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Evidence for the quantum birth of our Universe

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Abstract. We present evidence for a nonsingular origin of the Universe with intial conditions determined by quantum physics and relativistic gravity. In particular, we establish that the present temperature of the microwave background and the present density of the Universe agree well with our predictions from these intial conditions, after evolution to the present age using the Einstein–Friedmann equation. Remarkably, the quantum origin for the Universe naturally allows its evolution at exactly the critical density. We also discuss the consequences of these results to some fundamental aspects of quantum physics in the early Universe.

Keywords. Cosmological parameters; quantum cosmology; uncertainty principle; fundamental constants; gravity.

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The standard model of cosmology, the Big Bang theory, has been quite successful in describing the general features of the observed Universe. But the physics of the very early Universe is still uncertain. Solution of some of the serious problems of the classical Big Bang model requires input ‘beyond standard Big Bang’- like inflation. In the Big Bang theory of the Universe, reverse evolution via the Einstein equations yields a formal singularity in the finite past. It has been often suggested that quantum effects in the early Universe might cure many of the undesirable aspects of the Big Bang origin. However, there is no viable theory of quantum gravity at present.

In this paper we avoid the necessity of a formal quantum gravity theory to describe the origin of the Universe by making the assumption that the quantum origin would obey the uncertainty principle and relativistic gravity in an essential way. This allows us to deduce some important conclusions regarding observable features of the present Universe. While this approach cannot address the physical process of the origin itself, quantitative estimates of initial conditions can be made and their subsequent evolution and comparison with observational data in the present Universe can be made. We shall see that a comparison of the evolved data agrees well with the observations, suggesting the possibility that the origin of our Universe was indeed determined by quantum physics and relativistic gravity.

The most important cosmological parameters that we need to match are the present temperature (2.7 K), present density (very close to the critical density $\rho_0 = 3H_0^2c^2/8\pi G$, that makes the spatial curvature $k = 0$), and present Hubble parameter, $H_0 \simeq 68$ km/s-Mpc. The density of vacuum energy or a cosmological constant, if existing at all, is observed to be small on the Planck scale, and this can be ignored in the early Universe since the energy density will be dominated by that of radiation at very early times.

Processes such as quantum tunneling, quantum mechanical decay etc. are broadly characterized by the energy–time uncertainty relation, $\Delta E\Delta t \geq \hbar/2$. It is of particular interest to apply the uncertainty principle to physical systems that are 'materialized' in their lowest energy state, from some kind of a barrier. Then the energy itself, rather than its uncertainty could be estimated from the minimum uncertainty.

Our fundamental hypothesis is that the Universe took a quantum birth with characteristic time-scale $\tau_{\rm b}$ determined by fundamental principles of quantum physics and relativistic gravity [1]. We also assume that for times larger than this time-scale the Einstein equations and general relativity are valid, at least to a good approximation.

One might start with the assumption that dimensionally $\tau_{\rm b}$ would be the Planck time, $t_{\rm P} = (\hbar G/c^5)^{1/2} \simeq 5.4 \times 10^{-44}$ s. A consistent dimensional argument will give the values of the relevant quantities for the early Universe – its temperature, density, size etc. – all of the order of the Planck quantities derived from the fundamental constants G, c and $\hbar$. But this is in conflict with observations since an Universe with initial size of Planck radius and Planck density cannot evolve to the Universe of present size and present density [2]. Therefore, instead of starting with dimensional arguments, we choose to start with more fundamental physical principles.

We require that the minimal uncertainty principle connects the duration of the quantum birth $\tau_{\rm b}$ and the initial average energy $E_{\rm b}$ within a spatial size $r_{\rm b} \simeq c\tau_{\rm b}$. We have

$$E_{\rm b}\tau_{\rm b} \simeq \hbar/2. \tag{1}$$

We also require that the birth is characterized by a gravitationally relativistic process. This means that the energy content within a volume of radius $r_{\rm b}$ obeys

$$\frac{2GE_{\rm b}/c^2}{c^2 r_{\rm b}} = \frac{2GE_{\rm b}}{c^4\, c\tau_{\rm b}} \simeq 1. \tag{2}$$

We can solve for $\tau_{\rm b}$ from these fundamental assumptions and we get

$$\tau_{\rm b} \simeq \left(\frac{\hbar G}{c^5}\right)^{1/2}. \tag{3}$$

This turns out to be equal to the Planck time, with the natural numerical coefficients we have chosen. This justifies the usual dimensional argument that infers that the natural time-scale for a quantum origin is the Planck time.

We can estimate the initial average temperature from the uncertainty relation. We get

$$T_{\rm b} = E_{\rm b}/k_{\rm B} \simeq \hbar/2\tau_{\rm b}k_{\rm B} \simeq 7 \times 10^{31}\ {\rm K}.$$

In effect, our assumptions have fixed the initial temperature of the Universe to be approximately the Planck temperature.

It is important to note that *a priori* there is no reason why this temperature should be a unique point in the evolution of the Universe, though quantum gravitational effects are expected to be significant at this energy scale. The concepts of Planck time, energy (mass), temperature etc. are independent of the concept of an evolving Universe. We stress the fact that estimation of the temperature at Planck time starting from the present temperature and evolving backwards gives different results depending on the contents of the Universe, whereas our starting point is fixed uniquely by the fundamental principles of relativity and quantum mechanics. For estimating the present temperature, we chose the evolution of a Universe with its density close to the critical density, since this is supported by observations [3]. Also, the quantum birth scenario we present naturally singles out evolution at critical density, as we shall see.

The equations governing the evolution of the scale factor and the density in the Universe are the Einstein–Friedmann equations

$$\dot{R}^2 + kc^2 = \frac{8\pi G}{3}\rho R^2, \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}R}(\rho R^3) = -3pR^2. \tag{5}$$

The equation of state $p = p(\rho)$ will allow the determination of the evolution of the density as a function of the scale factor. With this, the scale factor $R(t)$ can be determined from the first equation. Here we consider a Universe that has been dominated by radiation and relativistic particles (labelled together as radiation) from the beginning to till about the time $t \simeq 10^{12}$ seconds, and then dominated by non-relativistic matter till present [4,5].

During the radiation-dominated era the scale factor $R(t)$ varies as $R(t) \sim t^{1/2}$, and during the matter-dominated era it varies as $R(t) \sim t^{2/3}$.

The temperature varies as the inverse of the scale factor throughout the whole evolution, $T(t) \sim 1/R(t)$.

Therefore, at the end of the radiation-dominated era ($t_\mathrm{r} \simeq 10^{12}$ s), we get for the temperature,

$$T(t_\mathrm{r}) = T_i/(t_\mathrm{r}/\tau_\mathrm{b})^{1/2} \simeq 5 \times 10^4 \text{ K}.$$

So, during $t_\mathrm{r} \simeq 10^{12}$ s to the present, $t_0 \simeq 5 \times 10^{17}$ s,

$$T(t_\mathrm{r}) = T(t_\mathrm{r})/(t_0/t_\mathrm{r})^{2/3} \simeq 1.7 \text{ K}. \tag{6}$$

A correction from the entropy increase from $e^+ - e^-$ annihilation [4,5] raises the temperature to 2.4 K. Our prediction matches with observed temperature with remarkable closeness. The present age of the Universe is not accurate to better than about 30%, and the numerical factors of order unity we use in the fundamental equations are to be treated as approximate. Fortunately, these uncertainties do not change the closeness of the prediction and the observation very much. What we have here is a prediction of the present temperature of the radiation, determined completely by the fundamental constants and the present age of the Universe.

Another interesting aspect of the scenario we are discussing is that the Universe originates and evolves naturally at the critical density. This feature is very attractive since inflation is the only other known mechanism that can endow the Universe with critical density naturally, without drastic fine tuning.

We expect that at the origin of the Universe an energy content of $E_{\rm b}$ will be present in a volume corresponding to the Planck length, if our theory of a quantum birth is at least approximately true. The scale comes into picture through the horizon size $c\tau_{\rm b}$ at Planck time. Note that we do not make any assumption regarding the actual size of the Universe at the origin, since this is not a well-defined concept. In fact, for a flat Universe the total size is infinite, though the observable Universe is finite at any epoch. Then the energy density is

$$\rho_{\rm U} = E_{\rm b}/V \simeq \frac{1}{2}\left(\frac{\hbar c^5}{G}\right)^{1/2} \Bigg/ \frac{4\pi\varepsilon}{3}\left(\frac{\hbar G}{c^3}\right)^{3/2} = \frac{3}{8\pi\varepsilon}\frac{c^7}{\hbar G^2}, \tag{7}$$

where ε is a numerical factor in the volume measure to take care of the possibility that the spatial curvature of the initial Universe could be non-zero. For a flat Universe, $\varepsilon = 1$, and for a $k = +1$ closed Universe, $\varepsilon = 3\pi/2$.

The Friedmann equation is

$$\dot{R}^2 + kc^2 = \frac{8\pi G\rho R^2}{3}, \tag{8}$$

and $\dot{R}/R \simeq c/c\tau_{\rm b} = 1/\tau_{\rm b}$, since the horizon scale at birth is $c\tau_{\rm b}$ and the horizon will expand at the speed of light. Then we have, in terms of the mass density $\rho = \rho_{\rm U}/c^2 = (3/8\pi\varepsilon)(c^5/\hbar G^2)$,

$$\frac{1}{\tau_{\rm b}^2} + \frac{kc^2}{R^2} = \frac{1}{\varepsilon\tau_{\rm b}^2}. \tag{9}$$

Multiplying through $\tau_{\rm b}^2$, we see that the Friedmann equation is satisfied with $k = 0$ and $\varepsilon = 1$. (Other possibilities are ruled out since ε depends on k, and $\varepsilon > 1$ for a closed Universe. The $k = -1$ Universe is also ruled out from similar reasoning.)

This solves the fundamental enigma why the observed Universe is evolving at the critical density. Our finding that the $k = 0$ Universe is natural without fine tuning agrees well with the physical expectation that anything created from nothing should have its total energy (dynamic energy + gravitational potential energy) zero, and hence $k = 0$.

We stress that the initial relativistic condition needs to hold only approximately for these conclusions to follow, and there is no fine tuning.

The density varies as $1/t^2$ during the radiation-dominated and the matter-dominated era and this gives the present density, $\rho_0 = \rho_{\rm U}/(t_0/\tau_{\rm b})^2 = 3c^2/8\pi G t_0^2$.

The numerical value of this expression is close to the observed density of the Universe.

The prediction for the initial Hubble parameter is $H_{\rm b} = \dot{R}/R \simeq 1/\tau_{\rm b}$. Evolved to present time we get a value close to 70 km/s-Mpc, again close to the observed value.

The quantity $2GM_{\rm H}/c^2 r_{\rm H} \simeq 1$ is preserved at its initial value naturally. Density of the Universe varies as $\rho(t) \sim 1/t^2$ at all times and the horizon size increases as $r_{\rm H} \sim t$. The mass contained in horizon size $r_{\rm H}$ is

$$M_{\rm H} \sim \rho(t) r_{\rm H}^3 \sim t. \tag{10}$$

This gives

$$GM_{\mathrm{H}}/c^2 r_{\mathrm{H}} = \text{constant} \sim 1. \tag{11}$$

This has deep implications for Mach's principle. The fundamental requirement for Mach's principle to determine inertia [6], eq. (11), is obtained naturally from the quantum birth scenario [7].

Now we discuss another important feature of our theory. Since the whole of the presently observable Universe is causally linked to the primordial Planck volumes in which the same fundamental constants determined the initial conditions, our theory predicts homogeneity and isotropy of the bare Universe, and all deviations from such a state have to occur through structure formation. The presently observable Universe of size scale 10^{28} cm would have been of size scale 10^{-3} cm at the Planck epoch (due to dominantly $t^{1/2}$ evolution of size scale) [5]. Thus we see that though the horizon size at the Planck time ($c\tau_{\mathrm{b}} \simeq 10^{-33}$ cm) was much smaller than the size of the Universe, homogeneity and isotropy over the whole Universe were assured at early times since the fundamental constants decided the initial conditions everywhere.

Our hypothesis fixes two points in the history of the evolution of the Universe, its birth and the present, and the possible trajectories consistent with various observations can be very limited. The present situation of Big Bang cosmology does not allow such tight constraining since no quantity at the singular origin is known. The two precisely known points in the thermal history we have outlined encompass all the interesting aspects of particle physics during the evolution of the Universe, from string physics to low-energy physics. Any particle physics phenomenon that can change the scale factor or the temperature is then constrained by the two end-points in the thermal history at which the temperatures are known precisely.

An interesting area of speculation that could be constrained by our analysis is that of the variation of the fundamental constants with time. If the values of the fundamental constants were drastically different at the Planck time, then the initial temperature, and therefore the present temperature would be very different from what we estimated. We can state with reasonable certainty that the fundamental constants had their present values during the Planck time to within a factor of 10 or so. The most important consequence of this assertion is that the gravitational constant is most likely not a running coupling constant. The unification idea that the gravitational constant was much stronger at the Planck scale does not seem to be consonant with the quantum birth scenario.

The good matching between the evolved cosmological parameters and the observed ones also suggests that the uncertainty principle, Einstein evolution etc. hold well for times larger than the Planck time. Even if there are corrections to the physical laws of quantum physics and relativity close to the Planck scale, these corrections are small. This is a very significant observation since it is difficult to imagine a way to test whether the physical laws as we know them remain valid at the highest energy scales we think about.

We conclude that there is observational evidence that the initial conditions for the evolution of the Universe were fixed by quantum physics, relativistic gravity, and associated fundamental constants. The observed background radiation and the present density close to the critical density are the relics of the quantum birth of the Universe. The high energy corrections to either the uncertainty relation or to Einstein gravity are likely to be small even at energy scales close to the Planck scale.

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