delay has not been very successful when the noise at the time period of expected time delay is non-negligible, even though the overall signal to noise ratio of the data could be very good. This was apparent in the case of $0957+561$ where time delay was around 417 days while some of the microlens events had similar duration. This problem is severe for $1422+231$ due to the low amplitude of variability in radio.

Our method is based on identifying single component of the intrinsic variability and using this component to cross correlate between the images (Narasimha 2001). The intrinsic variability will follow the same form in all the images, but with a time delay; any variability introduced en route, like the microlens effects, will be uncorrelated between the images. The smoothed cubic spline is used for the identification of the intrinsic time–variable feature. The details are discussed in Patnaik and Narasimha (2001). The essential idea of the method is as follows: In the conventional methods, some kind of cross–correlation is performed between the fluxes observed at the various images as function of time. Such a correlation is affected by two problems, namely, (1) Random variation introduced by microlensing. Since the shape of the microlens variability has the same form, these seemingly random variability appear to artificailly correlate with each other. (2) The calibration errors of the reference star or the standard radio source affects the time–delay measurements, if it is comparable to the amplitude of the intrinsic source variability. Using a suitably–smoothed light curve for computing the cross–correlation eliminates both these problems and has the added advantage of the shape of the light curve being compared between the images for testing the hypothesis of intrinsic source variability. Consequently, we smooth the given time series like the flux in the images,

Table 1. Estimates of Hubble constant (H_o) from time delay in some of the well–constrained gravitational lenses for two combinations of the parameters Ω and Λ (given in brackets).

System	Redshift (z_S,z_L)	Time delay (days)	D_{eff} (Mpc)	H_o for two (0.2, 0)	Cosmologies (0.3, 0.7)	Reference
Q0957+561	(1.4, 0.36)	423±3 [16]	1870	55±10	58±11	[13]
PG1115+080	(1.72, 0.31)	$25^{+3.3}_{-3.8}$ [15]	1550	54	57	[10]
PKS1830-211	(0.89, 2.5)	28 [18]	3550	56±9	57±10	[6]
B0218+357	(0.68, 0.95)	11.7±0.9 [1]	9900	41±9	43±10	[12]
B1600+434	(0.42, 1.59)	51				[5]
B1608+656	(0.63, 1.39)	76	~3000	65	67	[4]
B1422+231	(0.33, 3.62)	7.6±2.4 [14]	860	90	96	[11]
HE1104-1805	(0.77, 2.32)	~500				

[1]Biggs *et al* 1999. [4]Fassnacht *et al* 2000. [5]Koopmans *et al.* 2000. [6]Nair, Narasimha, Rao 1993. [10]Narasimha, Chitre 1991. [11]Narasimha, Patnaik 1993. [12]Narasimha it et al 2002. [13]Narasimha *et al* 1984. [14]Patnaik, Narasimha 2001. [15]Schecter *et al* 1997. [16]Schild, Thomson 1997. [18]Wiklind 2001.

their polarization angle or degree of polarization using smoothed cubic spline fit in such a way that short time scale fluctuations will be removed. The resulting profile of variability is compared in the images, to confirm that there is no undue smoothening and the profile in the various multiple images agree with each other, within the expected noise levels. When is this method superior to direct estimation of the time delay? If the smoothed cubic spline can isolate a single component of the intrinsic flux variability apart from the constant flux from the images, and any contribution due to microlens is filtered from the smooth function, then the signal in the correlation will improve compared to the noise by a factor of (n–4), where n is the number of epochs of observation and the number 4 is based on the minimum number of data points required for the computations of the smoothed spline. This method could be applied to half a dozen observed lenses. Most of the time, the results are essentially what is derived by the observers; in a few cases where the amplitude of the variability was small, this method gave good results, and the peak value of the cross–correlation could be used to test the change in flux introduced en route.

4. Estimation of Hubble Constant

We have constructed models of mass distribution in the lens for five well–constrained systems having time delay meausrements as well as considerable amount of multiwavelength data. The estimation of the effective distance for these systems based on time–delay is given in Table 1 (Narasimha 1999).

5. Summary

Time delay in gravitational lens systems is a powerful method to determine the Hubble constant and provide constraints on the other cosmological parameters as well as distribution of dark matter. However, it has still not provided firm results on the cosmological parameters, because of error in the observed time delay as well as insufficient constraints on the large and small scale mass distribution associated with the lens galaxy. There are still unresolved problems in almost all the lens systems and better observations will be necessary to resolve them. The main problem in modeling appears to be insufficient data to partition the lens mass into mass of the main lensing galaxy and either a larger scale mass like a dark halo or a smaller component like galactic bulge. However, the five well–constrained lens systems tabulated above do provide consistent value for the cosmological distance scale. It is tempting to put forward a value of 56 ± 8 km s^{-1} Mpc^{-1} for the Hubble Constant based on the table.

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