# GRAVITATING SYSTEMS WITH TORSION SELF-INTERACTION

*Letter to the Editor*

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The relation between the torsion vector $Q$ and the magnetic field $B$ it generates through the spin density $\sigma$ is known to be given (cf. De Sabbata and Sivaram, 1990a) by:

$$Q = 4\pi G\,\sigma/c^2 \tag{1}$$

where $B$ is given (cf. De Sabbata and Gasperini, 1982) by:

$$B = \left(\frac{8\pi}{3c}\right)(2\alpha G)^{1/2}\sigma. \tag{2}$$

Now in the case of magnetic or electric fields, it is known that $E^2$ or $B^2$ is proportional to the energy density of the field. The analogy between $Q$ and $B$ (which has been well established) suggests that we can interpret $Q^2$ as the energy density of the torsion field. This becomes more explicit in the case of propagating torsion, when we write

$$Q = \partial_\mu \phi, \tag{3}$$

so that

$$Q^2 = \partial_\mu \phi \partial^\mu \phi \tag{4}$$

can be interpreted as an energy density. The total torsional self energy would then be given by the volume integral

$$E_T = \int Q^2\,\mathrm{d}V. \tag{5}$$

Thus $Q^2 \,\alpha\, 4\pi G^2 \sigma^2/c^4$ can be interpreted as a contribution to the energy density in the Poisson equation, thus enabling us to write the modified Poisson equation as

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$$\nabla^2\phi \approx 4\pi G(\rho - G\sigma^2). \tag{6}$$

For the case of just gravitating torsional self-energy we would have

$$\nabla^2\phi_T \approx -4\pi G^2\sigma^2. \tag{7}$$

This has a solution

$$\phi_T \sim G^2\sigma^2 r^2. \tag{8}$$

If $S$ is the total intrinsic spin of the source we can write

$$\sigma \sim S/r^3, \tag{9}$$

so that

$$\phi_T \sim \frac{G^2S^2r^2}{r^6} \sim \frac{G^2S^2}{c^4r^4}, \tag{10}$$

or in units with $c$,

$$\phi_T \sim \frac{G^2S^2}{c^4r^4} \tag{11}$$

This would explain why the torsion potential $\phi_T$ falls off with distance as $\sim 1/r^4$, rather than $\sim 1/r$ for the ordinary gravitational potential $\phi_g \sim GM/r$. We note also that the $G^2$ dependence (rather than $G$) and also the proportionality to the source strength squared, i.e. $S^2$.

In general if a system has both mass $M$ and spin $S$, Eq. (7) has an equilibrium solution

$$R_e \approx \left(\frac{GS^2}{Mc^4}\right)^{1/3}. \tag{12}$$

This can also be roughly seen by matching the torsion potential $\phi_T$ as given by Eq. (12) with the ordinary gravitational potential $\phi_g$. Thus torsion would dominate at short distances. We can apply Eq. (13) in the context of strong gravity to a typical elementary particle, i.e, a proton with $M = m_p$, $S = \hbar/2$. Eq. (13) then gives with the strong gravity coupling $G_f \approx 6.7 \times 10^{30}$ cgs units, a radius of

$$R_p \approx \left(\frac{G_f\, h^{-2}}{4m_pc^4}\right)^{1/3} \approx 2 \times 10^{-14}\ \text{cm}, \tag{13}$$

close to the Compton wavelength of the proton. For the electron this gives a value close to the classical electron radius $r_e = e^2/m_ec^2 \approx 2 \times 10^{-13}$ cm. So we have the possibility of explaining both the stable elementary particles, i.e., the electron and proton in terms of strong gravity supported torsion self-interaction!

Eqs. (13) and (14), when applied to the proton and the electron, would give the ratio of strong and electromagnetic interactions coupling constants (De Sabbata and Sivaram, 1990b) as $g^2/e^2 \propto (m_p/m_e)^{2/3} \approx 150$, close to the observed value. Also the interpretation of unstable particles as torsion supported quantum vortices in an earlier paper (Garcia de Andrade and Sivaram, 1992; Sivaram and Sinha, 1977), would suggest the mass spin relation for hadronic resonances as $S\alpha(G_f/\hbar c)\,M^2$, or $S = aM^2$, where $a$ agrees with the observed Regge slope of $a \approx 1$ GeV$)^{-2}$ and the empirically observed $S$-$M^2$ relation for a multitude of resonances.

## References

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