Cool Stars, Stellar Systems and the Sun
ASP Conference Series, Vol. 154, 1998
eds. R. A. Donahue and J. A. Bookbinder

The Influence of Magnetic Flux Tubes on Their Environment

S.S. Hasan[1] and W. Kalkofen

Harvard-Smithsonian Center for Astrophysics, Cambridge MA

Abstract:

We present new calculations for model atmospheres in magnetic flux tubes extending vertically through the photosphere and convection zone of the Sun. This study is a continuation of earlier work by Hasan & Kalkofen (1994) on the equilibrium structure of intense magnetic flux tubes. We construct static models of flux tubes by solving the equations of energy transport and radiative transfer. The most significant aspect of our study is the inclusion of multidimensional radiative transfer in cylindrical geometry and the influence of the flux tube on the ambient medium. Our models satisfy the condition of radiative equilibrium; the lower boundary intensity includes the effect of convection. We determine the structure of the thermal boundary layer at the interface of the flux tube and the ambient medium. We find that the temperature does not change abruptly from its value on the flux tube axis to the ambient value far from the tube. Rather, there is a transition layer at the interface, where there is a significant horizontal temperature gradient. Detailed calculations have been carried out to examine the physical conditions in this layer as well as its horizontal extent.

1. Introduction

Numerous observations have shown that vertical magnetic fields in the form of flux tubes with kilogauss field strengths are ubiquitous in the solar photosphere. The tube diameters are typically sub-arc second and they are spatially unresolved by ground-based telescopes. Much of the earlier information about flux tube structure was indirectly derived from semi-empirical models. However, the development of new instrumentation has permitted observations with higher spatial resolution and greater sensitivity to the magnetic field strength.

The inputs from observations need to be incorporated into theoretical models, which eventually should be able to provide a quantitative understanding of the physical processes occurring in flux tubes. In the present study we undertake a theoretical investigation of this question and propose refinements which would contribute towards the above aim.

The present work is a continuation of earlier calculations by Hasan & Kalkofen (1994) on the equilibrium structure of a static flux tube. The main refinement in this investigation is the use of multidimensional radiative transfer to self-consistently treat the flux tube and the ambient medium. Previous work

[1]Permanent address: Indian Institute of Astrophysics, Bangalore 560034, India

on the equilibrium of flux tubes using multidimensional radiative transfer was carried out by Spruit (1976), Steiner & Stenflo (1989), and Pizzo, MacGregor & Kunasz (1993 a,b).

Briefly, the objectives of our investigation are to firstly, develop equilibrium models of flux tubes, such that the flux tube and the ambient medium are treated self-consistently; secondly, to determine the structure of the thermal boundary layer at the interface of the flux tube and the ambient medium; and finally to identify the direction of energy transport in the medium by calculating the radiative energy flux.

2. Equations

Let us consider a vertical magnetic flux tube of circular cross section embedded in a non-magnetized atmosphere. We adopt a cylindrical coordinate system and assume rotational symmetry about the tube axis. For simplicity we use the thin flux tube approximation to treat the magnetostatic equation. We assume that the pressure and magnetic field are specified and use the energy transport and radiative transfer equations to determine the thermal structure of the tube. We use β_0 to parameterize our models, where $\beta_0 = 8\pi p_0/B_0^2$ with p_0 and B_0 being the pressure and magnetic field strength respectively at $z = 0$.

The equations that we solve are:

$$\nabla\cdot \mathbf{F} = \nabla\cdot \mathbf{F}_{\mathrm{R}} = 4\pi\kappa(S - J) = 0, \tag{1}$$

and

$$(\mathbf{n}\cdot \nabla)I = \kappa(S - I), \tag{2}$$

where z is the height, $I = I(\mathbf{r}, \mathbf{n})$ is the specific radiation intensity at r in the direction $\mathbf{n}$, $S = \sigma T^4/\pi$ is the source function, T is the temperature, κ is the Rosseland mean opacity per unit distance, J is the mean radiation intensity, $\mathbf{F_R}$ is the radiative flux. In equation (1) we neglect the contribution of convection to the total flux. The mean intensity J and radiative flux $\mathbf{F_R}$ are related to I as follows:

$$J = \frac{1}{4\pi}\oint I(\mathbf{r}, \mathbf{n})d\Omega, \tag{3}$$

$$\mathbf{F_R} = 4\pi \oint I(\mathbf{r}, \mathbf{n})\mathbf{n}d\Omega, \tag{4}$$

3. Method of Solution

The formal solution of the transfer equation can be written as:

$$J(\mathbf{r}) = \Lambda(\mathbf{r}, \mathbf{r}')S(\mathbf{r}') + G(\mathbf{r}), \tag{5}$$

where $\Lambda(\mathbf{r}, \mathbf{r}')$ is an integral operator, and G represents the effect of the intensity at the boundary of the computational domain.

The constraint of radiative equilibrium ($J = S$) is achieved through a temperature correction procedure using a modified form of Λ-iteration (Steiner 1989). The temperature correction ΔT^i after the i-th iteration is

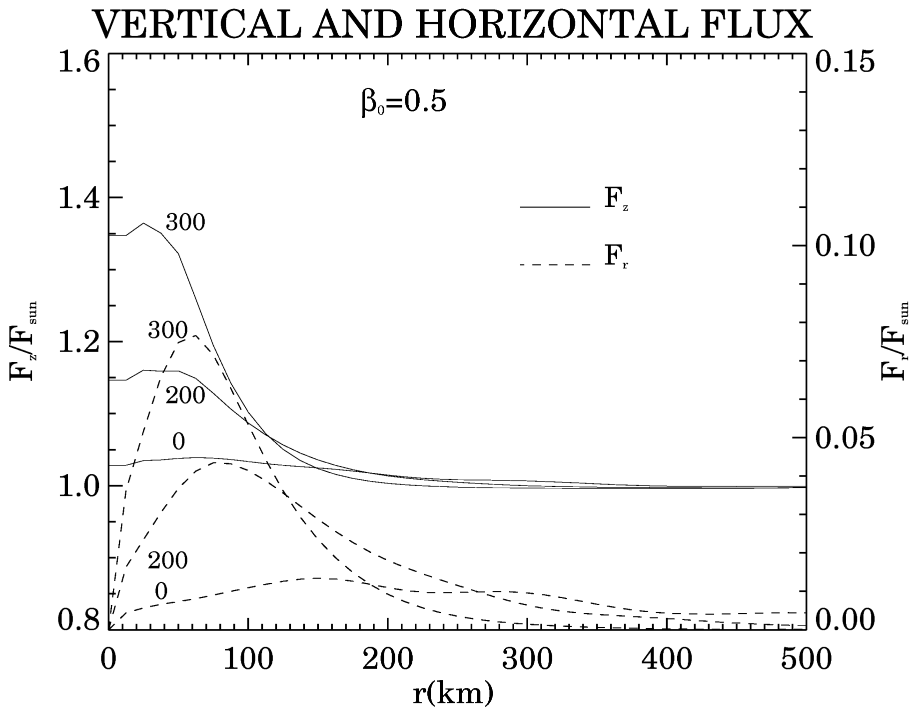

Figure 1. Variation with r of F_z (solid lines) and F_r (dashed lines) at a fixed height for $\beta_0 = 0.5$, in units of $F_{\rm sun}$ or $F_\odot$, the flux in the quiet Sun. The numbers beside the curves denote the height (in km).

$$(\mathbf{I} - \Lambda^*) \left(\frac{\partial S^i}{\partial T^i}\right)_{T^i} \Delta T^i = (\Lambda - \mathbf{I}) S^i + G, \tag{6}$$

where T^i and S^i are the temperature and source function respectively at the start of the iteration and $\mathbf{I}$ is the unit matrix.

The transfer equation is solved using the method of short characteristics (Kunasz & Auer 1988). The formal solution of equation (2) along with an efficient interpolation scheme are used to recursively calculate the specific radiation intensity I at all grid points for prescribed boundary conditions in 2-D planes, chosen to be tangent to various radial shells. The integration over polar angle in each plane is done using a double-Gaussian quadrature, with 8 angles per quadrant. In order to carry out the azimuthal integration, we exploit the rotational symmetry and sum the contribution from different planes. At the lower boundary we assume that the incident intensity (in the upward directions) is known. We locate the upper boundary in layers where the medium is optically thin and neglect incoming radiation from above. In the horizontal direction periodic boundary equations are used. Having determined I we can carry out the integration over solid angle to determine J and $\mathbf{F}_{\rm R}$.

Each iteration cycle consists of a solution of the transfer equation followed by a correction of the temperature. The hydrostatic pressure equation on the flux tube axis and in the external atmosphere is solved at the beginning of the calculation; thereafter the pressure is kept fixed at the initial value, but the density is updated along with the temperature. The effects of partial ionization

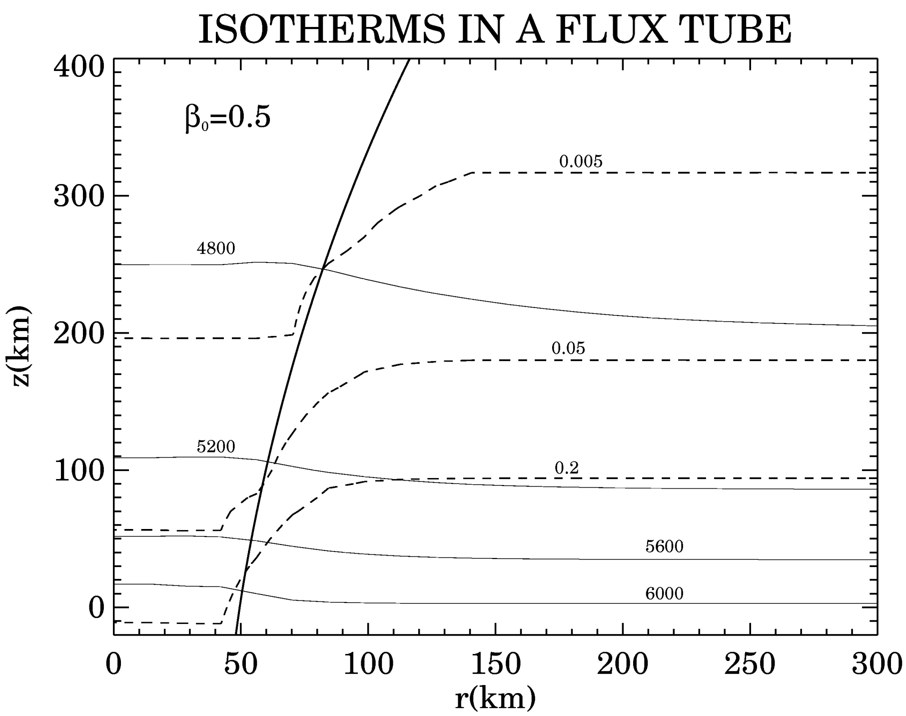

Figure 2. Isotherms (solid lines) and lines of constant optical depth (dashed lines) in a flux tube and the surrounding medium. The values of the contours are given above each curve. The heavy solid line denotes the boundary of the tube.

are also included. The Rosseland mean opacity is calculated by interpolating from tables.

4. Results

Figure 1 shows the variation with r (radial distance from the tube axis) of F_z (solid lines) and F_r (dashed lines) at a fixed height for $\beta_0 = 0.5$, where F_z and F_r are the vertical and horizontal components of the radiative flux, in units of $F_{\rm sun}$ or $F_\odot$, the flux in the quiet Sun (6.3×10^{10} erg cm^{-2} s^{-1}). The numbers besides the curves denote the height (in km). We find that at a height of 300 km, $F_z \approx 1.35 F_\odot$ and F_r is maximal near the tube boundary; in the lower layers ($z = 0$), F_r is negligible. At $z = 300$ km, F_r is an order of magnitude lower than $F_\odot$.

Figure 2 shows the isotherms (solid lines) and lines of constant optical depth (dashed lines) in a flux tube and the surrounding medium. The numbers above each curve denote the values of the contours. The heavy solid line denotes the boundary of the flux tube. The temperature is higher inside the flux tube than in the undisturbed ambient atmosphere. At the tube boundary, the temperature shows a gradual transition to the undisturbed ambient value. The region of influence of the flux tube on the ambient medium increases with height (reflecting the decreased density, hence increased photon mean free path). The lines of constant optical depth reflect mainly the sudden change in density, hence opacity,

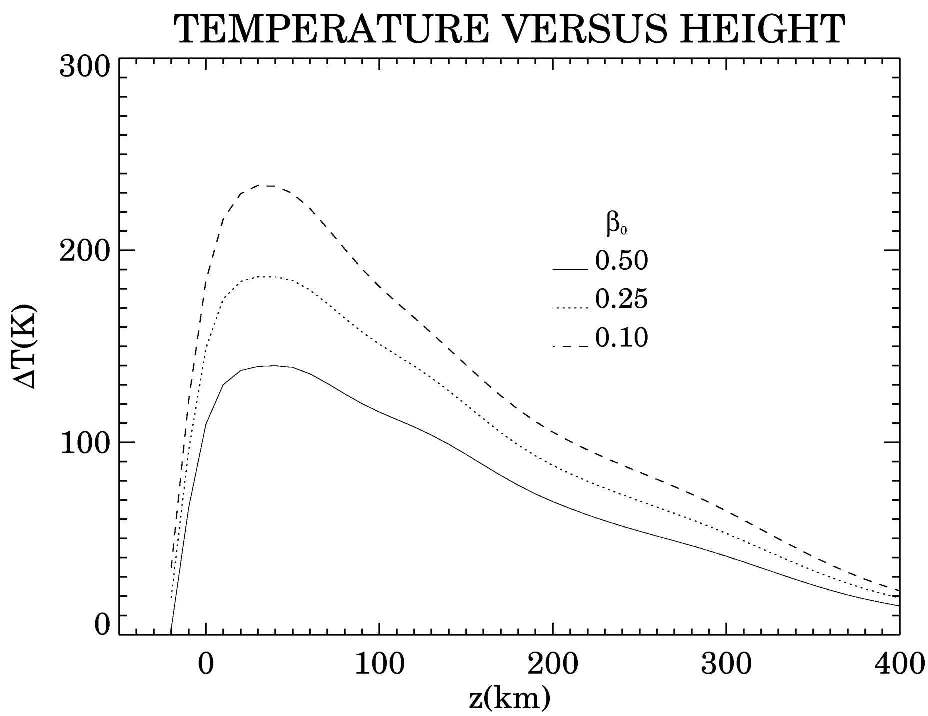

Figure 3. ΔT, the temperature difference between the value on the axis and that in the undisturbed atmosphere (at the same height), as a function of height z for various values of β_0.

at the tube boundary. The temperature increase acts in the same direction as a density reduction, but is of much smaller magnitude.

Figure 3 shows ΔT, the temperature difference between the value on the axis and that in the undisturbed atmosphere at the same height, as a function of z for different values of β_0. The temperature in the flux tube (with respect to the undisturbed atmosphere) is depressed in the deepest layers ($z < 30$ km) and ΔT reaches a maximum at $z \approx 40$ km. The lower the value of β_0 (the stronger the magnetic field), the greater is its effect on the temperature.

5. Discussion and Summary

Our results indicate that in the photosphere, the temperature inside the flux tube is generally higher than in the undisturbed ambient atmosphere — however, the temperature at the tube-external medium interface is approximately the same as that on the tube axis. We find that at the tube boundary, the temperature shows a gradual transition to the undisturbed value.

The vertical optical depth changes sharply at the tube boundary due to the abrupt change in density and hence opacity at the interface — the effect of temperature on the optical depth is of much smaller magnitude. Furthermore, the size of the thermal boundary layer (i.e., the distance over which the temperature changes from its value at the tube boundary to that in the undisturbed external atmosphere) increases with height (reflecting the decreased density and consequently increased photon mean free path).

We have also considered the effect of changing β_0. The lower the value of β_0 (or stronger the magnetic field), the higher the temperature inside the tube and the larger the size of the thermal boundary layer.

6. Conclusions

The main conclusions to emerge from our investigation are:

- We have demonstrated an efficient method for constructing equilibrium models of axially symmetric static magnetic flux tubes in the photosphere;
- We find that the influence of the flux tube on the ambient medium is important and needs to be considered when making comparison with observations, especially for strong lines formed in the upper photosphere — for weak lines in the deep photosphere the main effect is due to the reduced density in the tube (since the isotherms are nearly horizontal);
- A new feature of the results is that the temperature excess on the tube axis has a maximum in the low photosphere;
- A combination of the present calculations with our earlier work provides realistic model atmospheres for flux tubes extending into the convection zone and in which there is a continuous transition between convective and radiative energy transport.

Acknowledgments. We are grateful to NASA and the National Science Foundation for support. SSH is thankful to the Smithsonian Institution for travel assistance under SMIN grant number 9411FR00007 and the award of a Fellowship.

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