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Proof of absence of spooky action at a distance in quantum correlations

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Abstract. I prove that there is no spooky action at a distance and nonlocal state-reduction during measurements on quantum entangled systems. The prediction of quantum theory as well as experimental results are in conflict with the concept of nonlocal state-reduction, as conclusively shown here under very general assumptions. This has far-reaching implications in the interpretation of quantum mechanics in general, and demands a radical change in its present interpretation of measurements on entangled multiparticle systems. Motivated by these results we re-examine Bell's theorem for correlations of entangled systems and find that the correlation function used by Bell fails to incorporate phase correlations at source. It is the use of such an unphysical correlation function, and not failure of locality, that leads to the Bell's inequalities.

Keywords. Quantum nonlocality; EPR state; Bell's inequalities; entangled states.

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1. Introduction

The standard folklore in quantum measurement theory is that a local measurement on one of the several subsystems of a quantum entangled system results in the state-reduction of the associated space-like separated systems as well. In the context of a two-particle entangled system, this means that a measurement and the state-reduction of one of the particles result in the instantaneous state-reduction of the other particle as well, however space-like separated the 'unmeasured' particle may be. While no one has been able to identify the physical mechanism resulting in such a nonlocal state-reduction, the experimental fact that the state of the second particle 'is always found to agree with the prediction made on the basis of the first measurement' has led to the belief that the state of the distant particle has collapsed to a definite eigenstate as a result of the first measurement.

In this paper I prove that this conclusion is not valid and that there is no state-reduction at a distance. Such nonlocal state-reduction is in conflict with the basic mathematical formalism of quantum mechanics as I rigorously show. It is also in conflict with experimental results. Simple applications to the foundations of quantum information theory is outlined.

In fact, it is surprising to realize that almost the entire physics community subscribes to the view of nonlocal state-reduction despite the total failure to find the influence responsible for such a phenomenon. Conceptually nonlocal state-reduction goes against the spirit of special relativity, and smells strongly of irrationality. Therefore, it is very significant that a rigorous proof can be given for absence of nonlocal state-reduction.

2. Proof of absence of spooky action at a distance

A state-reduction in quantum systems is characterized by two essential aspects. (a) The state possesses a definite value for an observable or a set of observables. (b) The dispersions of noncommuting observables satisfy the uncertainty principle. In classical systems the second aspect is irrelevant. Conversely, if the second condition is violated the system cannot be in a definite quantum state.

Consider an experiment in which a source emits pairs of particles entangled in transverse position and momentum. This is very similar to the EPR entangled state [1], except that the entanglement here, for convenience, is in the transverse position and transverse momentum. The operators corresponding to relative position and total momentum commute, and there are simultaneous eigenstates in which the two particles are correlated through the conservation law for momentum.

In order to keep the discussion as close as possible to an experimental realization, and to use the convenience of a finite set of eigenvalues, we write the initial entangled state as

$$|\psi\rangle_{12} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |x_i\rangle_1 \, |x_i'\rangle_2 \, . \tag{1}$$

We use the correlation information in the entangled state, $x_i' = -x_i$, with the line of symmetry chosen as $x = 0$, to write this as a superposition of product states

$$|\psi\rangle_{12} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |x_i\rangle_1 \, |{-x_i}\rangle_2 \, . \tag{2}$$

This is similar to writing the singlet spin-entangled state in terms of the spin eigenvalues as

$$|\psi'\rangle_{12} = \frac{1}{\sqrt{2}} \left[|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2 \right] . \tag{3}$$

A measurement on one of the particles, say particle 1, reduces the state (2) to a simple product state

$$|\psi\rangle_{12} \to |\varphi\rangle_{12} = |x_j\rangle_1 |-x_j\rangle_2 , \tag{4}$$

where the index j is one of the values in the range $\{1, N\}$. The standard interpretation of quantum mechanics then states the following:

(a) The measurement on the first particle reduced its state to a definite eigenstate with a particular eigenvalue of the relevant observable (position x_j, in this case).

(b) Since the total state is reduced to the product state $|x_j\rangle_1 |-x_j\rangle_2$, the second particle has reduced instantaneously and nonlocally to the eigenstate $|-x_j\rangle_2$, with eigenvalue $-x_j$.

This is how nonlocality enters the quantum description.

I now rigorously prove that this interpretation is not correct, and that there is no nonlocal state-reduction.

The proof is very simple and uses just the fundamental lemma of quantum mechanics.

Fundamental Lemma. An eigenstate of position is superposition of eigenstates of momentum, obeying the Fourier relation. For the state we have written this can be stated as

$$\xi(x)_1 = \langle x|x_j\rangle_1 \propto \int_{-2h/\delta x}^{2h/\delta x} \mathrm{d}p\ \exp(ipx). \tag{5}$$

In general, an eigenstate of one observable is a superposition of eigenstates of a noncommuting observable.

Comments. The uncertainty principle connecting position and momentum is a consequence of the noncommuting relation contained in the lemma above. The immediate physical effect is that a localized eigenstate of position starts spreading and evolves into a dispersed state after finite time. If we had used continuous eigenvalues with arbitrarily large precision of position measurement, this Fourier integral will range over all momenta from $-\infty$ to ∞.

The state of the first particle immediately after the measurement obeys this fundamental lemma, and the momentum states that make up the position eigenstate reveal in the spreading and dispersion of the position eigenstate $|x_j\rangle_1$ after the measurement (see figure 1a). If the second particle is truly in an eigenstate of position as a result of the measurement on the first particle, then its true state is identical in properties regarding dispersion to the state of the first particle. This is because the dispersion implied by the relation in eq. (5) is independent of the shift in x. Therefore, standard interpretation of quantum mechanics implies that

$$\xi'(x)_2 = \langle x|-x_j\rangle_2 \propto \int_{-2h/\delta x}^{2h/\delta x} \mathrm{d}p\ \exp(ipx). \tag{6}$$

The implication of this fundamental result is startling. The second particle that is unmeasured should then behave like the first particle in dispersion, since both are in eigenstates of position with similar uncertainties in their position eigenvalue. Therefore the wave function of the second particle should start spreading spontaneously, instantaneously and nonlocally. But this contradicts the mathematical prediction of quantum mechanics in too drastic a manner. For, quantum mechanics predicts that the state of the particle *will be found to be correlated with the first particle, irrespective of the time delay after which the first particle's position was measured* (figure 1b). The state in eq. (6) does not allow this. Therefore, we have proved that there is no nonlocal state-reduction. Experimental results support this proof [2].

The mathematical formalism of quantum mechanics predicts that the second particle will be found in an angular width defined by $\delta x/L$, centred around the angle $-x_j/L$, where L is the distance from the source to the detectors of the first measurement. But, the position eigenstate implied by the present interpretation contradicts this prediction. It starts spreading, and the probability to find the second particle at larger angles now depends on whether there is a measurement made on the first particle. This clearly violates signal locality, since it is possible to determine statistically whether a measurement has been made on the first particle by observing the average dispersion of the second particle. On the

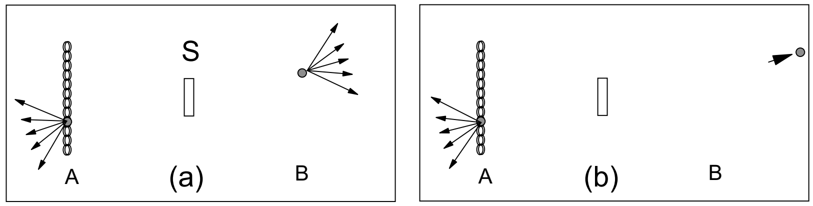

Figure 1. The left panel shows the consequence of a measurement on the first particle on the dispersion of the second particle, if the present interpretation of nonlocal state-reduction in quantum mechanics is valid. S is the source and there is an array of detectors at A. The right panel shows what is consistent with the mathematical prediction of quantum mechanics. It shows what would be measured if a measurement is actually made on the second particle. Till a real measurement is made there is no state-reduction.

other hand, if we did not interpret the prediction of quantum mechanics in terms of the projection postulate for the second particle, we would have merely said that the product state implies that the second particle *will be found* to be in an eigenstate of position $|-x_j\rangle$, with eigenvalue $-x_j$ *iff a measurement is actually made on the second particle*. Then, we cannot write the state of the second particle as given in eq. (6), even after the measurement on the first particle, and there is no nonlocal state-reduction. There is only a prediction, using prior information on correlation, of what would be found if a measurement is made. Then there is no conflict between the prediction of quantum mechanics and our revised interpretation. But then the aspect of nonlocal state-reduction is blown away! However disappointing this may be for advocates of nonlocality, the conclusion that there is no nonlocality during measurement on entangled systems is inescapable, since it relies only on the fundamental lemma of quantum mechanics relating eigenstates of one observable to that of a noncommuting conjugate observable.

Figure 1 shows the difference between the standard nonlocal interpretation of quantum mechanics and the situation without nonlocality advanced in this paper. What happens in a real experiment is what is depicted in the second panel, and not the one in the first panel.

It is worth restating this proof in a different but equivalent way. When the first particle is found in an eigenstate of position with eigenvalue x_1 and uncertainty Δx_2, as a result of a real measurement, its subsequent evolution obeys the uncertainty principle. The dispersion in momentum as measured at far field will be given by $\Delta p_1 \geq \hbar/\Delta x_1$. This can be made arbitrarily large by increasing the precision of the position measurement. If the position of the first particle is measured to be x_1 with spread Δx_1, the position at which the second particle will be found can be predicted with good certainty, using the conservation law, as some x_2, with $\Delta x_2 \sim \Delta x_1$. If this amounts to a true state-reduction to a definite position x_2, then its momentum should spread out to satisfy $\Delta p_2 \geq \hbar/\Delta x_2$ even without a real measurement. This does not happen, since if it did, signal locality could be violated – we could send signals faster than light. This becomes possible since $\Delta x_1 \sim \Delta x_2$ can be made as small as one wishes and then Δp_2 will become larger than any initial spread of the original state. This feature is not observable in experiments with spin, since the variance in spin is a bounded variable. The variance in noncommuting observables does not increase beyond $\hbar^2/4$ for the case of spin.

Note that there is no need to assume any specific model of state-reduction for the proof to be valid. The feature used in the proof is the requirement that the behaviour in dispersion should be similar for two particles that are in similar eigenstates. It is an argument that compares the behaviour of the two particles, and hence it is completely independent of models of state-reduction.

Remarkably, Popper [3] had proposed such an experiment in a different context. Recently this experiment was performed [4], and consistent with our assertion based on signal locality [5], no additional spread of momentum of the second particle was seen. This clearly proves that there is no state-reduction at distance. It also suggests that terms like quantum teleportation are inappropriate. What is seen in those experiments are prior correlations encoded in the initial phase coherence at source as I have shown earlier [2,6,7].

3. Discussion and summary

3.1 *What is unphysical in Bell's theorem?*

Since the result that there cannot be any nonlocality in quantum correlations is established rigorously, we have to re-examine the standard claim that the conflict between Bell's theorem and observed quantum correlations implies violation of local realism. Most of the physicists interpret this as a violation of locality itself, forgetting that locality and physical reality are two distinct and independent concepts. In any case, I have already shown that there cannot be any nonlocality. Therefore, there must be some other step in the derivation of the Bell's theorem [8] that conflicts with the observed physical results. It turns out to be easy to spot the physical flaw in the formulation of Bell's theorem. The correlation function used by Bell, and all local realists, is of the form [8,9]

$$P(\mathbf{a},\mathbf{b}) = \int \mathrm{d}h\rho(h)A(\mathbf{a},h)B(\mathbf{b},h), \quad \text{where} \int \mathrm{d}h\rho(h) = 1. \tag{7}$$

Here A and B are eigenvalues of measurements at space-like separated points and $\mathbf{a}$ and $\mathbf{b}$ represent local measurement settings. h represents hidden variables that determine individual outcomes, and $\rho(h)$ is the probability distribution of the hidden variables. Unfortunately, such a correlation function does not incorporate the possibility of phase coherence at source. The Bell correlation function is an average over products of eigenvalues, and an eigenvalue does not have a phase. Therefore, such correlation functions ignore any phase coherence at source. This is equivalent to ignoring a crucial wave property. Imagine trying to reproduce an interference pattern theoretically by adding up only intensities (just because what is observed by the experimenter is intensity) and not amplitudes! Bell's theorem suffers from similar physical fallacy. In other words, Bell's theorem can be restated as the impossibility to reproduce observed quantum correlations if phase coherence at source is ignored. In this sense Bell's theorem is trivial, since we know that wave-like features and properties are crucial in explaining any quantum mechanical result. Physically it is clear why the discrepancy between Bell's result and quantum correlations could be cured by introducing concepts like negative probabilities, since it is possible to get an interference pattern even in a ray picture that ignores phase if one is willing to introduce negative intensities [10].

It is therefore clear why the joint detection probabilities for entangled systems are nonseparable. The local probabilities are not independent due to correlation at source, and not due to nonlocality. But the correlation is encoded in the phase coherence or relative phase and not in particle-like properties such as the relative spin direction or relative position. This possibility was not noticed by EPR when they formulated a definition of physical reality [1]. Similar situation is familiar in an almost classical context – that of the Hanbury Brown–Twiss intensity correlations [11]. Conceptually, quantum correlations are just a quantum step away from the classical wave intensity–intensity correlations.

This also means that all tests of Bell's inequalities are repeatedly trying to verify whether quantum correlations can be reproduced if quantum phase is ignored, and only particle-like properties (spin direction, position etc.) and statistical hidden variables are used in the theory. Obviously they are bound to find that Bell's inequalities are violated, purely because Bell's inequalities were derived in the first place after ignoring the physical reality of phase and phase coherence. Correlations and interference are the two sides of the quantum coin whose dynamics depends crucially on phase and phase coherence. Testing Bell's inequalities is then in the same spirit as testing whether an interference pattern can be produced using only light rays without phase (no experimenter is motivated to do this for obvious reasons). Bell's theorem has made its important contribution in making us realize that quantum mechanical probabilities are different from coin-tossing probabilities and that a local realistic theory based on concepts derived from gambling statistics with hidden variables cannot reproduce quantum correlations. But the realization that the quantum correlations are due to phase coherence at source makes the Bell's theorem redundant and solves the issues raised by the EPR query [2,6].

3.2 *Some simple applications*

For a multiparticle system containing N particles, $(N-1)$ relative phases are required to reproduce the quantum correlations [2], and in this sense this is the information content in entanglement. This approach suggests that the maximal violation of the Bell's inequalities is a factor of $\sqrt{2}$ for each relative phase, corresponding to each ignored phase coherence at source. Then we get the general result that the maximal violation for an N-particle entangled state is $(\sqrt{2})^{(N-1)} = 2^{(N-1)/2}$. This exponential violation is directly related to the entropy of entanglement, proportional to $N-1$.

Another application is regarding the robustness of information encoded using multiparticle entanglement. In the picture I have advocated, the entire mechanism of loss of entanglement is due to the local decoherence and the resultant diffusion of individual phases. Therefore, the larger the number of relative phases that is used to 'share' the coherence, the more robust the state is, since each relative phase has correspondingly lower weight in decohering the information. The formalism elaborated in ref. [2] also allows a simple understanding of phenomena that involve EPR pairs, like entanglement swapping, teleportation etc. without the aspect of nonlocal state-reduction. In fact, concepts like teleportation loses its quantum spice entirely since the whole exercise amounts to reconstructing a state given its correlation with a state that itself has a prior coherence in phase with the second particle of the EPR pair, available to the distant observer.

3.3 *Summary*

I have rigorously proved that there is no nonlocal state-reduction during measurements on entangled multiparticle systems. The physical reason why the Bell's inequalities are violated is traced to ignoring phase coherences at source in the correlation function used, and the violation has nothing to do with nonlocality. This result calls for a drastic change in the present interpretation of quantum mechanics with regard to the projection postulate applied to entangled systems.

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