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An Estimation of the Amount of Heating for Some Solar Coronal Loops

G. Elwert

Lehrsthul fuer Theoretische Astrophysik der Universitaet Tuebingen, Auf der Morgenstelle 12,7400 *Tuebingen, West Germany*

and

Udit Narain

Astrophysics Research Group, Meerut College, Meerut 250 001.

(*Received July* 31,1979; *Revised December* 13, 1979 *and April* 11, 1980)

Abstract

Based on the standard idea that short cooling times imply continued heating, a method of estimation of the amount of heating is developed and applied to the five examples of Krieger (1978) assuming conduction to be the dominant cooling mechanism in plane parallel and line dipole geometries. As expected, one requires more heating in former geometry than in the latter. The required total energy supplied by heating is found to be comparable to the total thermal energy of the events under consideration.

I. INTRODUCTION

In recent years, many investigators (Neupert et al. 1974; Cheng and Widing 1975; Pallavicini et al. 1975; Widing and Cheng 1974; Vorpahl et al. 1977) have noticed that the observed cooling times of the solar coronal features are longer than the conductive cooling times. Mainly two proposals have been put forward to explain this discrepancy. In one, the conduction is inhibited either geometrically (Neupert et al. 1974) or through plasma turbulence (Cheng and Widing 1975; Widing and Cheng 1974). In the other, the observed feature is somehow continuously heated (Cheng and Widing 1975; Pallavicini et al. 1975; Vorpahl et al. 1977).

In a recent paper, Krieger (1978) thoroughly discussed the discrepancy of observed and calculated cooling times. Whether conduction is inhibited or not, this discrepancy can always be removed by postulating a source of continued heating, the indirect observational evidence of which is now becoming available (Levine and Withbroe 1977; Gerassimenko et al. 1978). All the studies listed above lead one to arrive at the same conclusion as stated above. Here, based on this standard idea, we give a method of estimating the required amount of heating with and without geometrical inhibition.

II. THEORY

We make following simplifying assumptions :

1. Flow of energy takes place along the magnetic field lines which are current-free above the chromosphere and the flow across the field-lines can be ignored.
2. Temperatures are large so that the gravitational effects can be ignored.
3. Conductive losses dominate over the radiative ones.
4. All the variables can be written as a product-of-two functions, one depending on time and the other on arclength(s) along the field lines.
5. There is no exchange of plasma between flux-tube and chromosphere, which implies that the electron density is a function of arc-length only.

Under the assumptions made above, the equation of energy transfer in presence of a source of heating (Strauss and Papagiannis 1971) is

$$\frac{\partial}{\partial t}(3nkT)=\frac{1}{A}\,\frac{\partial}{\partial s}\left(A\kappa\frac{\partial T}{\partial s}\right)+Q(o,t)\,e^{-\frac{s^2}{\gamma^2R^2}}, \tag{1}$$

in which the coefficient of thermal conductivity is given by (Spitzer 1962; Antiochos and Sturrock 1976)

$$\kappa = \alpha\, T^{5/2},\quad \alpha \simeq 10^{-6}. \tag{2}$$

Here n is the electron density, k the Boltzmann constant, T the plasma temperature, A the normalized area of cross section of the flux-tube, $R=H+D$, H being the

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height of the loop above the chromosphere and D the depth (Antiochos and Sturrock, 1976), Q (o,t) is a function of time and γ is a measure of the breadth of the source. The nature of the source is left unspecified for a comment to be made later. Using the hydrogen plasma pressure (assuming it fully ionized)

$$p = 2nkT, \tag{3}$$

Eq. (2) and assumption (5), we obtain from (1) the following for the pressure p

$$\frac{3}{2} p(t)^{-3.5} \frac{dp}{dt} = \frac{\alpha}{A} \frac{d}{ds} \left(A \frac{dG}{ds} \right) + Q(o,t) p(t)^{-3.5} e^{-\frac{s^2}{\gamma^2 R^2}} \tag{4}$$

with

$$G(s) = \frac{2}{7} [2 k n(s)]^{-3.5}. \tag{5}$$

The general solution of (4) is possible only by numerical methods. In order to simplify the approach, we seek an analytical solution. A convenient choice of the time-part of the heating term, which leads to such a solution, is

$$Q(o, t)\, p(t)^{-3.5} = Dp\,, \tag{6}$$

where Dp is a constant, independent of time. For simplicity, we assume it independent of arc-length also. A discussion of this point will be given later.

Condition (6) makes (4) soluble by the method of separation of variables. The time-part gives

$$p(t) = p_o \left[1 - \frac{5}{3} K_s\, p_o^{2.5}\, t\right]^{-0.4}, \tag{7}$$

where $p_o = p\,(t{=}o)$ and K_s is the constant of separation. This constant is chosen such that the heating decreases with time. A particular choice is

$$K_s = -\frac{3}{5} p_o^{-2.5}\, \tau^{-1}, \tag{8}$$

where τ is the characteristic time of decay of a coronal loop. Now (7) takes the form

$$p(t) = p_o \left(1 + \frac{t}{\tau}\right)^{-0.4}. \tag{9}$$

The part depending on arc-length 's' gives

$$\frac{\alpha}{A} \frac{d}{ds} \left(A \frac{dG}{ds}\right) + Dp \cdot e^{-\frac{s^2}{\gamma^2 R^2}} = K_s. \tag{10}$$

Depending on the area of the cross section parameter A, two cases arise :

(a) Plane Parallel Geometry :

In this case the area of cross section of the magnetic fluxtube is throughout the same which implies

$$A = 1. \tag{11}$$

On integrating (10) once, under the condition $\frac{dG}{ds} = 0$ at $s = 0$, one obtains

$$\frac{dG}{ds} + \frac{Dp}{\alpha} I_1(\gamma s) = \frac{K_s}{\alpha} s, \tag{12}$$

with

$$I_1(\gamma s) = \int e^{-\frac{s^2}{\gamma^2 R^2}} ds. \tag{13}$$

The second integration gives

$$G(s) = G_o + \frac{K_s}{2\alpha} s^2 - \frac{Dp}{\alpha} I_2(\gamma s), \tag{14}$$

with

$$I_2(\gamma s) = \int (I_1(\gamma s))\, ds, \tag{15}$$

and

$$Go = G(s = o).$$

Let the foot-points of the flux-tube be situated at a distance of s_b from its top (s=0). Then $G(s_b) \approx 0$, (5) and (8) lead to the following expression for the decay time:

$$\tau_p = \frac{1.05\ \alpha^{-1} p_o T_{oo}^{-3.5} s_b^2}{1 - 3.5 \dfrac{Dp}{\alpha} \left(\dfrac{p_o}{T_{oo}}\right)^{3.5} I_2(\gamma s_b)}, \tag{16}$$

with

$$T_{oo} \equiv T(s = 0,\ t = 0).$$

(b) Line Dipole Geometry :

Here the area of the cross-section is proportional to angle (see, e. g. Fig. 1 of Antiochos and Strurrock (1976). In particular,

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$$A = \cos^2 \theta , \quad (17)$$

$$s = R\,\theta . \quad (18)$$

Proceeding in the same way as in the constant cross-section case, the final expressions are

$$G(\theta) = Go + \frac{K_s R^2}{2\alpha}\,\theta \tan\theta - \frac{DpR^2}{\alpha}\, I_2(\gamma\theta), \quad (19)$$

with

$$I_2(\gamma\theta) = \int I_1(\gamma\theta)\sec^2\theta\, d\theta, \quad (20)$$

where

$$I_1(\gamma\theta) = \int e^{-\frac{\theta^2}{\gamma^2}} \cos^2\theta\, d\theta. \quad (21)$$

and

$$\tau_{\text{dipole}} = \tau_p \frac{\tan\theta_b}{\theta_b}. \quad (22)$$

III. METHOD AND RESULTS

The heating function, in view of (1), (6) and (9), is

$$Q(\theta, t) = Q_0 \left(1 + \frac{t}{\tau}\right)^{-1.4} \exp\left(-\frac{\theta^2}{\gamma^2}\right), \quad (23)$$

$$Q_0 \equiv Q\,(\theta = 0, t = 0) = Dp\,p_0^{3.5}, \quad (24)$$

It is obvious from (23) and (24) that one requires Dp, p_0, τ, γ and R for the evaluation of the desired amount of heating.

Krieger (1978) reported observed cooling times (τ) assuming an exponential decay. Our Figure 1 shows that to a good approximation the values of τ reported by him could be used in (23).

p_0 can be calculated easily, using equation (3) and the values of electron density (n), and plasma temperature (T_{00}), also reported by Krieger (1978).

Following Antiochos and Strurrock (1976), the compression factor Γ is given as

$$\Gamma = \frac{A\,(\theta = 0)}{A\,(\theta = \theta_b)} = \frac{B\,(\theta = \theta_b)}{B\,(\theta = 0)} = \sec^2\theta_b, \quad (25)$$

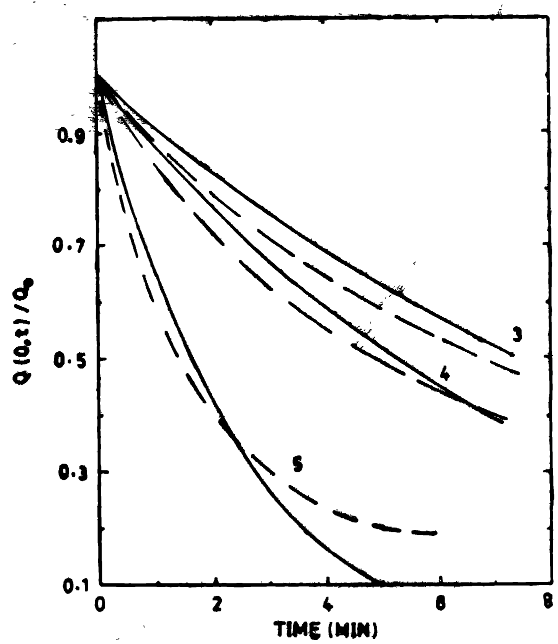

Fig. 1 : Comparison of the decay rate of heating based on an exponential law (solid line) and equation (23) (broken line).

3. Flare loop (June 15)
4. Flare kernel (Sept. 1)
5. Flare kernel (Aug. 9).

in which B is the strength of the magnetic field. Using (25), Krieger (1978) obtained the maximum possible values of the compression factor (Γ_{max}) for all the 5 events considered by him. Then equations (18), (25) and $2\,s_b = l$ give R in a straightforward manner. The total length (l) of the loop has been taken from Krieger (1978). The parameter Dp is obtained from equations (13), (15), (16), (20)-(22) and the observed values of the cooling times.

Corresponding to whether the whole loop, or a half of it, or a small part of it is heated, we choose γ = 1.0, 0.5 and 0.1, respectively. One can estimate the amount of heating for any other reasonable value of γ in the same way as in these representative cases.

In order to discuss the time-dependence of the source of heating in the present model, we exhibit present $Q\,(0, t)\,/\,Q_0$ against time (t) together with that for an exponential decay, graphically.

The maximum amount of heating (Q_0) estimated for all the 5 events, for the three above-mentioned values of γ, are presented in Table 1, along with the values of R, for constant cross-section and line dipole geometries.

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TABLE 1

Q_0 (in erg cm^{-3} sec^{-1})

X-ray event type (1973)	R † (cm)	$\gamma = 0.1$		$\gamma = 0.5$		$\gamma = 1.0$	
		$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$
Filament disappearance ..	4.6^{9}[a]	2.0^{-3}	..	4.0^{-4}	..	2.1^{-4}	..
Flare loop (Nov. 26)	6.9^{8}	2.3	3.1^{-1}	0.46	7.0^{-2}	0.24	4.5^{-2}
Flare loop (June 15)	1.1^{9}	1.1	1.0^{-2}	0.22	1.0^{-2}	0.11	1.0^{-2}
Flare kernel (Sep. 1)	5.6^{7}	1.1^{3}	3.5^{2}	2.2^{3}	7.8^{1}	1.2^{2}	5.1^{1}
Flare kernel (Aug. 9)	1.4^{8}	3.0^{2}	9.3^{1}	6.0^{1}	2.1^{1}	3.3^{1}	1.4^{1}

a The superscript denotes the power of ten by which the number is to be multiplied.

† The height of the loop above chromosphere differs from R by only about 10 per cent.

TABLE 2

Total amount of heating Q_T^* (in erg)

X-ray event type (1973)	Area of cross section at s=0 (km^2)	$\gamma = 0.1$		$\gamma = 0.5$		$\gamma = 1.0$	
		$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$
Filament disappearance ..	6.2^{8}[a]	1.9^{29}	..	1.9^{29}	..	1.9^{29}	..
Flare loop (Nov. 26)	1.3^{7}	4.5^{28}	6.0^{27}	4.5^{28}	6.1^{27}	4.5^{28}	6.0^{27}
Flare loop (June 15)	3.2^{7}	2.7^{28}	2.4^{26}	2.7^{28}	1.1^{27}	2.6^{28}	1.7^{27}
Flare kernel (Sept. 1)	7.1^{4}	2.1^{27}	6.7^{26}	2.1^{27}	6.6^{26}	2.2^{27}	6.6^{26}
Flare kernel (Aug. 9)	4.1^{5}	2.4^{27}	7.4^{26}	2.4^{27}	7.4^{26}	2.4^{27}	7.6^{26}

a The superscript denotes the power of ten by which the number is to be multiplied.

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The total amount of heating may be estimated by integrating the expression (23) as follows :

$$Q_T = Q_o \int_0^{\tau} \left(1+\frac{t}{\tau}\right)^{-1.4} dt \int_{-s_b}^{s_b} e^{-\frac{\theta^2}{\gamma^2}} F.A.ds, \quad (26)$$

where F is the actual area of cross section of the flux-tube at the top. Since the value of F, in general, is not available, we make the assumption that the radius r of the flux tube at the top is one tenth of the length. In general, it is

$$F = \pi\,(0.1l)^2 \left(\frac{r}{0.1l}\right)^2 = \pi\,(0.2\,s_b)^2 \left(\frac{r}{0.2s_b}\right)^2. \quad (27)$$

Now (26) and (27) give :

$$Q_T = \frac{\pi\, Q_o \tau}{5}(1-2^{-0.4})\, s_b^2 \int_0^{s_b} e^{-\frac{\theta^2}{\gamma^2}} A.ds \left(\frac{r}{0.1l}\right)^2 \quad (28)$$

$$= Q_T^* \left(\frac{r}{0.1l}\right)^2.$$

Equation (28) can now be used to estimate the total amount of heating for constant cross section (equation (11)), and line dipole (equation (17)) geometries. The results of these calculations are presented in Table II.

IV. DISCUSSION AND CONCLUSION

It is obvious from Tables 1 and 2 that Q_o and Q_T^* in constant cross section geometry are larger than those in line dipole geometry. This is quite expected. In constant cross section geometry, no inhibition of conduction takes place; consequently, flow of mass and energy takes place freely. In order to observe the same decay time, more heating is required. This may be considered as the upper limit of heating, becaus- the presence of some inhibiting mechanism cannot be ruled out (Neupert et al. 1974; Cheng and Widing 1975; Widing and Cheng 1974; Vorpahl et al. 1977). The inhibition of conduction can take place either geometrically or through plasma turbulence. For the events considered here the plasma turbulence generated by temperature gradient has been ruled out (Krieger 1978). Consequently we consider here geometrical inhibition only. We have used maximum possible values of compression factor which implies that the geometrical inhibition is maximum. As a result, the amount of heating in line-dipole case represents the lower limit. This means that the direct observations, if possible, are likely to give values in between the two limits mentioned above.

Table 1 also shows that Q_o for small γ is larger than that for large γ. This is also quite expected. $\gamma = 0.1$ represents that roughly 1/10 of the flux-tube is heated, whereas $\gamma = 0.5$ means that approximately half of it is heated. Thus, to observe the same decay time in the former case, one needs more energy at the top than in the latter case.

Fig. 1 shows that at the beginning of the cooling phase, the rate of decay of heating according to our formula (23) is a little faster than that for an exponential decay, which was assumed by Krieger (1978) while deducing the values of observed decay times. As the time progresses, the latter becomes faster than that in the present case. This behaviour is quite similar to that of Culhane et al. (1970) for the decay of temperature (their Figure 2).

The time-dependence in the present model comes from our assumptions (3) and (5). Assumption (3) is true only when the plasma temperature is appreciably high. As the temperature decreases, other processes, e.g., conduction driven evaporation (Antiochos and Sturrock 1978) and radiation overtake it (Krall et al. 1978). In the light of the above statements, the present rate of decay of heating is quite expected.

Strauss and Papagiannis (1971), using a similar expression for the space part of the heating arrived at a form of $Q(o, t)$ (see, e.g., their Figure 4) by fitting it to the observational data for the flare of June 20, 1968. Consequently, we cannot compare our results with theirs. A more realistic model will perhaps give a better description of the rate of decay of heating.

Various mechanisms have been proposed for the heating of solar coronal loops (Athay 1976; Svestka 1976). We do not consider our present model capable of discriminating among them ; consequently it is left to a later and more sophisticated study. It is obvious from Table 2 that the required total energy supplied by heating is in the range 10^{26} to 10^{29} ergs. This is comparable to the total thermal energy of the events under consideration.

All static models including the present one predict the transmission of high heat fluxes to the transition zone and the upper chromosphere (Antiochos and Sturrock 1976, 78; Krieger 1978), the observational evidence of which does not seem to be available. This discrepancy can be removed by taking exchange of plasma between the coronal loop and the chromosphere into account (Antiochos and Sturrock 1978). This amounts to relaxing assumption (5). Further, the numerical approach is the most appropriate way to solve equation (4) or its modified versions. Our efforts in these directions are in progress and the results will be reported later.

ACKNOWLEDGEMENTS

The authors thank Dr. L. Elzner and Dr. R. R. Rausaria for helpful discussions. They express their gratefulness to Professor P. A. Sturrock, Dr. A. S.

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Krieger and Dr. S. K. Antiochos for helpful correspondence. Special thanks are due to Dr. R. Mewe, Professor C. de Jager and Professor Z. Svestka of the Space Research Laboratory of the Astronomical Institute at Utrecht for pointing out an error in Table 2. One of them (U. N.) acknowledges the receipt of a German Academic Exchange Service Fellowship during the period of this work. He also wishes to thank the Meerut College authorities for granting necessary leave.

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